Online Appendix to "Capital Reallocation"

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1 Data Description: Stylized Facts

This section provides a detailed description for all data used in Section 2, as well as in Figures 1 and 2 and Tables 1 and 2.

Macroeconomic data: Annual GDP data is from the FRED database at the Federal Reserve Bank in St. Louis (`http://research.stlouisfed.org/fred`). We use data from 1963 to 2016. Annual CPI data for all urban consumers are from the Bureau of Labor Statistics (`http://www.bls.gov`).

Assets, property, plant and equipment, capital expenditures, acquisitions and property, plant and equipment sales data: Data on assets, property, plant and equipment, and capital expenditures are generated from Compustat variables AT, PPENT and CAPX, respectively. Acquisitions and sales of property, plant and equipment are reported in Compustat as AQC and SPPE. We use data from 1971-2016. We calculate aggregate acquisitions by summing over firms by year. We exclude firm year observations in which the acquisitions entry contained a combined data code. For the aggregate sales of property, plant and equipment, firm year observations in which the property, plant and equipment entry contained a combined data code were excluded. For the reallocation to asset turnover rate, total assets are summed over firms by year using the same inclusion rule.

Tobin's q data: Following Eisfeldt and Rampini (2006), the data used to calculate a proxy for Tobin's q were collected from Compustat. We define Tobin's q as the market to book ratio. The book value of assets is given by annual data item total assets (AT). The market value of assets was computed as the book value of assets (AT) plus the market value of common stock less the sum of book value of common stock (CEQ) and balance sheet deferred taxes (TXDB). The market value of common stock is calculated as closing price (PRCC_C) multiplied by shares outstanding (CSHO). We use data from 1963-2016. We exclude firm year observations where total assets (AT) were non-positive, or where the book value of common stock (CEQ) or deferred taxes (TXDB) were negative. Missing values for balance sheet deferred taxes were set to zero. For all dispersion calculations, firm year observations where q was negative were excluded. The standard deviation of q is market value weighted. The standard deviations of q's being greater than 0 and less than 5 are standard deviations of q's weighted by market value for all q's in that range.

Total factor productivity data at the three digit NAICS code level: The annual data on industry multifactor productivity change and value of sectoral output are from the Bureau of Labor Statistics (`http://www.bls.gov/`). We use data for 18 durable and non-durable manufacturing industries at the three digit NAICS code level (NAICS 311-316, 321-327, 331-337, 339) from 1988 to 2015. The standard deviation of the productivity growth across sectors is computed by weighting the industries by the value of sectoral output at the end of the year.

Capacity utilization data: The annual industry capacity utilization data are constructed from the monthly data provided by the Federal Reserve Board (`http://www.federalreserve.gov`) by computing the average capacity utilization for the year in each industry. We use data for 18 durable and non-durable manufacturing industries at the three digit NAICS code level (NAICS 311-316, 321-327, 331-337, 339) from 1988 to 2015. The standard deviation of capacity utilization across sectors is computed by weighting the industries by the value of sectoral output at the end of the year.

Job flows data: The annual data on the gross job creation rate and the gross job destruction rate are from Census (`https://www.census.gov/ces/dataproducts/bds/index.html`). We used the data from 1977 through 2015. Excess job reallocation is job reallocation (sum of creation and destruction) minus the absolute value of the net change in employment. See also Davis and Haltiwanger (1992) and Davis et al. (1998).

External financing data: Construction of financial flows follows that in Eisfeldt and Muir (2016). We exclude financial, utility and government related firms. We also exclude firms with missing assets, equity, debt, and those with missing or negative PPE and cash balances. Please refer to the online appendix of Eisfeldt and Muir (2016) for detailed construction of debt and equity financing flows or each firm. We aggregate debt and equity financing over firms in the bottom 90% of assets by year. See the online appendix to **?** for details. See also Covas and Den Haan (2011) for the anomalous effect of large firms on aggregates in their study of equity and debt issuance over the business cycle. We then normalize aggregate financing data with lag aggregate total asset for firms with bottom 90% of assets. Total external financing is sum of debt and equity financing.

VIX: We use VIX as CBOE S&P 500 Volatility Index from 1990-2016. *Uncertianty Shock*: Following Gilchrist et al. (2014), we use data from the Center for Research in Security Prices (CRSP) data base. We utilize daily stock returns data for U.S. nonfinancial corporations with at least 1,250 trading days of data. For each firm-year, we get OLS residuals running a four factor model regression using the Fama and French (1992) 3-factor model with the momentum risk factor proposed by Jegadeesh and Titman (1993) and Carhart (1997). We then calculate volatility for these residuals for each firm-year. Finally we weight market value firm volatility by market value for each year to get the time series of uncertainty measure.

2 Theoretical Appendix

We show detailed solutions for each case for the static model, namely the baseline frictionless case, the model with a collateral constraint, and the model with a capital liquidity cost. The models are nested, however we analyze each case separately for ease of exposition. To solve each model, we first solve the optimizing problem for each firm, then we aggregate all capital reallocation and find the market clearing price for capital. Finally, we get comparative statics from the equations defining the equilibrium market clearing prices and quantities of capital reallocation.

2.1 Analytical Solution: Baseline Model

In the baseline model, in which there are no frictions to trading capital, firm i will choose optimal capital level $\hat{k}_i$ to optimize their output given their initial k_i and a_i as follows:

$$\max_{\hat{k}_i} \ a_i \hat{k}_i^{\theta} - P(\hat{k}_i - k_i). \tag{1}$$

The first order condition is:

$$\hat{k}_i = (\frac{P}{\theta a_i})^{\frac{1}{\theta-1}}. \tag{2}$$

Equation 2 holds for all the firms in the economy. Thus, firms whose initial capital $k_i \geq \left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}}$ will be sellers. Note that we can rewrite the marketing clearing condition equation as:

$$\sum_i \hat{k}_i = \sum_i k_i. \tag{3}$$

We then use this condition to find the marketing clearing capital price. The market clearing price of capital will have to satisfy the following equation[1]:

$$\int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \int_{k_m}^{\infty} (\frac{P}{a\theta})^{\frac{1}{\theta-1}} \frac{c k_m^c}{k^{c+1}} dkda = \int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \int_{k_m}^{\infty} k \frac{f k_m^c}{k^{c+1}} dkda. \tag{4}$$

Simplifying the above equation, we get:

$$\frac{f}{f + \frac{1}{\theta-1}} (\frac{P}{a_{agg}\theta})^{\frac{1}{\theta-1}} = \frac{c k_m}{c-1}. \tag{5}$$

After solving for the market clearing price, we get the quantity of reallocation:

$$R = \int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \int_{\left(\frac{P}{\theta a}\right)^{\frac{1}{\theta-1}}}^{\infty} \left(k - (\frac{P}{\theta a})^{\frac{1}{\theta-1}}\right) \frac{c k_m^c}{k^{c+1}} dkda \tag{6}$$

$$= \frac{1}{c-1} k_m^c (\frac{P}{\theta a_{agg}})^{\frac{1-c}{\theta-1}} \frac{f}{\frac{1-c}{\theta-1} + f} \tag{7}$$

We get comparative statistics for the price of capital and the quantity of reallocation as aggregate productivity changes using Equation (5):

$$\frac{\partial P}{\partial a_{agg}} = \frac{P}{a_{agg}} > 0 \tag{8}$$

$$\frac{\partial R}{\partial a_{agg}} = 0 \tag{9}$$

[1] We require $f + \frac{1}{\theta-1} > 1$ to ensure that the left hand side of Equation 4 is finite.

We can get comparative statistics for the price of capital and the quantity of reallocation with respect to changes in productivity dispersion as:

$$\frac{\partial P}{\partial f} = -\frac{P}{f\left(f + \frac{1}{\theta-1}\right)} < 0 \tag{10}$$

$$\frac{\partial R}{\partial f} = -\frac{1}{(1-\theta)^2} k_m^c \left(\frac{P}{\theta a_{agg}}\right)^{\frac{1-c}{\theta-1}} \frac{c}{\left(f + \frac{1-c}{\theta-1}\right)^2 \left(f + \frac{1}{\theta-1}\right)} < 0 \tag{11}$$

Finally, using these results, we can get the cross comparative statics:

$$\frac{\partial\left(\frac{\partial P}{\partial f}\right)}{\partial a_{agg}} = -\frac{1}{f\left(f + \frac{1}{\theta-1}\right)} \frac{\partial P}{\partial a_{agg}} < 0 \tag{12}$$

$$\frac{\partial\left(\frac{\partial R}{\partial f}\right)}{\partial a_{agg}} = 0 \tag{13}$$

2.2 Analytical Solution: Collateral Constraint Model

In the model with a collateral constraint, firm i solves the following problem

$$\max_{\hat{k}_i} \; a_i \hat{k}_i^\theta - P(\hat{k}_i - k) \tag{14}$$

$$\text{s.t. } \hat{k}_i - k \leq \xi k_i. \tag{15}$$

We make an additional assumption without loss of generality that even for the lowest productivity a_{agg}, there are some firms that are constrained by the collateral constraint. We will solve the model under the assumption and later validate that the statement is true in equilibrium when $c \to 1$, which is consistent with the empirical distribution of firm sizes.[2]

Solving the optimization problem, we know firms choose their optimal capital level according to

$$\hat{k}_i = \begin{cases} \left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}}, & \text{if } \frac{1}{1+\xi}\left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}} \leq k_i \\ (1+\xi)k_i, & \text{if } k_m \leq k_i < \frac{1}{1+\xi}\left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}}. \end{cases} \tag{16}$$

Thus the market clearing condition will be

$$\int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \left(\int_{\frac{1}{1+\xi}\left(\frac{P}{\theta a}\right)^{\frac{1}{\theta-1}}}^{\infty} \left(\frac{P}{a\theta}\right)^{\frac{1}{\theta-1}} \frac{c k_m^c}{k^{c+1}} dk + \int_{k_m}^{\frac{1}{1+\xi}\left(\frac{P}{\theta a}\right)^{\frac{1}{\theta-1}}} (1+\xi) k \frac{c k_m^c}{k^{c+1}} dk \right) da \tag{17}$$

$$= \int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \int_{k_m}^{\infty} k \frac{c k_m^c}{k^{c+1}} dk da. \tag{18}$$

[2]See Axtell (2001).

After simplifying, we get:

$$\frac{k_m}{\frac{1}{1+\xi}\big(\frac{P}{\theta a_{agg}}\big)^{\frac{1}{\theta-1}}} = \Big(\frac{\xi c\big(\frac{1-c}{\theta-1}+f\big)}{f(1+\xi)}\Big)^{\frac{1}{c-1}}. \tag{19}$$

We now prove that there exist $c^* > 1$ such that when $c \in (1, c^*]$, even for the lowest productivity level, some small firms are constrained. This is equivalent to finding an $c^* > 1$ such that

$$\frac{\xi c^*\big(\frac{1-c^*}{\theta-1}+f\big)}{f(1+\xi)} = 1. \tag{20}$$

Solving this quadratic equation we get that

$$\begin{aligned} c^* &= \frac{-\xi(f+\frac{1}{\theta-1}) + \Big(\xi^2\big(f+\frac{1}{1-\theta}\big)^2 + \frac{4f\xi}{1-\theta}\Big)^{\frac{1}{2}}}{\frac{2\xi}{1-\theta}} && (21)\\ &\geq \frac{-\xi(f+\frac{1}{\theta-1}) + \xi\big(f+\frac{1}{1-\theta}\big)}{\frac{2\xi}{1-\theta}} = 1 \qquad \square && (22)\end{aligned}$$

After validating the market clearing condition, we know that the quantity of reallocation is:

$$\begin{aligned} R &= \int_{a_{agg}}^{\infty} \frac{f a_{agg}^f}{a^{f+1}} \int_{\big(\frac{P}{\theta c}\big)^{\frac{1}{\theta-1}}}^{\infty} \Big(k - \big(\frac{P}{\theta c}\big)^{\frac{1}{\theta-1}}\Big)\frac{c k_m^c}{k^{c+1}} dk da && (23)\\ &= \frac{1}{c-1} k_m^c \big(\frac{P}{\theta a_{agg}}\big)^{\frac{1-c}{\theta-1}} \frac{f}{\frac{1-c}{\theta-1}+f}. && (24)\end{aligned}$$

We can compute the relevant comparative statistics as:

$$\begin{aligned} &\frac{\partial P}{\partial a_{agg}} = \frac{P}{a_{agg}} > 0 && (25)\\ &\frac{\partial P}{\partial \xi} = \frac{P\Big(\frac{1}{\xi} - \frac{c}{1+\xi}\Big)}{\frac{1-c}{\theta-1}} > 0 && (26)\\ &\frac{\partial R}{\partial a_{agg}} = 0 && (27)\\ &\frac{\partial R}{\partial \xi} = \frac{k_m^c}{c-1}\frac{1-c}{\theta-1}\frac{1}{f+\frac{1-f}{\theta-1}}\big(\frac{P}{a_{agg}}\big)^{\frac{1-c}{\theta-1}}\frac{1}{P}\frac{\partial P}{\partial \xi} > 0 && (28)\\ &\frac{\partial P}{\partial f} = -\frac{P}{f(f+\frac{c-1}{1-\theta})} < 0 && (29)\\ &\frac{\partial R}{\partial f} \propto -\frac{1}{\big(f+\frac{1-c}{\theta-1}\big)^2} P^{\frac{1-c}{\theta-1}}(1+\frac{1-c}{(\theta-1)f}) < 0 && (30)\end{aligned}$$

Equation (26) and (28) hold when $c < \frac{1+\xi}{\xi}$. This is satisfied by our choice of c, as it is easy to show that $c^* < \frac{1+\xi}{\xi}$. Further we have the following cross comparative statics:

$$\frac{\partial\left(\frac{\partial P}{\partial f}\right)}{\partial a_{agg}} = -\frac{1}{f\left(f+\frac{c-1}{1-\theta}\right)}\frac{\partial P}{\partial a_{agg}} < 0 \tag{31}$$

$$\frac{\partial\left(\frac{\partial P}{\partial \xi}\right)}{\partial f} = \frac{\left(\frac{1}{\xi}-\frac{c}{1+\xi}\right)}{\frac{1-c}{\theta-1}}\frac{\partial P}{\partial f} < 0 \tag{32}$$

$$\frac{\partial\left(\frac{\partial P}{\partial \xi}\right)}{\partial a_{agg}} = \frac{\left(\frac{1}{\xi}-\frac{c}{1+\xi}\right)}{\frac{1-c}{\theta-1}}\frac{\partial P}{\partial a_{agg}} > 0 \tag{33}$$

$$\frac{\partial\left(\frac{\partial R}{\partial f}\right)}{\partial a_{agg}} = 0 \tag{34}$$

$$\frac{\partial\left(\frac{\partial R}{\partial \xi}\right)}{\partial f} \propto \frac{1}{f+\frac{1-c}{\theta-1}} + \frac{1-c}{\theta-1}\frac{1}{P}\frac{\partial P}{\partial f} < 0 \tag{35}$$

$$\frac{\partial\left(\frac{\partial R}{\partial \xi}\right)}{\partial a_{agg}} = 0 \tag{36}$$

2.3 Analytical Solution: Liquidity Cost Model

In the liquidity cost model, firm i solves the following problem:

$$\max_{\hat{k}_i}\ a_i\hat{k}_i^{\theta} - P(\hat{k}_i - k_i)\mathbb{1}_{k_i<\hat{k}_i} + P\frac{k_i-\hat{k}_i}{1+\gamma}\mathbb{1}_{k_i>\hat{k}_i} \tag{37}$$

Firms choose:

$$\hat{k}_i = \begin{cases} \left(\frac{P}{\theta k_i}\right)^{\frac{1}{\theta-1}}, & \text{if } k_m \le k_i < \left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}} \\ k_i, & \text{if } \left(\frac{P}{\theta a_i}\right)^{\frac{1}{\theta-1}} \le k_i < \left(\frac{P}{\theta a_i(1+\gamma)}\right)^{\frac{1}{\theta-1}} \\ \left(\frac{P}{\theta a_i(1+\gamma)}\right)^{\frac{1}{\theta-1}}, & \text{if } \left(\frac{P}{\theta a_i(1+\gamma)}\right)^{\frac{1}{\theta-1}} \le k_i \end{cases} \tag{38}$$

We assume that even for the lowest productivity firms, the smallest firms will be net buyers, and verify this below. Given this assumption, we have the market clearing condition:

$$\int_{a_{agg}}^{\infty}\frac{fa_{agg}^f}{a^{f+1}}\int_{\left(\frac{P}{\theta a(1+\gamma)}\right)^{\frac{1}{\theta-1}}}^{\infty}\frac{1}{1+\gamma}\left(k-\left(\frac{P}{\theta k(1+\gamma)}\right)^{\frac{1}{\theta-1}}\right)\frac{ck_m^c}{k^{c+1}}dkda \tag{39}$$

$$=\int_{a_{agg}}^{\infty}\frac{fa_{agg}^f}{a^{f+1}}\int_{k_m}^{\left(\frac{P}{\theta a}\right)^{\frac{1}{\theta-1}}}\left(\left(\frac{P}{\theta a}\right)^{\frac{1}{\theta-1}}-k\right)\frac{ck_m^c}{k^{c+1}}dkda \tag{40}$$

Simplifying:

$$\frac{f}{f+\frac{1}{\theta-1}}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{1}{\theta-1}}+\frac{f}{f+\frac{1-c}{\theta-1}}\frac{1}{c-1}k_m^a\left(\frac{P}{\theta a_{agg}}\right)^{\frac{1-c}{\theta-1}}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)=\frac{c}{c-1}k_m \tag{41}$$

To ensure the market clearing condition has a solution, we choose parameter values such that:

$$\frac{f}{f+\frac{1}{\theta-1}}+\frac{f}{f+\frac{1-c}{\theta-1}}\frac{1}{c-1}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)<\frac{c}{c-1} \tag{42}$$

Given such parameter values, we have two positive solutions to Equation (41) for $k_m/\left(\frac{P}{\theta a_{agg}}\right)^{\frac{1}{\theta-1}}$. One is greater than 1, the other is less than 1. The one that is less than one validates our assumption that even for the lowest productivity level, some firms are net capital purchasers. Next, we obtain the quantity of reallocation:

$$R=\int_{a_{agg}}^{\infty}\frac{fa_{agg}^f}{a^{f+1}}\int_{\left(\frac{P}{\theta a(1+\gamma)}\right)^{\frac{1}{\theta-1}}}^{\infty}\frac{1}{1+\gamma}\left(k-\left(\frac{P}{\theta k(1+\gamma)}\right)^{\frac{1}{\theta-1}}\right)\frac{ck_m^c}{k^{a+1}}dkda \tag{43}$$

$$=\frac{f}{f+\frac{1-c}{\theta-1}}\frac{1}{c-1}k_m^c\left(\frac{P}{\theta a_{agg}}\right)^{\frac{1-c}{\theta-1}}\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}} \tag{44}$$

The comparative statistics can then be computed directly as follows:

$$\frac{\partial P}{\partial a_{agg}}=\frac{P}{a_{agg}}>0 \tag{45}$$

$$\frac{\partial P}{\partial \gamma}=\frac{\frac{f}{f+\frac{1-c}{\theta-1}}\frac{1}{c-1}k_m^c\left(\frac{P}{\theta a_{agg}}\right)^{\frac{-c}{\theta-1}}\frac{\theta-c}{\theta-1}(1+\gamma)^{\frac{2\theta-c-1}{1-\theta}}P}{\frac{1}{1-\theta}\left(\frac{f}{f+\frac{1}{\theta-1}}-\frac{f}{f+\frac{1-c}{\theta-1}}k_m^c\left(\frac{P}{\theta a_{agg}}\right)^{\frac{-c}{\theta-1}}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)\right)}>0 \tag{46}$$

$$\frac{\partial R}{\partial a_{agg}}=0 \tag{47}$$

$$\frac{\partial R}{\partial \gamma}\propto\frac{c-1}{c-\theta}(1+\gamma)\frac{1}{P}\frac{\partial P}{\partial \gamma}-1 \tag{48}$$

$$\propto-\frac{f}{f+\frac{1}{\theta-1}}+\frac{f}{f+\frac{1-c}{\theta-1}}k_m^c\left(\frac{P}{\theta a_{agg}}\right)^{\frac{-c}{\theta-1}}<0 \tag{49}$$

$$\frac{\partial P}{\partial f}=\frac{P}{f}\frac{\frac{1}{(f+\frac{1}{\theta-1})^2}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}-\frac{k_m^c}{(f+\frac{1-c}{\theta-1})^2}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)}{\frac{k_m^c}{f+\frac{1-c}{\theta-1}}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)-\frac{1}{f+\frac{1}{\theta-1}}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}}<0 \tag{50}$$

$$\frac{\partial R}{\partial f}\propto\frac{1}{f+\frac{1-c}{\theta-1}}+\frac{f}{P}\frac{\partial P}{\partial f} \tag{51}$$

$$=\frac{\frac{\frac{c}{1-\theta}}{(f+\frac{1}{\theta-1})^2}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}}{k_m^c\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right)-\frac{f+\frac{1-c}{\theta-1}}{f+\frac{1}{\theta-1}}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}}<0 \tag{52}$$

Further we have the cross comparative statics:

$$\frac{\partial\left(\frac{\partial P}{\partial f}\right)}{\partial a_{agg}} = \frac{1}{a_{agg}}\frac{\partial P}{\partial f} < 0 \tag{53}$$

$$\frac{\partial\left(\frac{\partial P}{\partial \gamma}\right)}{\partial a_{agg}} = \frac{1}{a_{agg}}\frac{\partial P}{\partial \gamma} > 0 \tag{54}$$

$$\frac{\partial\left(\frac{\partial R}{\partial f}\right)}{\partial a_{agg}} = 0 \tag{55}$$

$$\frac{\partial\left(\frac{\partial R}{\partial \gamma}\right)}{\partial f} \propto \partial \frac{P^{\frac{1}{\theta-1}}\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}}{k_m^c\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right) - \frac{f+\frac{1-c}{\theta-1}}{f+\frac{1}{\theta-1}}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}}/\partial\gamma < 0 \tag{56}$$

$$\frac{\partial\left(\frac{\partial R}{\partial \xi}\right)}{\partial a_{agg}} = 0 \tag{57}$$

Note that:

$$\frac{\partial\left(\frac{\partial P}{\partial \gamma}\right)}{\partial f} = \frac{\partial P}{\partial f \partial \gamma} \tag{58}$$

$$= \frac{\partial P}{\partial \gamma}\frac{\partial P}{\partial f}\frac{1}{P} + \frac{P}{f}\frac{\partial\left(\frac{\partial P}{\partial f}\frac{f}{P}\right)}{\partial \gamma} \tag{59}$$

The first term is negative. The second term:

$$\frac{\partial\left(\frac{\partial P}{\partial f}\frac{f}{P}\right)}{\partial \gamma} \propto \partial \frac{1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}}{\frac{k_m^c}{b+\frac{1-c}{\theta-1}}\left(1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}\right) - \frac{1}{f+\frac{1}{\theta-1}}\left(\frac{P}{\theta a_{agg}}\right)^{\frac{c}{\theta-1}}}/\partial\gamma \tag{60}$$

$$\propto \partial \frac{P^{\frac{c}{\theta-1}}}{1-\left(\frac{1}{1+\gamma}\right)^{\frac{\theta-c}{\theta-1}}}/\partial\gamma < 0 \tag{61}$$

Thus

$$\frac{\partial\left(\frac{\partial P}{\partial \gamma}\right)}{\partial f} < 0. \tag{62}$$

2.4 Analytical Solution: Output Based Collateral Constraint

Li (2015) argues that leverage constraints that allow for borrowing against output lead to less misallocaiton than constraints based on size alone. This is an important point; the precise form of the constraint matters for the degree of misallocation. We illustrate this point in the context of our simple static model. To do so, we replace the size-based collateral constraint, with a constraint that allows for borrowing against output:

$$\xi a_i \hat{k}_i^\theta \geq P(\hat{k}_i - k_i) \tag{63}$$

We compare then compare the properties for capital reallocation in two economies. The first has a capital based borrowing constraint as in the main text, based on initial capital , the other has a production based borrowing constraint. We verify that constraints based on size alone allow for less productive reallocation than constraints that allow for borrowing against output.

Reallocation Amount We show that allowing for borrowing against production results in more capital reallocation in equilibrium. To prove this, we first show that when the same ξ applies to both the size-based and output-based constraints, the production-based constraint generates a higher capital price in equilibrium. As there is no friction on sellers, regardless of the constraint type, equation (24) holds under both constraints. Thus, a higher price will result in a higher quantity of capital reallocation.

We use proof by contradiction to show that the equilibrium price will be higher when firms are faced with a borrowing constraint based on production. Assume under both types of constraint, assume that the market clears at the same price P. We first show that there will be fewer constrained firms with a production based constraint. For the economy with a production based borrowing constraint (we denote variables in this case with subscript p), firms will be constrained if

$$k_i \leq (\frac{P}{\theta a_i})^{\frac{1}{\theta-1}}(1-\frac{\xi}{\theta}) \equiv \bar{k}_{i,p}. \tag{64}$$

For the economy with a capital based borrowing constraint (we denote variables in this case with subscript c), firms will be constrained if

$$k_i \leq (\frac{P}{\theta a_i})^{\frac{1}{\theta-1}}(1-\frac{\xi}{1+\xi}) \equiv \bar{k}_{i,c}. \tag{65}$$

It is easy to see that for any a_i, $\bar{k}_{i,c} > \bar{k}_{i,p}$, which is equivalent to stating that more firms will be constrained in a model with a capital-based constraint. We next show that for firms that are constrained in both economies, a production-based constraint enables firms to acquire more capital. For any initial level of capital k_i, if the firm is constrained, it will choose $\hat{k}$ in each of the two economies as follows:

$$\hat{k}_{i,c} = (1+\xi)k_i \tag{66}$$

$$\hat{k}_{i,p} = \frac{\xi a_i}{P}\hat{k}_{i,p}^{\theta} + k_i \tag{67}$$

As the firm always has the (sub-optimal) choice to sell all their initial capital, we know that

$$a_i\hat{k}_i^{\theta} > a_i\hat{k}_i^{\theta} - P(\hat{k}_i - k_i) > Pk_i. \tag{68}$$

Thus $\hat{k}_p > \hat{k}_c$ for all firms that are constrained in both economies. This is a contradiction. The supply of capital is the same in both economy while in the production-based constraint economy there is greater capital demand. Thus, the price of capital must be higher in the economy with a production based constraint to be able to clear the market. The higher price has two effects which ensure that markets to clear: First the higher price leads to more sales, second, the production based constraint gets tighter. Figure 1 illustrates this. The blue line represents the production based constraint. The demand for capital is weakly greater than it is with a capital based constraint, holding the price of capital fixed. It is easy to prove a contradiction using a similar argument if we assume that the market clearing price is lower in the economy with a production based constraint. Note that a constraint on production is much looser than one based on size, since, to ensure that some firms are constrained requires that $\xi < \theta$.

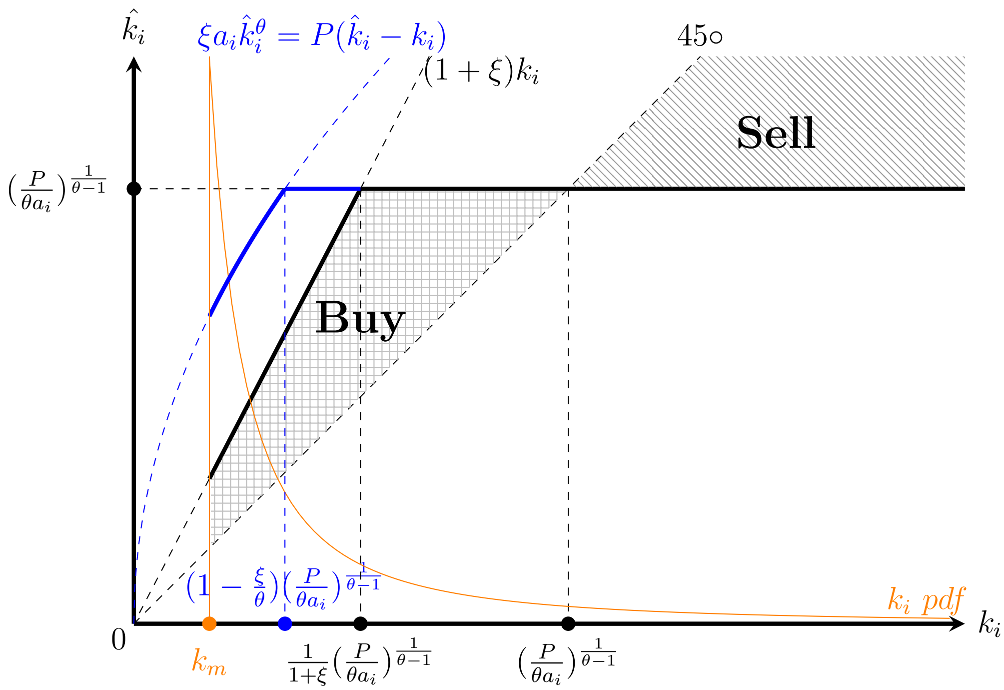

Figure 1: Same Financial Paramter for Two Types of Constraint

Reallocation Sensitivity We can also show that reallocation is more sensitive to the financial constraint parameter in a model with a production-based financial constraint. In other words, we claim that:

$$\frac{\partial R_p}{\partial \xi} > \frac{\partial R_c}{\partial \xi} \tag{69}$$

To show this, we first find the ξ_p and ξ_c such that under both constraints, reallocation and the price of capital are the same.[3]. We know that $\xi_p << \xi_c$. Then, we relax both constraints by δ. We show that if the price of capital is unchanged, a production based constraint will result in more incremental purchases of capital, and thus more incremental reallocation after market clearing. First, we show that for any productivity level a_i, for each

[3]The capital that buyers demand is a decreasing function of ξ in both cases. Thus there will always exist such pair of ξ_p and ξ_c.

$\hat{k}_i \in (0, (\frac{P}{\theta a_i})^{\frac{1}{\theta-1}}]$, there will be more incremental buyers participating in capital reallocation given a production-based constraint when we relax ξ. Then we show that for $k_i \in (0, k_m)$, which is outside of our capital support, there will be less incremental buying. Thus, we know that firms within the set of producers will acquire more capital under a production based constraint. We illustrate the proof in figure 2. The blue lines represent production based constraints while the black lines denote the capital based constraints. What we want to show is that when ξ changes by the same amount in each case, the blue dotted area is larger than the black dotted area, meaning that the incremental demand for capital is higher in an economy with a production based constraint. We show that first, the solid plus the dotted area for the production-based constraint (blue) is larger than the same areas for capital based constraint (black). We then show that the solid blue area is smaller than the solid black area. For the first statement, we want to show that:

$$(\frac{1}{1+\xi_c} - \frac{1}{1+\xi_c+\delta})\hat{k}_i < \frac{\delta a_i \hat{k}_i^{\theta}}{P} \tag{70}$$

$$\frac{1}{(1+\xi_c)(1+\xi_c+\delta)} < \frac{a_i \hat{k}_i^{\theta-1}}{P} \tag{71}$$

Because

$$\hat{k}_i \leq (\frac{P}{\theta a_i})^{\frac{1}{\theta-1}} \tag{72}$$

we have that:

$$\frac{1}{(1+\xi_c)(1+\xi_c+\delta)} < \frac{1}{\theta} \leq \frac{a_i \hat{k}_i^{\theta-1}}{P} \qquad \square \tag{73}$$

We show the second statement by contradiction. Assume that there are still some firms that are constrained after relaxing the constraint. Then:

$$\xi_p a_i \hat{k}_{i,p}^{\theta} = P(\hat{k}_{i,p} - k_i) \tag{74}$$

for $k_i \in (0, k_m)$. For the capital based constraint, the addtional demand for capital for firm i will be δk_i. Assume firms can get greater than δk_i capital under the production-based borrowing constraint. We then have that

$$(\xi_p + \delta) a_i (\hat{k}_{i,p} + \delta k_i)^{\theta} \geq P(\hat{k}_{i,p} + \delta k_i - k_i) \tag{75}$$

We fix the price at the initial level

$$P = \frac{\xi_p a_i \hat{k}_{i,p}^{\theta}}{\hat{k}_{i,p} - k_i}. \tag{76}$$

Then we need to show that

$$(\xi_p + \delta)(\hat{k}_{i,p} + \delta k_i)^\theta \geq \frac{\xi_p \hat{k}_{i,p}^\theta}{\hat{k}_{i,p} - k_i}(\hat{k}_{i,p} + \delta k_i - k_i). \tag{77}$$

However,

$$LHS < \xi_p \hat{k}_{i,p}^\theta \Big(1 + \frac{\delta k_i}{\hat{k}_{i,p}}\Big)^\theta \tag{78}$$

$$< \xi_p \hat{k}_{i,p}^\theta \Big(1 + \frac{\delta k_i}{\hat{k}_{i,p}}\Big) \tag{79}$$

$$< \xi_p \hat{k}_{i,p}^\theta \Big(1 + \frac{\delta k_i}{\hat{k}_{i,p} - k_i}\Big) = RHS \qquad \square \tag{80}$$

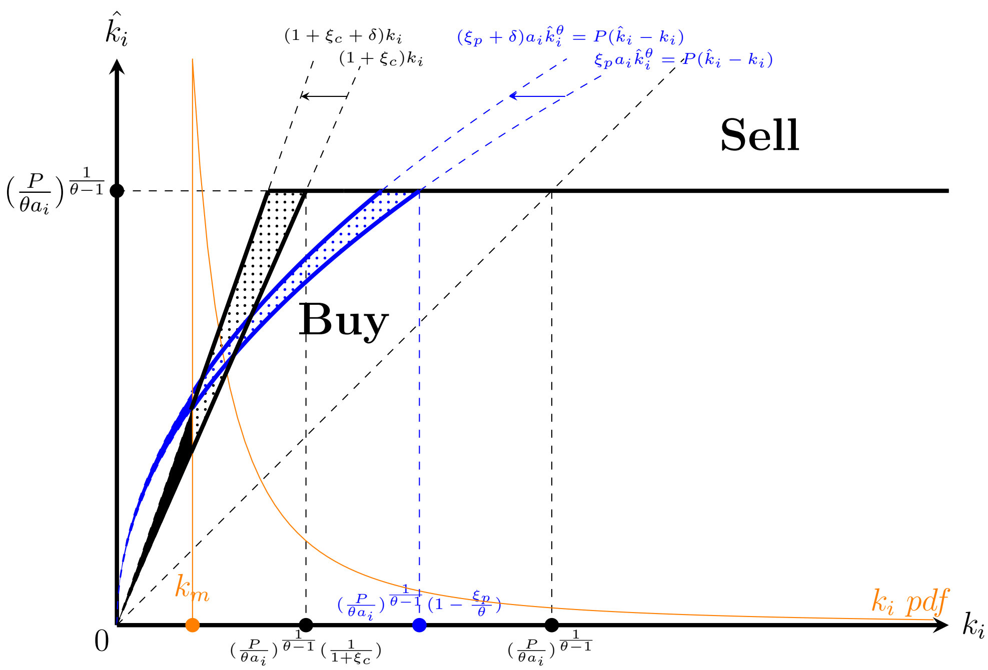

Figure 2: Same change for Two Types of Constraint

3 Data and Methodology for Misallocation Effects

See the text for details, as well as the code posted online.

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