Online Appendix

Marco Del Negro, Domenico Giannone, Marc P. Giannoni, Andrea
Tambalotti

A   Gibbs Sampler for VARs with Common Trends

Let us use the notation $x_{i:j}$ to denote the sequence $\{x_i, .., x_j\}$ for a generic variable x_t. The
Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

    (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

    (b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

    (a) $\Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

    (b) $\varphi | \Sigma_\varepsilon, \Sigma_e, \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

  Details of each step follow:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
   This is given by the product of the marginal posterior distribution of λ (conditional
   on the other parameters) times the distribution of $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}$ conditional on λ (and
   the other parameters).

   (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
       The marginal posterior distribution of λ (conditional on the other parameters) is
       given by

       $$p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}) \propto L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e) p(\lambda),$$

       where $L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)$ is the likelihood obtained from the Kalman filter applied
       to the state space system (1) through (5). $p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T})$ does not have a
       known form so we will use a Metropolis-Hastings step.

(b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

Given λ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$. Note that in addition to $\bar{y}_{1:T}$ and $\tilde{y}_{1:T}$ we also need to draw the initial conditions $\bar{y}_0$ and $\tilde{y}_{-p+1:0}$ in order to estimate the parameters of (3) and (2) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector y_{t_0} has some missing elements, the corresponding rows of the observation equation (1) are simply deleted for $t = t_0$.

2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

This step is straightforward because for given $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$ equations (2) and (3) are standard VARs where in case of (2) we actually know the autoregressive matrices. The posterior distribution of Σ_e is given by

$$p(\Sigma_e | \bar{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_e + \hat{S}_e, \kappa_e + T)$$

where $\hat{S}_e = \sum_{t=1}^{T} (\bar{y}_t - \bar{y}_{t-1})(\bar{y}_t - \bar{y}_{t-1})'$. The posterior distribution of φ and Σ_ε is given by

$$p(\Sigma_\varepsilon | \tilde{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_\varepsilon + \hat{S}_\varepsilon, \kappa_\varepsilon + T),$$
$$p(\varphi | \Sigma_\varepsilon, \tilde{y}_{0:T}) = \mathcal{N}\left(vec(\hat{\Phi}), \Sigma_\varepsilon \otimes \left(\sum_{t=1}^{T} \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \right),$$

where $\tilde{x}_t = (\tilde{y}_{t-1}', .., \tilde{y}_{t-p}')'$ collects the VAR regressors,

$$\hat{\Phi} = \left(\sum_{t=1}^{T} \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \left(\sum_{t=1}^{T} \tilde{x}_t \tilde{y}_t' + \underline{\Omega}^{-1} \underline{\Phi} \right), \ \hat{S}_\varepsilon = \sum_{t=1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t' + (\hat{\Phi} - \underline{\Phi})' \underline{\Omega}^{-1} (\hat{\Phi} - \underline{\Phi}),$$

and $\hat{\varepsilon}_t = \tilde{y}_t - \hat{\Phi}' \tilde{x}_t$ are the VAR residuals.

B Additional Tables and Figures

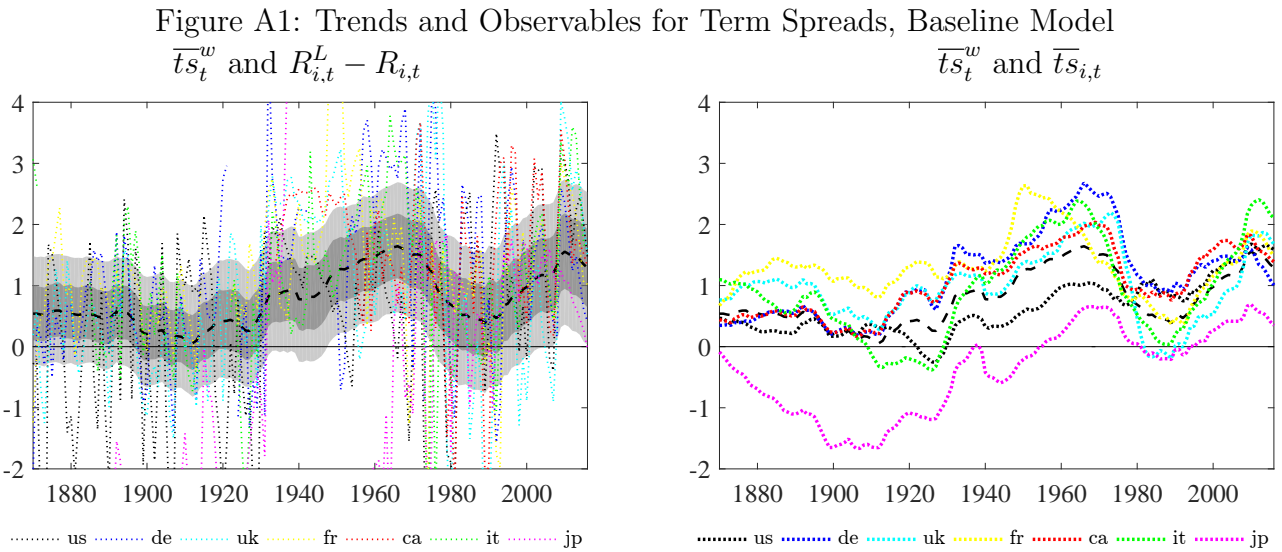Table A1: Change in $\bar{r}_t^w$ and Its Components – Additional Details

	1980-2016	1990-2016	1997-2016
	Baseline Model		
	-1.88	-1.64	-1.03
	$[-2.68, -1.11]$	$[-2.38, -0.93]$	$[-1.59, -0.45]$
$\bar{r}_t^w$	$(-3.24, -0.61)$	$(-2.84, -0.47)$	$(-1.96, -0.09)$
	$\langle -3.47, -0.38 \rangle$	$\langle -3.03, -0.25 \rangle$	$\langle -2.18, 0.07 \rangle$
	$\{0.99\}$	$\{0.99\}$	$\{0.96\}$
	Convenience Yield Model		
	-1.71	-1.73	-1.13
	$[-2.45, -1.00]$	$[-2.39, -1.10]$	$[-1.67, -0.61]$
$\bar{r}_t^w$	$(-2.94, -0.55)$	$(-2.84, -0.67)$	$(-2.01, -0.26)$
	$\langle -3.16, -0.33 \rangle$	$\langle -3.08, -0.48 \rangle$	$\langle -2.19, -0.11 \rangle$
	$\{0.99\}$	$\{1.00\}$	$\{0.99\}$
	-0.82	-0.97	-0.66
	$[-1.29, -0.35]$	$[-1.38, -0.54]$	$[-1.02, -0.30]$
$-\overline{cy}_t^w$	$(-1.60, -0.05)$	$(-1.65, -0.24)$	$(-1.25, -0.06)$
	$\langle -1.74, 0.12 \rangle$	$\langle -1.79, -0.08 \rangle$	$\langle -1.36, 0.06 \rangle$
	$\{0.96\}$	$\{0.98\}$	$\{0.96\}$
	-0.90	-0.78	-0.48
	$[-1.48, -0.31]$	$[-1.32, -0.25]$	$[-0.91, -0.06]$
$\overline{m}_t^w$	$(-1.89, 0.08)$	$(-1.66, 0.09)$	$(-1.17, 0.22)$
	$\langle -2.07, 0.28 \rangle$	$\langle -1.83, 0.26 \rangle$	$\langle -1.34, 0.35 \rangle$
	$\{0.94\}$	$\{0.93\}$	$\{0.87\}$

Consumption Model

	-1.93	-1.94	-1.22
	$[-2.66, -1.16]$	$[-2.63, -1.25]$	$[-1.78, -0.65]$
$\overline{r}_t^w$	$(-3.18, -0.69)$	$(-3.10, -0.82)$	$(-2.18, -0.29)$
	$\langle -3.44, -0.39 \rangle$	$\langle -3.35, -0.60 \rangle$	$\langle -2.38, -0.11 \rangle$
	$\{0.99\}$	$\{1.00\}$	$\{0.98\}$
	-0.71	-0.92	-0.65
	$[-1.20, -0.22]$	$[-1.37, -0.47]$	$[-1.01, -0.26]$
$-\overline{cy}_t^w$	$(-1.51, 0.11)$	$(-1.67, -0.19)$	$(-1.25, -0.02)$
	$\langle -1.68, 0.27 \rangle$	$\langle -1.80, -0.03 \rangle$	$\langle -1.37, 0.10 \rangle$
	$\{0.92\}$	$\{0.98\}$	$\{0.96\}$
	-1.22	-1.02	-0.58
	$[-1.83, -0.60]$	$[-1.57, -0.48]$	$[-1.05, -0.15]$
$\overline{m}_t^w$	$(-2.23, -0.19)$	$(-1.95, -0.12)$	$(-1.33, 0.16)$
	$\langle -2.45, 0.03 \rangle$	$\langle -2.16, 0.10 \rangle$	$\langle -1.48, 0.31 \rangle$
	$\{0.97\}$	$\{0.97\}$	$\{0.90\}$
	-0.74	-0.61	-0.35
	$[-1.20, -0.31]$	$[-1.00, -0.21]$	$[-0.66, -0.03]$
$\overline{g}_t^w$	$(-1.50, -0.03)$	$(-1.28, 0.06)$	$(-0.88, 0.19)$
	$\langle -1.66, 0.10 \rangle$	$\langle -1.42, 0.17 \rangle$	$\langle -0.99, 0.29 \rangle$
	$\{0.96\}$	$\{0.93\}$	$\{0.86\}$
	-0.47	-0.42	-0.24
	$[-0.91, -0.01]$	$[-0.82, -0.02]$	$[-0.56, 0.08]$
$\overline{\beta}_t^w$	$(-1.21, 0.31)$	$(-1.07, 0.25)$	$(-0.78, 0.30)$
	$\langle -1.35, 0.46 \rangle$	$\langle -1.22, 0.38 \rangle$	$\langle -0.89, 0.44 \rangle$
	$\{0.84\}$	$\{0.85\}$	$\{0.77\}$
	-1.84	-1.31	-0.72
	$[-2.53, -1.14]$	$[-2.00, -0.66]$	$[-1.31, -0.14]$
$\overline{\Delta c}_t^w$	$(-2.99, -0.67)$	$(-2.41, -0.21)$	$(-1.71, 0.25)$
	$\langle -3.20, -0.43 \rangle$	$\langle -2.63, 0.03 \rangle$	$\langle -1.91, 0.46 \rangle$
	$\{0.99\}$	$\{0.97\}$	$\{0.89\}$

Note: For a variable x, where $x = \{\overline{r}_t^w, \overline{m}_t^w, -\overline{cy}_t^w, \overline{g}_t^w, \overline{\beta}_t^w, \overline{\Delta c}_t^w\}$ depending on the model, the table shows the posterior median of $\Delta x = x_{2016} - x_{t_0}$ for t_0 being equal to 1980 (left column), 1990 (middle column), and 1997 (middle column), and the 68 ($[.,.]$), 90 ($(.,.)$), and 95 68 ($\langle .,. \rangle$) percent posterior coverage interval for Δx (in parenthesis). The figures in curly brackets ($\{.\}$) show $Pr\{x \leq 0\}$.

## B.1   Baseline Model (Section 4.1) – Additional Results

Figure A1: Trends and Observables for Term Spreads, Baseline Model

$\overline{ts}_t^w$ and $R_{i,t}^L - R_{i,t}$ $\overline{ts}_t^w$ and $\overline{ts}_{i,t}$

Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{ts}_t^w$ (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\overline{ts}_t^w$ (dashed-and-dotted black line).

Figure A2: Country-Specfic Trends r_t^i and Observables, Baseline Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n}\sum_{i=1}^{n}(R_{i,t} - \pi_{i,t})$$
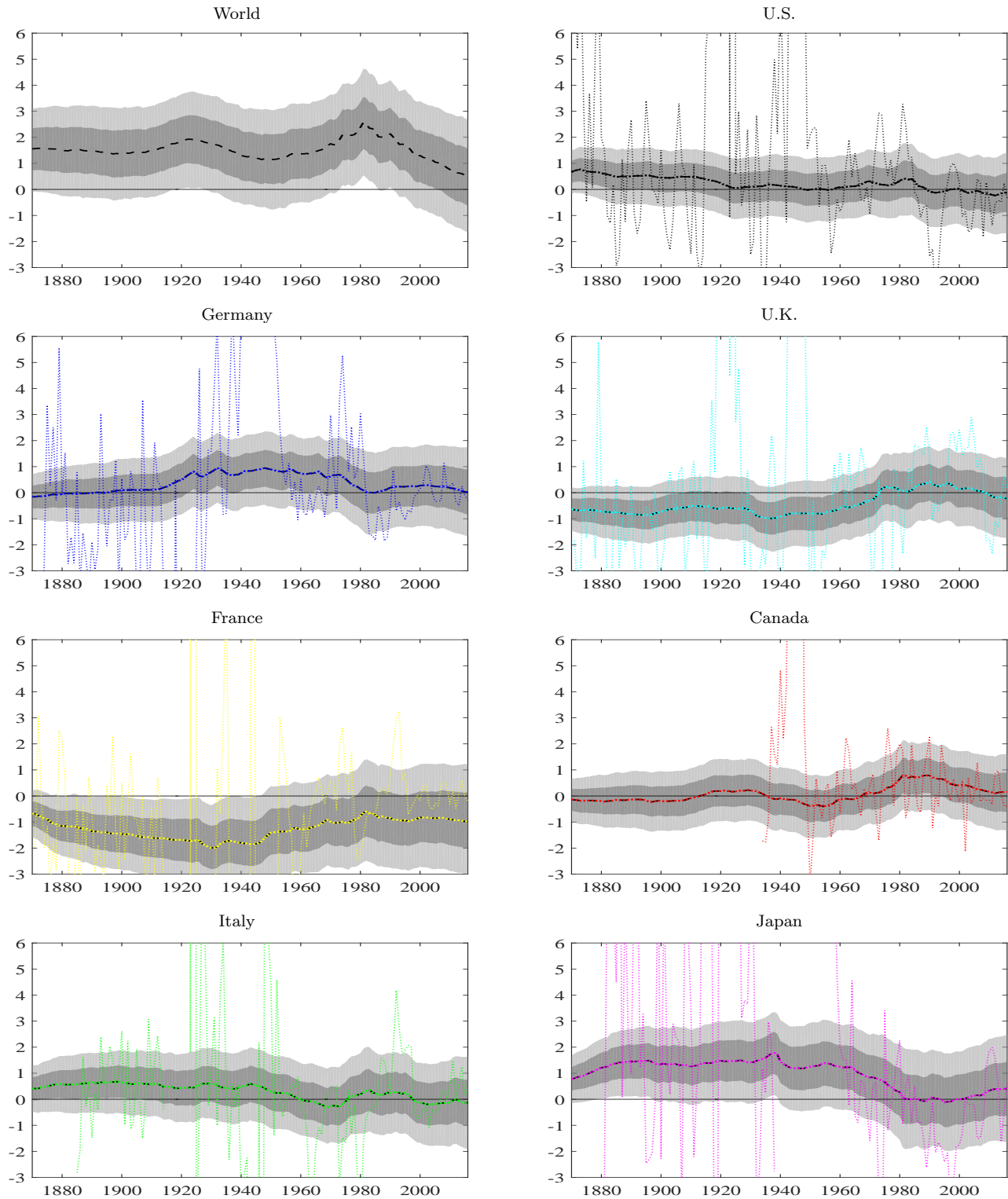
Note: The upper-left panel reproduces the estimates of $\bar{r}_t^w$, so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect $\bar{r}_t^i$ for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate a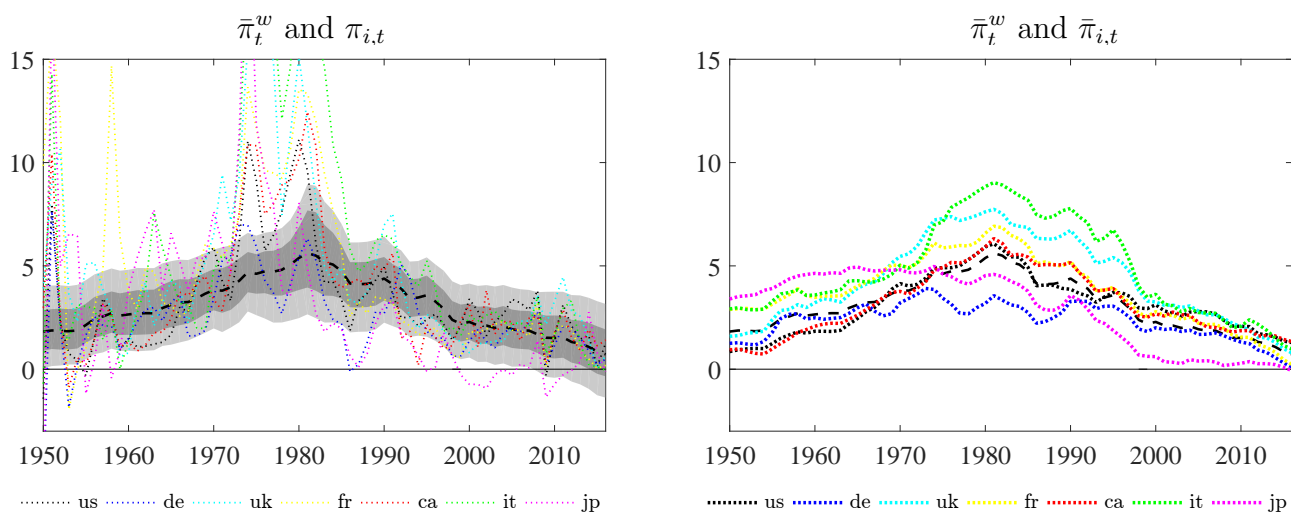cross countries $\frac{1}{n}\sum_{i=1}^{n}(R_{i,t} - \pi_{i,t})$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

B.2   Alternative Priors for the Standard Deviation of Innovations to the Trend

Figure A3: Alternative Priors for the Standard Deviation of Innovations to the Trend, Baseline Model

(prior for the variance of innovations to the trend centered at σ_0)
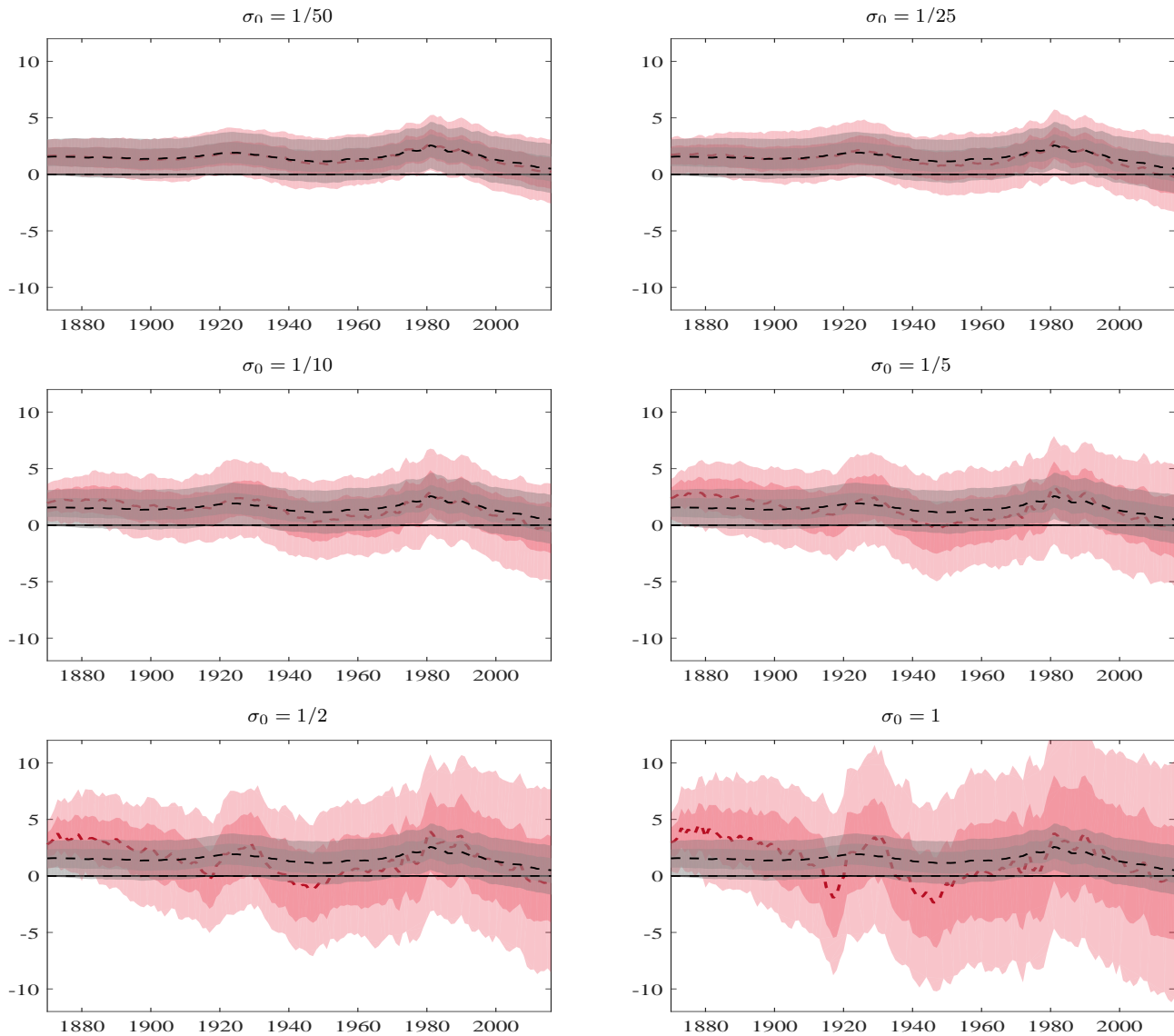
Note: The dashed black line is the posterior median and the shaded gray areas are the 68 and 95 percent posterior coverage intervals for $\bar{r}_t^w$ in the baseline model. The dashed red line is the posterior median and the shaded red areas are the 68 and 95 percent posterior coverage intervals for $\bar{r}_t^w$ obtained when centering the prior for the variance of innovations to the trend equal to 1/50, 1/25, 1/10, 1/5, 1/2, and 1 as opposed to 1/100 as in the baseline specification.

B.3   Baseline Model Estimated from 1950

Figure A4: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model Estimated from 1950
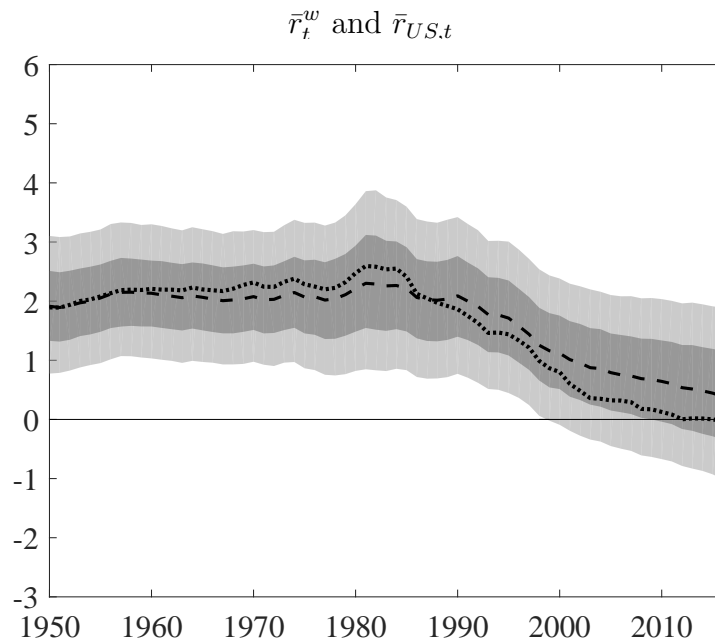
$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$

Note: The dashed black line shows the posterior median of $\bar{r}_t^w$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A5: Trends and Observables for Short-Term Real Rates, Baseline Model Estimated from 1950
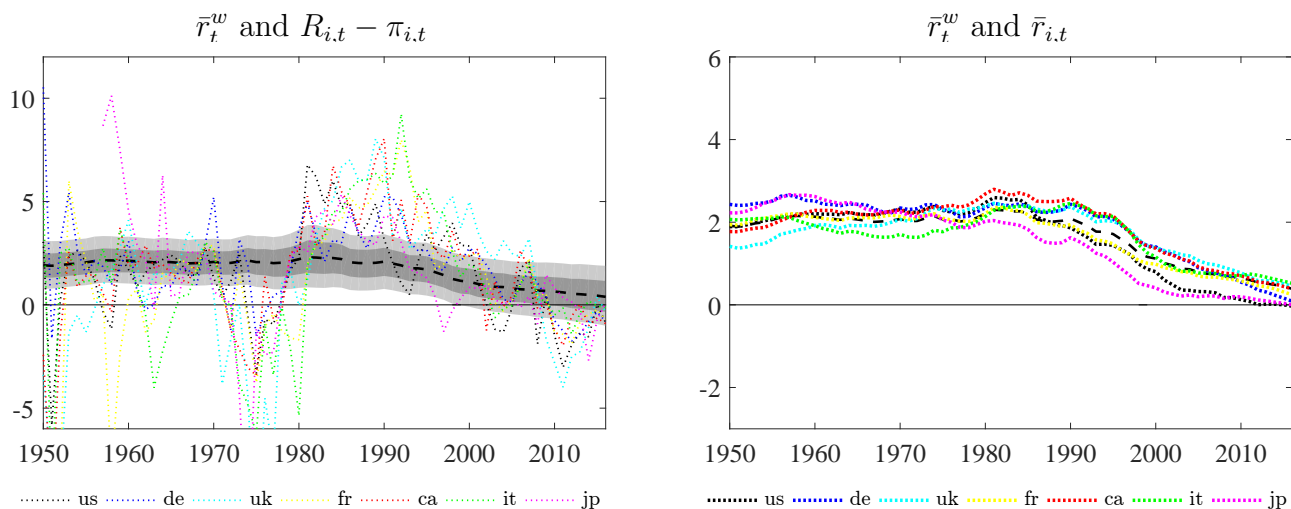
$$\bar{r}_t^w \text{ and } R_{i,t} - \pi_{i,t} \qquad \qquad \bar{r}_t^w \text{ and } \bar{r}_{i,t}$$

Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{r}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{r}_t^w$ (dashed black line).
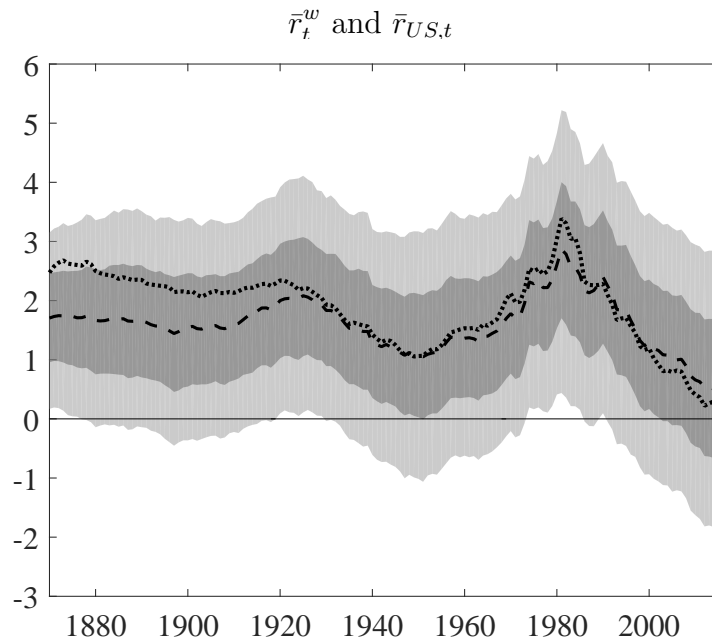
Figure A6: Trends and Observables for Inflation, Baseline Model Estimated from 1950

$$\bar{\pi}_t^w \text{ and } \pi_{i,t} \qquad \qquad \bar{\pi}_t^w \text{ and } \bar{\pi}_{i,t}$$

Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Figure A7: Trends and Observables for Term Spreads, Baseline Model Estimated from 1950
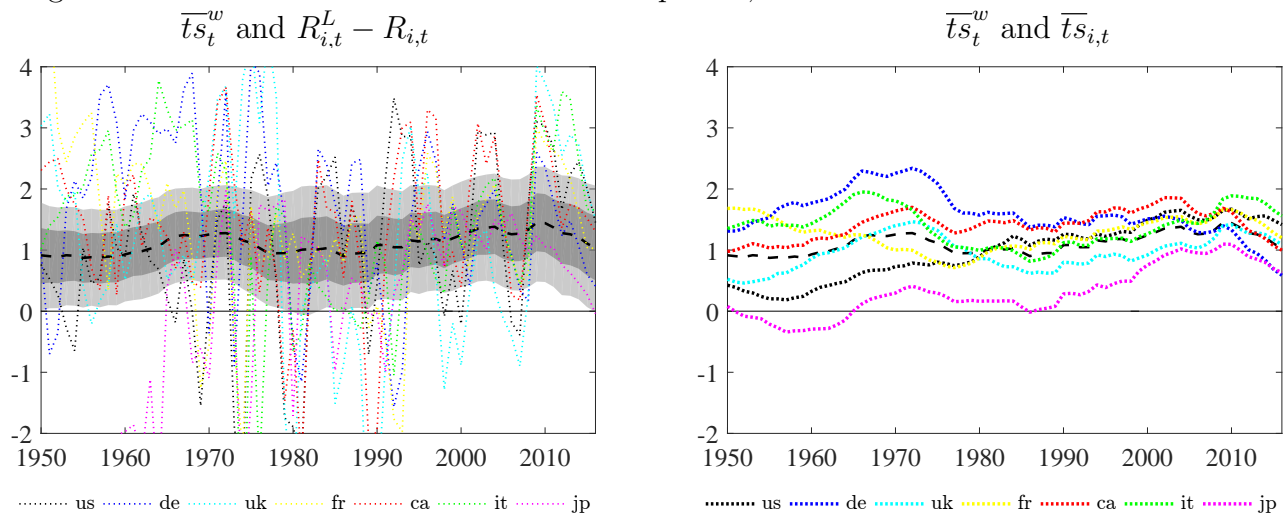
$\overline{ts}_t^w$ and $R_{i,t}^L - R_{i,t}$ $\overline{ts}_t^w$ and $\overline{ts}_{i,t}$
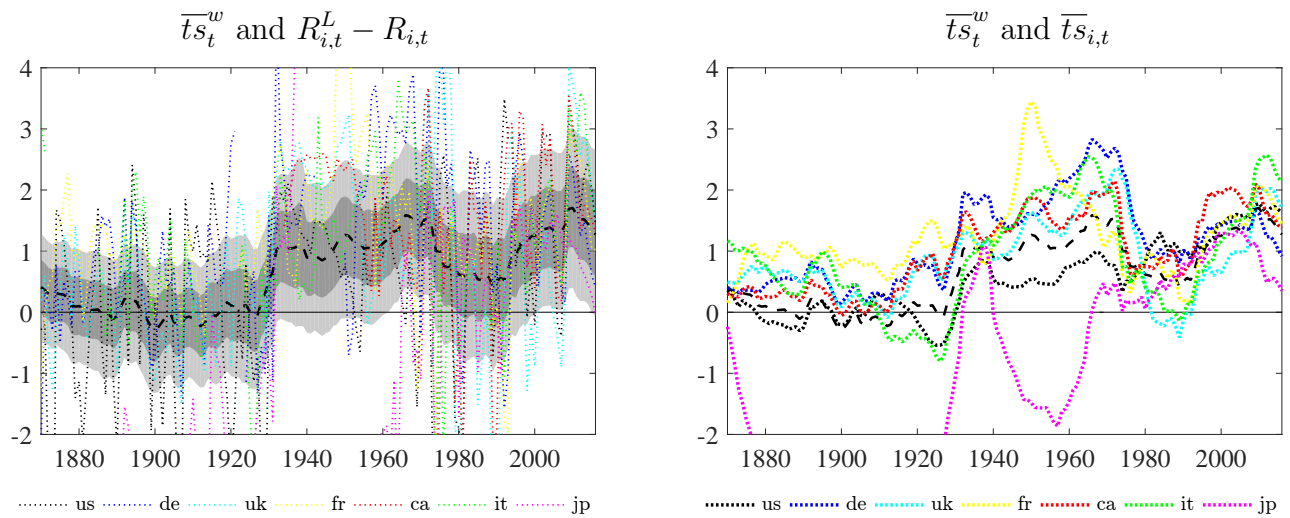

Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{ts}_t^w$ (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\overline{ts}_t^w$ (dashed-and-dotted black line).

B.4   Baseline Model with 50 Degrees of Freedom

Figure A8: Trends in Global and U.S. Real Rates: 1870-2016, Baseline Model with 50 Degrees of Freedom

$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$

Note: The dashed black line shows the posterior median of $\bar{r}_t^w$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A9: Trends and Observables for Short-Term Real Rates, Baseline Model with 50 Degrees of Freedom
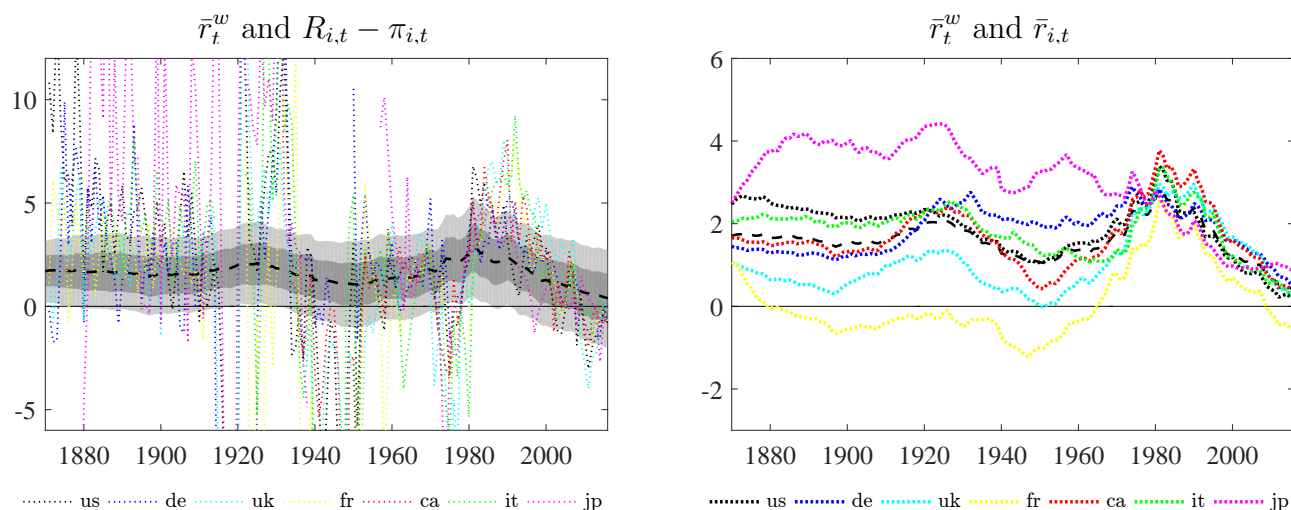
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{r}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{r}_t^w$ (dashed black line).

Figure A10: Trends and Observables for Inflation, Baseline Model with 50 Degrees of Freedom
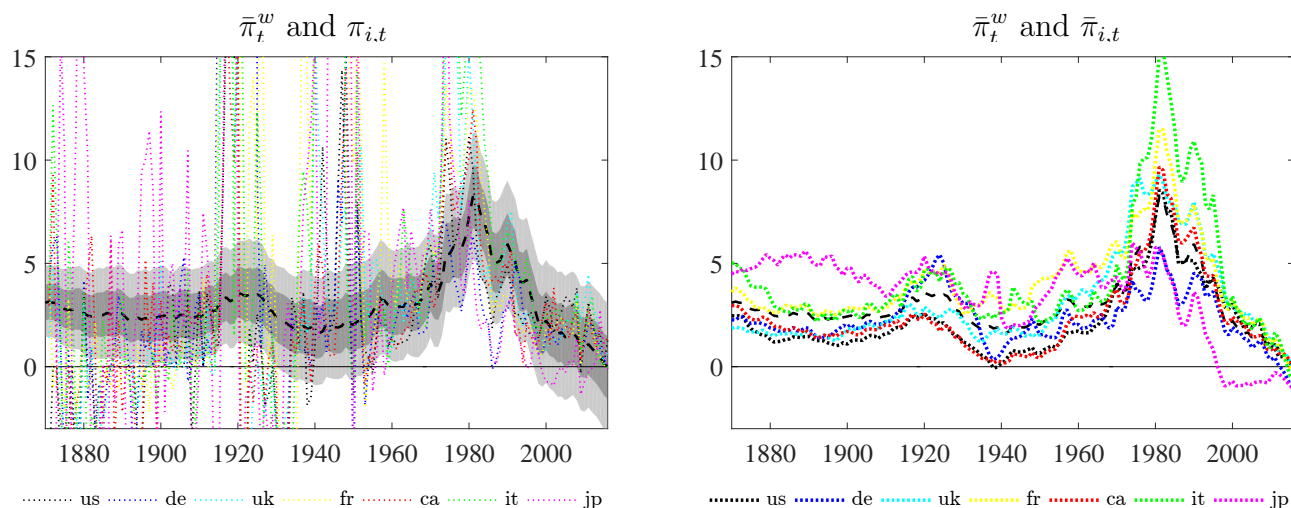
Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Figure A11: Trends and Observables for Term Spreads, Baseline Model with 50 Degrees of Freedom

$\overline{ts}_t^w$ and $R_{i,t}^L - R_{i,t}$

$\overline{ts}_t^w$ and $\overline{ts}_{i,t}$

Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{ts}_t^w$ (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\overline{ts}_t^w$ (dashed-and-dotted black line).

B.5   Baseline Model – Unrestricted Version

Figure A12: Trends in Global and U.S. Real Rates: 1870-2016, Unrestricted Version of the Baseline Model (Section 4.1)
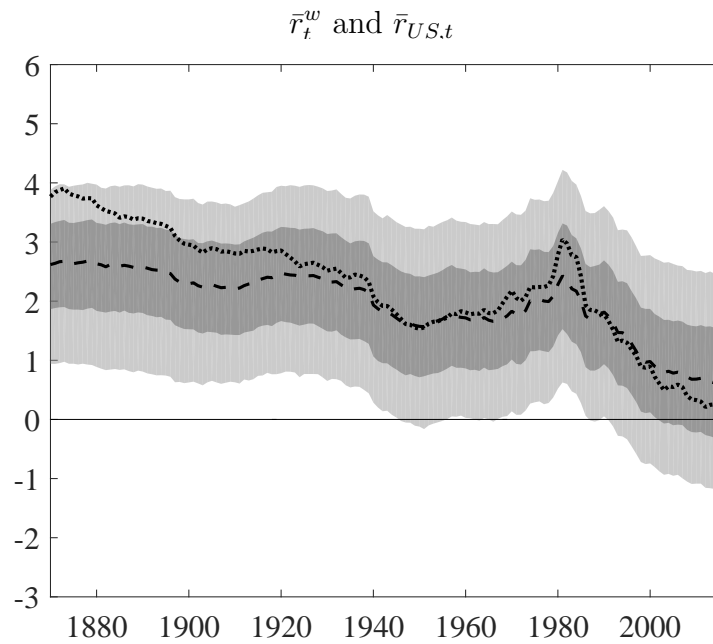
$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$

Note: The dashed black line shows the posterior median of $\bar{r}_t^w$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Table A2: Estimates of λ_i^r for the Unrestricted Version of the Baseline Model (Section 4.1)

us	1.23
	$[1.02, 1.45]$
	$\langle 0.80, 1.68 \rangle$
de	0.86
	$[0.68, 1.07]$
	$\langle 0.43, 1.36 \rangle$
uk	0.72
	$[0.50, 0.90]$
	$\langle 0.32, 1.18 \rangle$
fr	0.85
	$[0.61, 1.08]$
	$\langle 0.38, 1.35 \rangle$
ca	1.05
	$[0.84, 1.22]$
	$\langle 0.67, 1.43 \rangle$
it	1.24
	$[0.94, 1.50]$
	$\langle 0.68, 1.73 \rangle$
jp	1.37
	$[1.15, 1.53]$
	$\langle 0.89, 1.75 \rangle$

Note: The table shows the posterior median of λ_i^r and the 68 and 95 percent posterior coverage intervals.

B.6   Convenience Yield Model (Section 4.2) − Additional Results

Figure A13: Trends in Global and U.S. Real Rates: 1870-2016, Convenience Yield Model
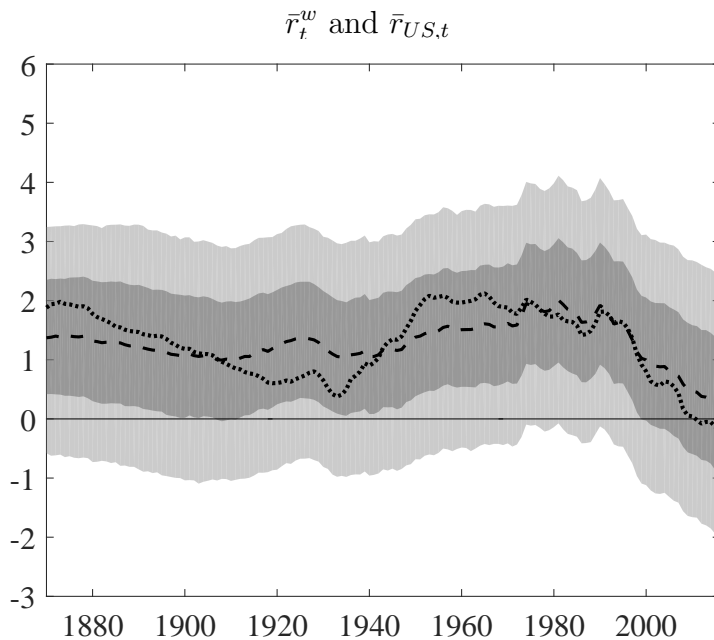
$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$

Note: The dashed black line shows the posterior median of $\bar{r}_t^w$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

Figure A14: Trends and Observables for Short-Term Real Rates, Convenience Yield Model
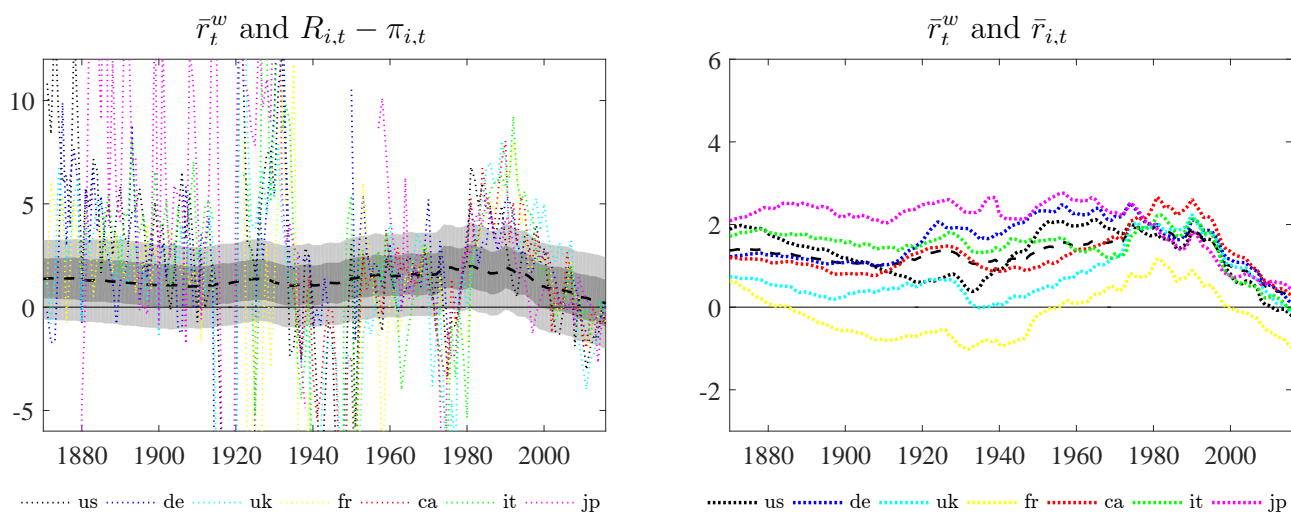
Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{r}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{r}_t^w$ (dashed black line).

Figure A15: Trends and Observables for Inflation, Convenience Yield Model
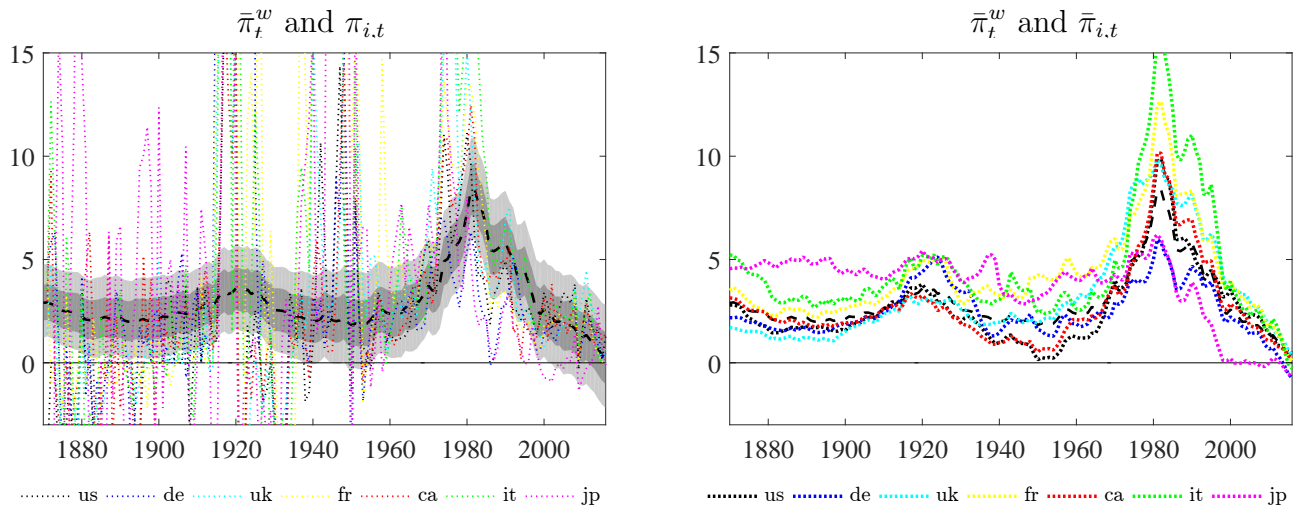
$\bar{\pi}_t^w$ and $\pi_{i,t}$

$\bar{\pi}_t^w$ and $\bar{\pi}_{i,t}$

Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Figure A16: Trends and Observables for Term Spreads, Convenience Yield Model
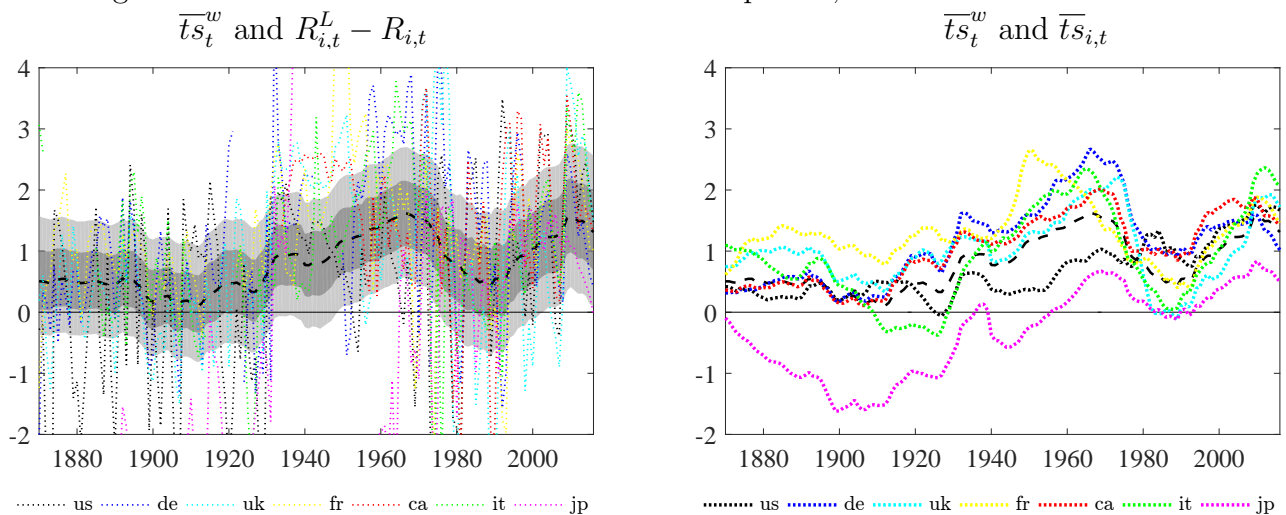
$\overline{ts}_t^w$ and $R_{i,t}^L - R_{i,t}$

$\overline{ts}_t^w$ and $\overline{ts}_{i,t}$

Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{ts}_t^w$ (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\overline{ts}_t^w$ (dashed-and-dotted black line).

Figure A17: Country-Specfic Trends r_t^i and Observables, Convenience Yield Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n}\sum_{i=1}^{n}(R_{i,t} - \pi_{i,t})$$
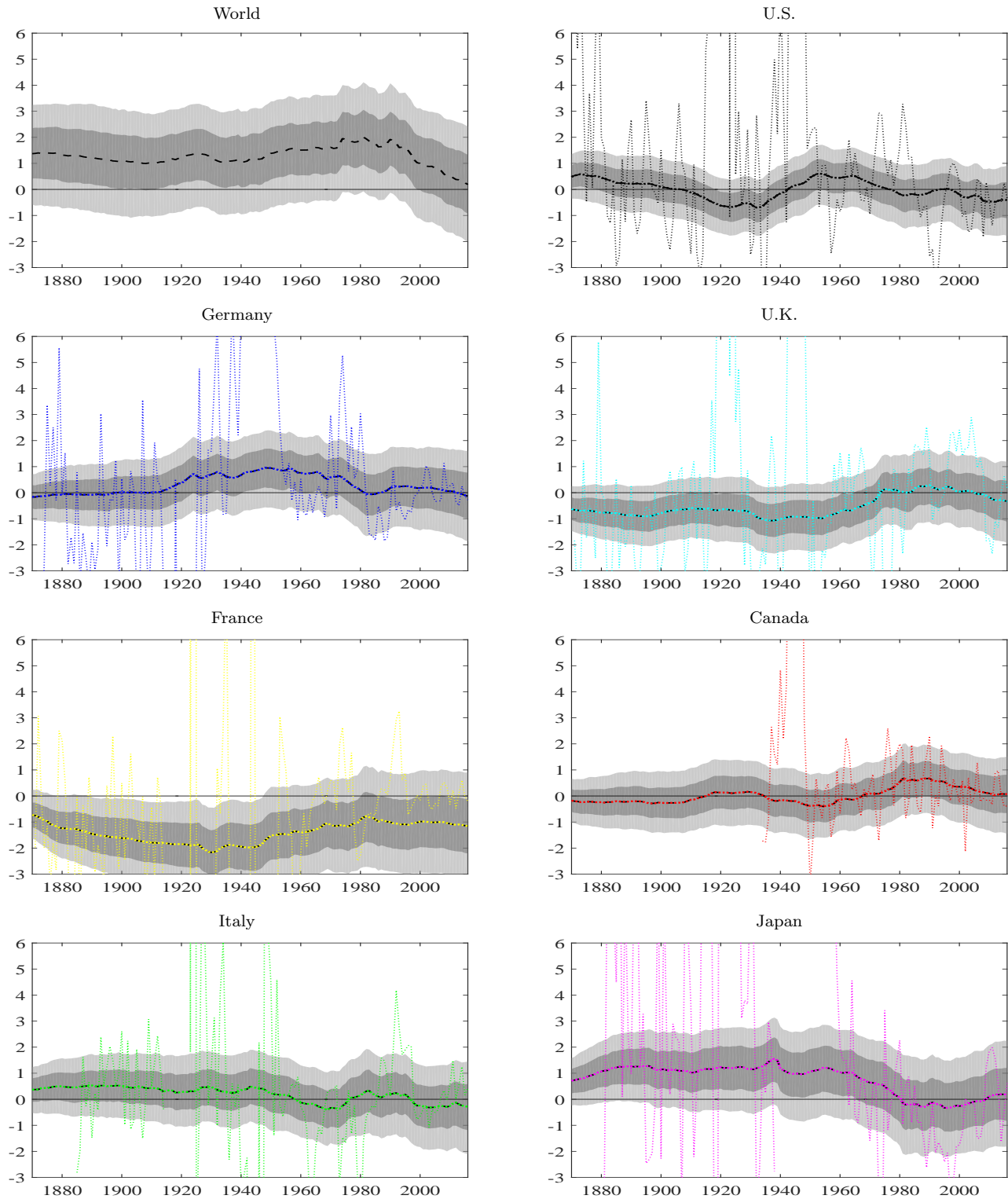
Note: The upper-left panel reproduces the estimates of $\bar{r}_t^w$, so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect $\bar{r}_t^i$ for each country together with the ex-post real rates $R_{i,t} - \pi_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n}\sum_{i=1}^{n}(R_{i,t} - \pi_{i,t})$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

B.7 Consumption Model (Section 4.3) − Additional Results

Figure A18: Trends in Global and U.S. Real Rates: 1870-2016, Consumption Model
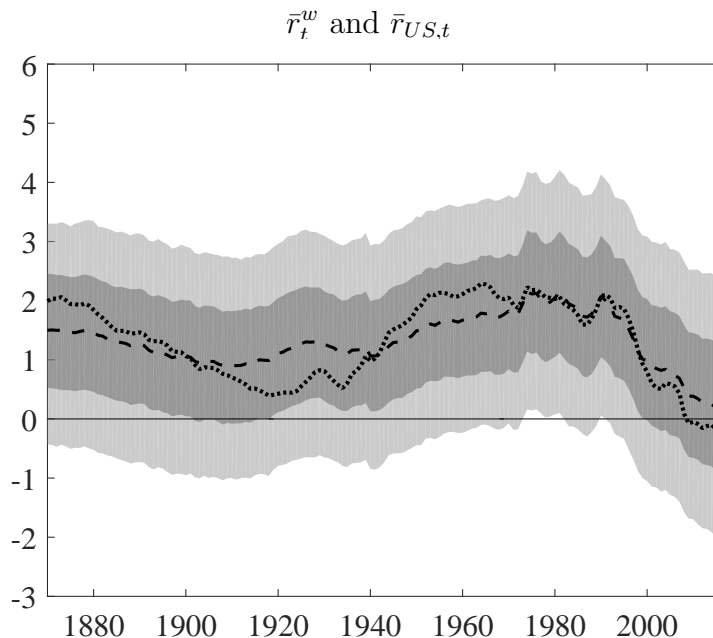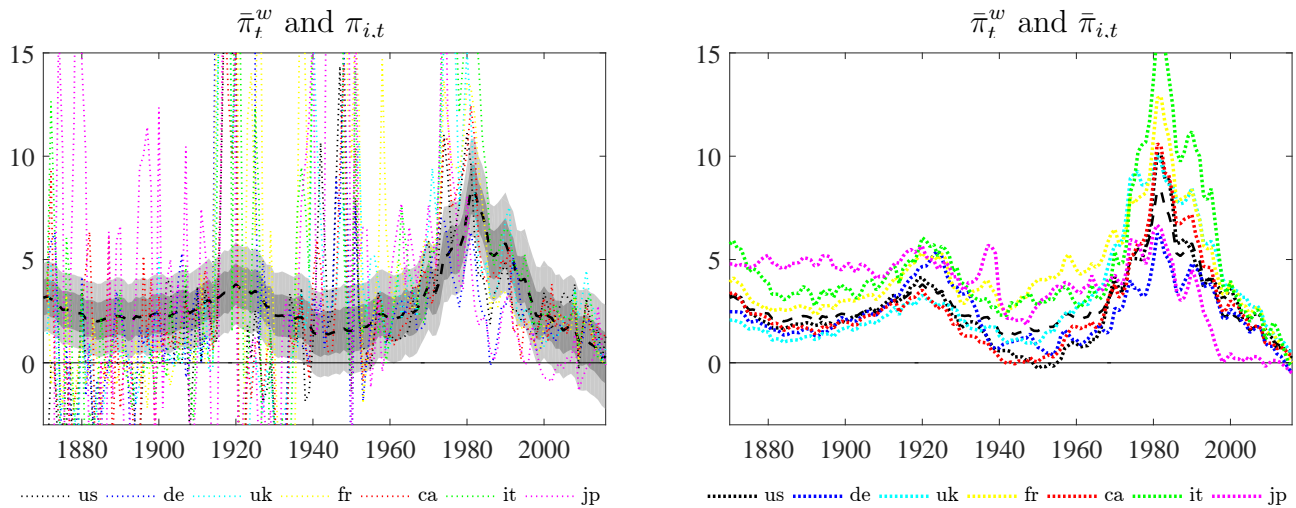
$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$

Note: The dashed black line shows the posterior median of $\bar{r}_t^w$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The dotted black line shows the posterior median of $\bar{r}_{US,t} = \bar{r}_t^w + \bar{r}_t^{US}$.

### Figure A19: Trends and Observables for Short-Term Real Rates, Consumption Model
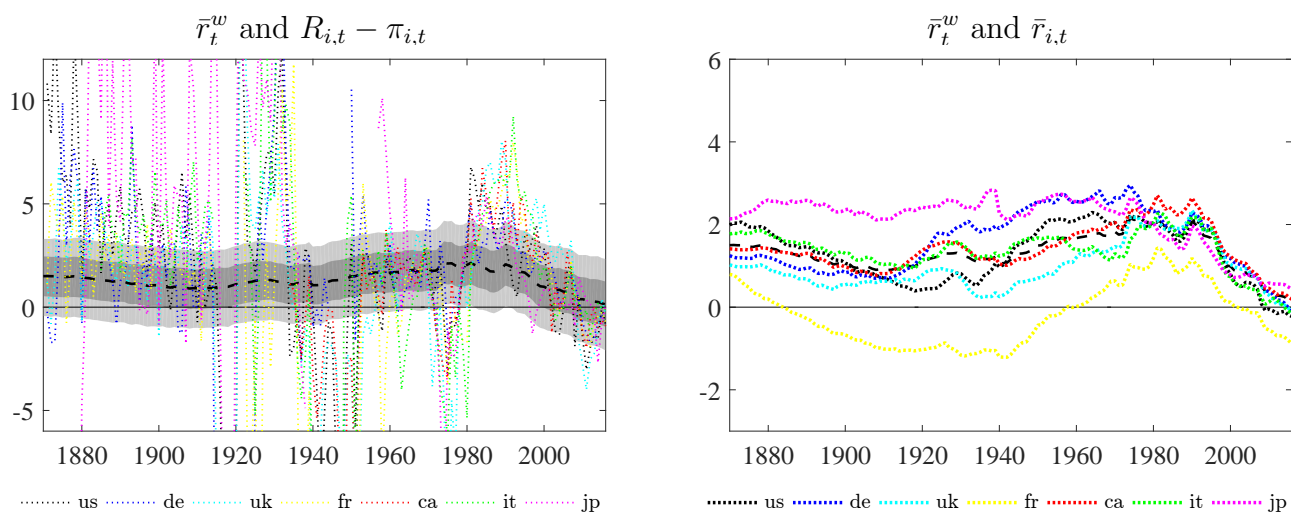
$$\bar{r}_t^w \text{ and } R_{i,t} - \pi_{i,t} \qquad\qquad \bar{r}_t^w \text{ and } \bar{r}_{i,t}$$

Note: The left panel shows $R_{i,t} - \pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{r}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{r}_{i,t} = \bar{r}_t^w + \bar{r}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{r}_t^w$ (dashed black line).

Figure A20: Trends and Observables for Inflation, Consumption Model

$\bar{\pi}_t^w$ and $\pi_{i,t}$

$\bar{\pi}_t^w$ and $\bar{\pi}_{i,t}$

Note: The left panel shows $\pi_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\bar{\pi}_t^w$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\bar{\pi}_{i,t} = \lambda_i^\pi \bar{\pi}_t^w + \bar{\pi}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\bar{\pi}_t^w$ (dashed black line).

Figure A21: Trends and Observables for Term Spreads, Consumption Model
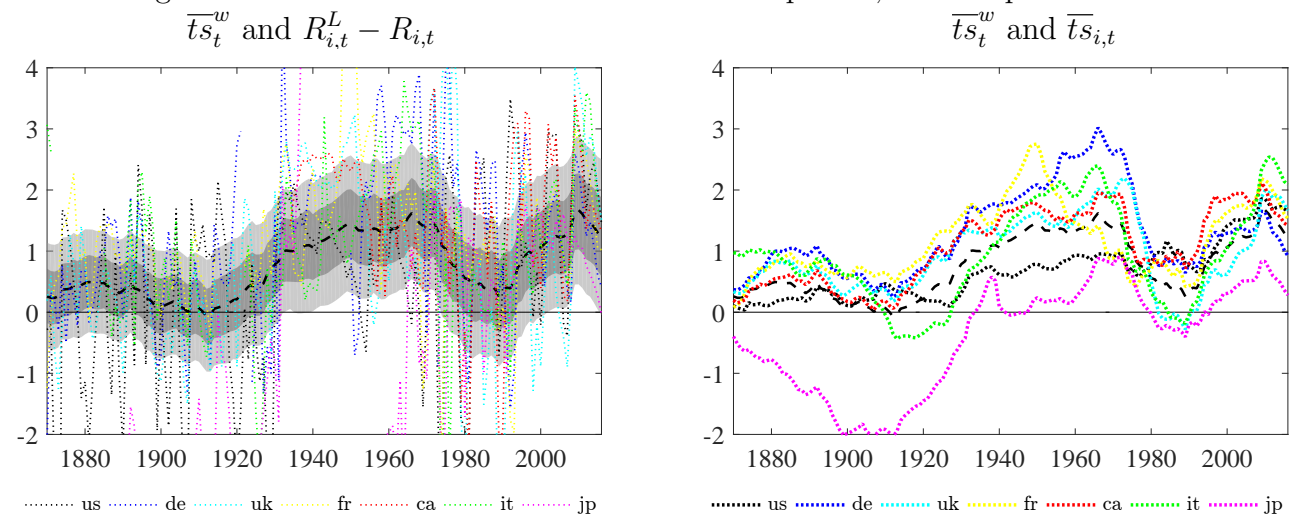
$\overline{ts}_t^w$ and $R_{i,t}^L - R_{i,t}$

$\overline{ts}_t^w$ and $\overline{ts}_{i,t}$

Note: The left panel shows $R_{i,t}^L - R_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{ts}_t^w$ (the dashed-and-dotted black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals). The right panel shows the posterior median of $\overline{ts}_{i,t} = \overline{ts}_t^w + \overline{ts}_t^i$ for each country i (dotted lines; see legend), together with the posterior median of the trend $\overline{ts}_t^w$ (dashed-and-dotted black line).

Figure A22: Country-Specfic Trends r_t^i and Observables, Consumption Model

$$\bar{r}_t^i \text{ and } R_{i,t} - \pi_{i,t} - \frac{1}{n}\sum_{i=1}^{n}(R_{i,t} - \pi_{i,t})$$
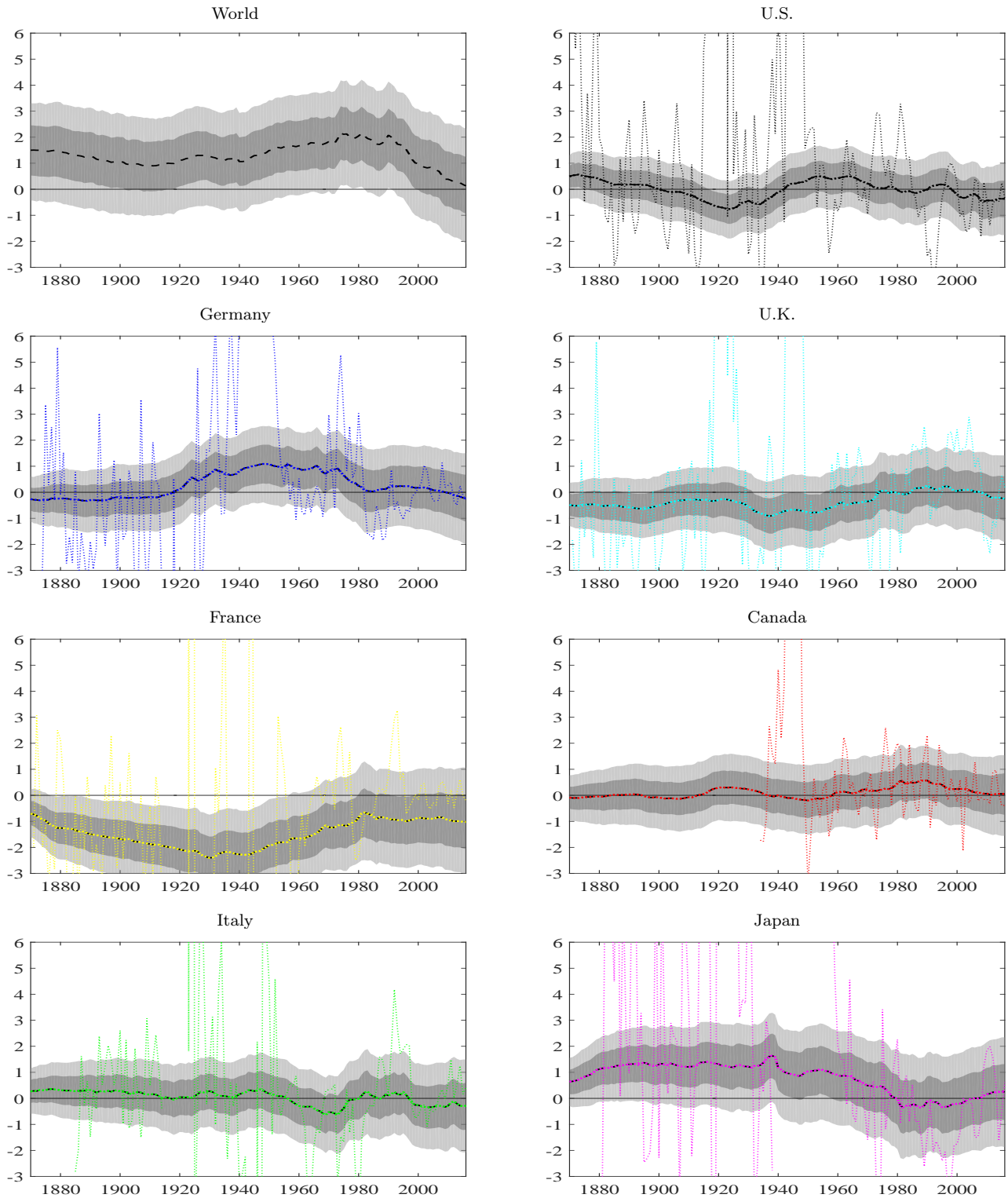
B.8   The VAR with Common Trends Applied to Changes in the Real Exchange Rate

In this section we present the results of the model

$$\Delta q_{i,t} = \overline{\Delta q_t^w} + \overline{\Delta q_t^i} + \widetilde{\Delta q}_{i,t}, \tag{A-1}$$

for $i = 2, \ldots, n$ ($i = 1$ is the U.S.), where $\Delta q_{i,t}$ is the first difference in the log real exchange rate in country i relative to the U.S., $\overline{\Delta q_t^w}$ captures trends in the real exchange rate changes relative to the U.S. for all countries, $\overline{\Delta q_t^i}$ captures country-specific trends, and $\widetilde{\Delta q}_{i,t}$ measures cyclical variations in the real exchange rate changes. The system is estimated jointly for all countries in the sample using the same priors as for the model described in section 4.1 of the paper.

Figure A23: Common Trend and Observables for Changes in the Real Exchange Rate
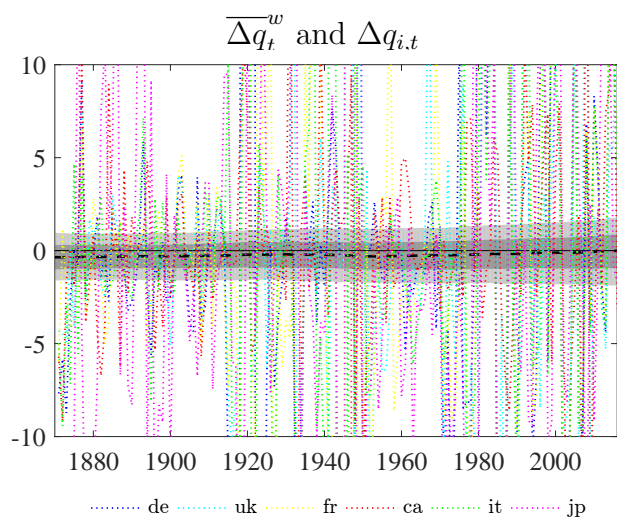
Note: The panel shows $\Delta q_{i,t}$ for each country i (dotted lines; see legend), together with the trend $\overline{\Delta q_t^w}$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals).

Figure A24: Country-Specfic Trends and Observables for Changes in the Real Exchange Rate, Exchange-Rate-Only Model

$$\overline{\Delta q}_t^i \text{ and } \Delta q_{i,t} - \frac{1}{n}\sum_{i=1}^{n}\Delta q_{i,t}$$
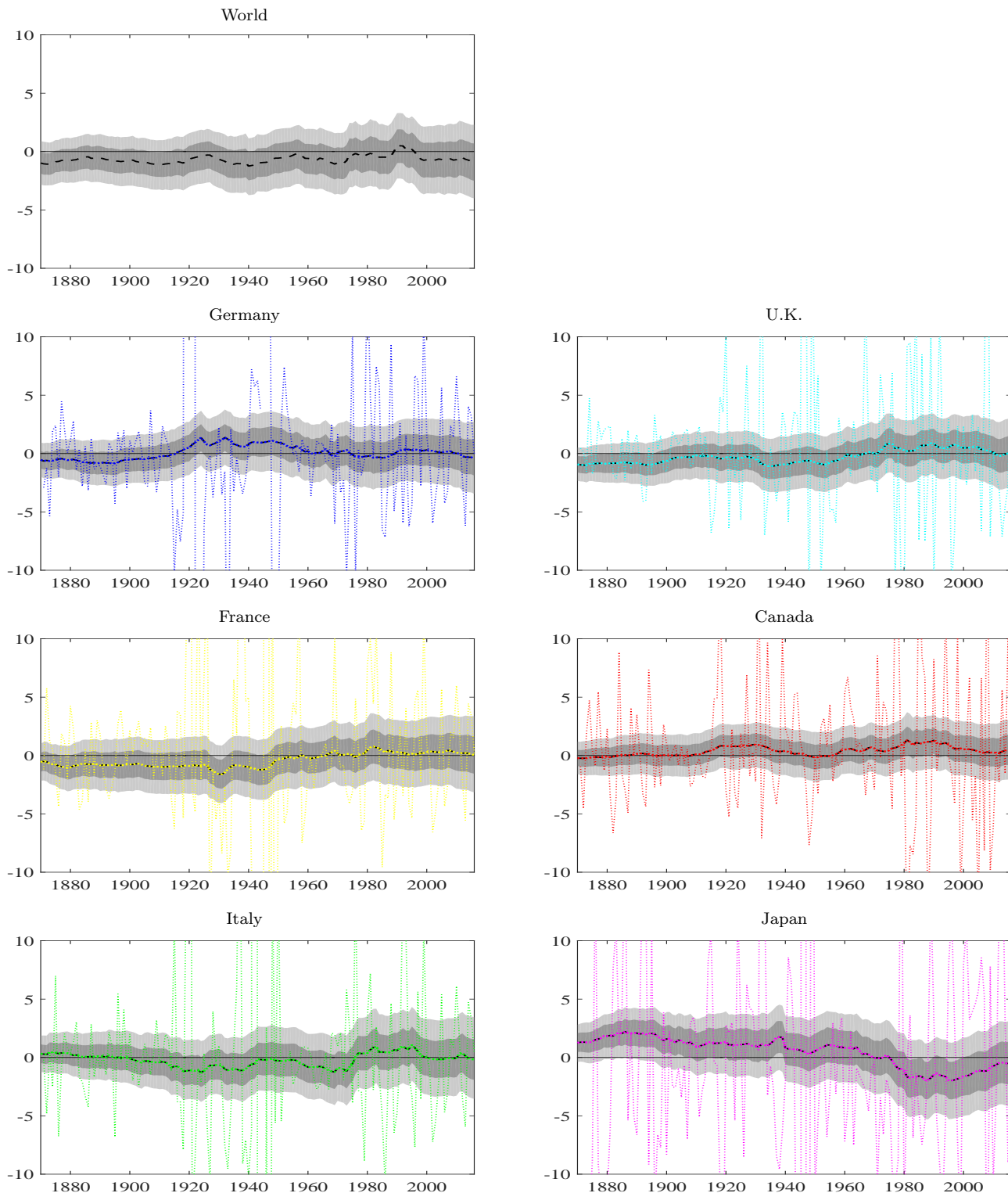
Note: The upper-left panel reproduces the estimates of $\overline{\Delta q}_t^w$, so as to put in context the magnitude of the fluctuations for the remaining panels, which show the estimates of the country-specific effect $\overline{\Delta q}_t^i$ for each country together with the ex-post real rates $\Delta q_{i,t}$ in deviation from the average ex-post real rate across countries $\frac{1}{n}\sum_{i=1}^{n}\Delta q_{i,t}$. In all panels, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.
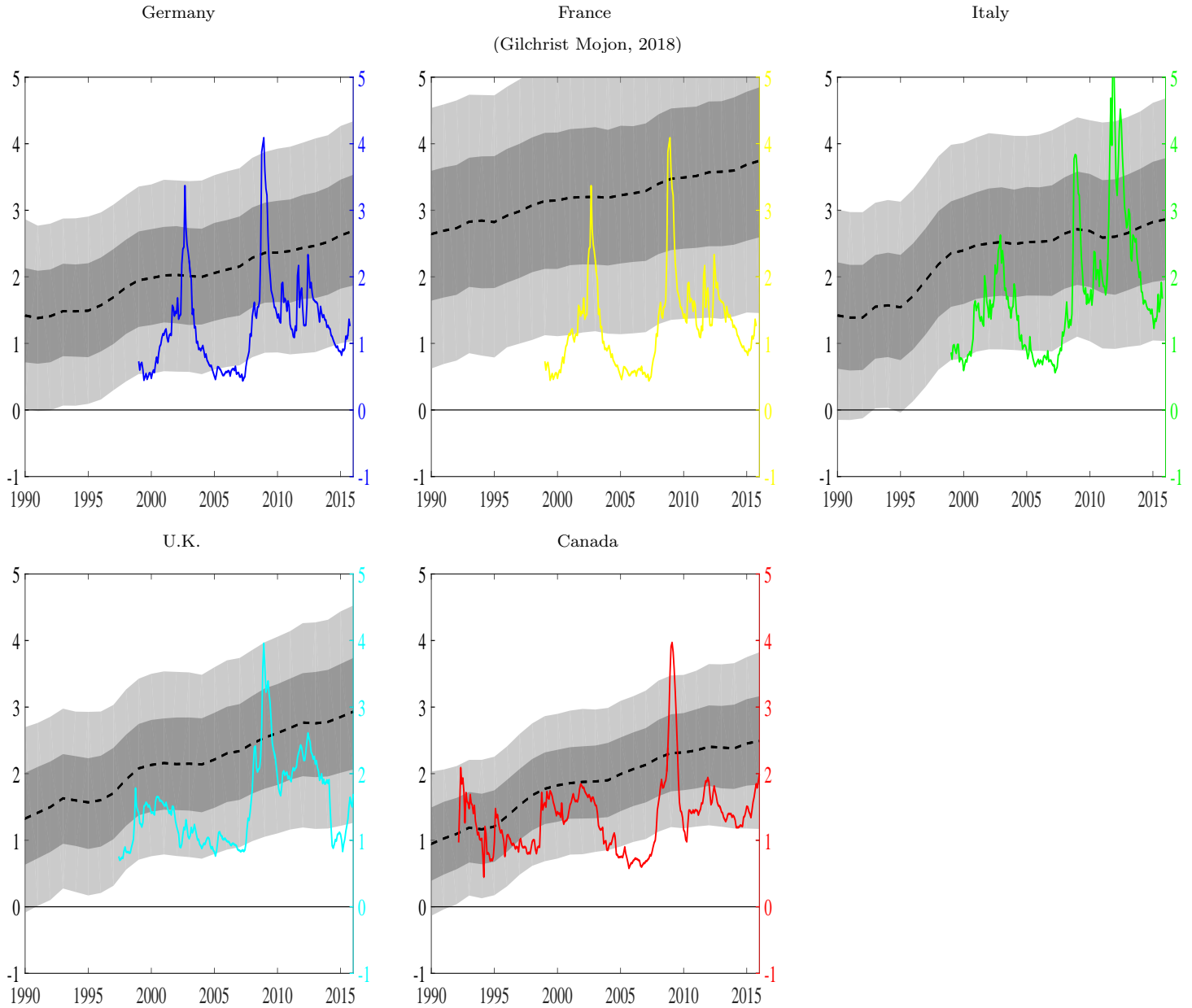
B.9   Spreads

Figure A25: Baa-Treasury Spread and Convenience Yield: US

$$\bar{r}_t^w \text{ and } \bar{r}_{US,t}$$
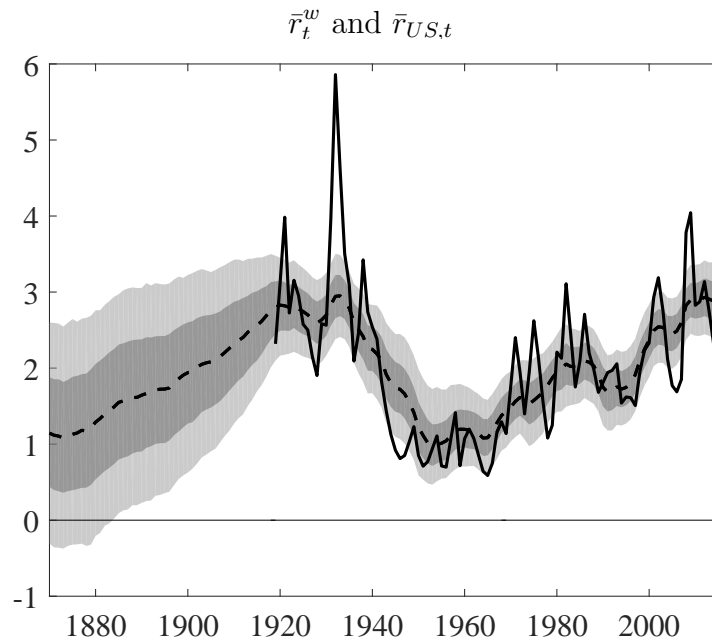
Note: The dashed black line shows the posterior median of $\bar{c}y_{US,t} = \bar{c}y_t^w + \bar{c}y_t^{US}$ and the shaded areas show the 68 and 95 percent posterior coverage intervals. The solid black line shows Baa-Treasury spread.

Figure A26: Spreads

Germany   France   Italy

(Gilchrist Mojon, 2018)

U.K.   Canada

Note: In each panel, the dashed black line shows the posterior median of $\bar{c}y_{i,t} = \bar{c}y_t^w + \bar{c}y_t^i$ (convenience yield for country i) and the shaded areas show the 68 and 95 percent posterior coverage intervals. The solid lines show the spread between yields on corporate and government bonds. The spreads data for EMU countries (Germany, France, and Italy) are obtained from Gilchrist and Mojon (2018) and are computed with respect to German government bonds. U.K. corporate spreads are obtained from the Bank of England and are computed as the difference between corporate bond yields and the rate on 10-year government bonds. For Canada the spreads are obtained from the Federal Reserve Board's financial markets database and are the difference between corporate yields and five-year government bonds. We thank Ozge Akinci for providing us with these data.