Online Appendix

Globalization and the Increasing Correlation
Between Capital Inflows and Outflows

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This online Appendix has 3 sections. Section I derives the linearized model equilibrium conditions. Section II derives expressions for net capital outflows, saving minus investment, and the current account (trade account plus net investment income) and confirms that their equality corresponds to market equilibrium conditions. Section III derives the model solution.

I Model Linearization

We will linearize the model around the equilibrium in the absence of shocks. We set $A = 1/\beta$, which gives us a non-shock equilibrium where consumption is identical across both periods. The model only has a solution for relative prices. We normalize the price levels to 1 in the absence of shocks: $P_{H1} = P_{H2} = P_{F1} = P_{F2} = 1$. The equilibrium in the absence of shocks is:

$$C_1 = C_2 = Y_{H1} = Y_{H2} = AK \tag{1}$$

$$C_H^H = C_F^F = \psi AK \tag{2}$$

$$C_F^H = C_H^F = (1-\psi)AK \tag{3}$$

$$W^n = (1-m)(1+A)K \tag{4}$$

$$W = m(1+A)K \tag{5}$$

$$R = A \tag{6}$$

$$I_H = I_F = 0 \tag{7}$$

$$z_H = z_F = 0.5 \tag{8}$$

$$Q_H = Q_F = 1 \tag{9}$$

$$\Pi_H = \Pi_F = 0 \tag{10}$$

We now log linearize the market clearing conditions around this equilibrium. The logs of variables in deviation from their no-shock equilibrium will be denoted with lower case letters. We abstract from the installment firm profits in what follows as they are zero in the no-shock equilibrium and their linearization is zero as well.

I.A Asset Market Equilibrium Conditions

After substituting the wealth expressions into the Home asset market clearing condition, dividing by $[\beta/(1+\beta)]K = K/(1+A)$, we have

$$\begin{aligned}
&Ae^{p_{H1}+\varepsilon_{H1}+\varepsilon_H^\beta}(1-m+0.5mz_H) + Ae^{p_{H1}+\varepsilon_{H1}+\varepsilon_F^\beta}0.5mz_F + \\
&Ae^{p_{F1}+\varepsilon_{F1}+\varepsilon_H^\beta}0.5mz_H + Ae^{p_{F1}+\varepsilon_{F1}+\varepsilon_F^\beta}0.5mz_F + \\
&e^{q_H+\varepsilon_H^\beta}(1-m) + 0.5m\left(e^{q_H}+e^{q_F}\right)m\left(z_H e^{\varepsilon_H^\beta} + z_F e^{\varepsilon_F^\beta}\right) = \\
&(1+A)e^{q_H} + (1+A)\frac{I_H}{K} \qquad (11)
\end{aligned}$$

Portfolio shares will remain as shares rather than logs of shares. Log-linearizing gives (in deviation from no-shock equilibrium)

$$\begin{aligned}
&(1-0.5m)A(p_{H1}+\varepsilon_{H1}) + 0.5mA(p_{F1}+\varepsilon_{F1}) + (A+1)m(z_H+z_F) + \\
&(1-0.5m)(1+A)\varepsilon_H^\beta + 0.5m(1+A)\varepsilon_F^\beta + mq^A = \\
&(A+m)q_H + \frac{1}{\xi}(1+A)(q_H - p_{H1} + \varepsilon_H^I) \qquad (12)
\end{aligned}$$

where $q^A = 0.5(q_H + q_F)$. The log-linearized market clearing condition for the Foreign asset is analogous:

$$\begin{aligned}
&(1-0.5m)A(p_{F1}+\varepsilon_{F1}) + 0.5mA(p_{H1}+\varepsilon_{H1}) - (A+1)m(z_H+z_F) + \\
&(1-0.5m)(1+A)\varepsilon_F^\beta + 0.5m(1+A)\varepsilon_H^\beta + mq^A = \\
&(A+m)q_F + \frac{1}{\xi}(1+A)(q_F - p_{F1} + \varepsilon_F^I) \qquad (13)
\end{aligned}$$

We will normalize prices such that the average of the log prices in both periods is zero: $p_1^A = 0.5(p_{H1}+p_{F1}) = 0$ and $p_2^A = 0.5(p_{H2}+p_{F2}) = 0$. Taking the average and the difference of the asset market clearing conditions, we then have

$$-(1+\beta)\varepsilon^{\beta,A} + \left(1+\frac{1+\beta}{\xi}\right)q^A + \frac{1+\beta}{\xi}\varepsilon^{I,A} = \varepsilon_1^A \qquad (14)$$

$$\begin{aligned}
&\left(\frac{1-m}{1+\beta}+\frac{1}{\xi}\right)p_1^D + \frac{1-m}{1+\beta}\varepsilon_1^D + 4mz^A + (1-m)\varepsilon^{\beta,D} = \\
&\left(\frac{A+m}{A+1}+\frac{1}{\xi}\right)q^D + \frac{1}{\xi}\varepsilon^{I,D} \qquad (15)
\end{aligned}$$

Here $\varepsilon_1^A = 0.5(\varepsilon_{H1}+\varepsilon_{F1})$, $\varepsilon^{\beta,A} = 0.5(\varepsilon_H^\beta+\varepsilon_F^\beta)$, $\varepsilon^{I,A} = 0.5(\varepsilon_H^I+\varepsilon_F^I)$, and superscripts D denote the analogous difference of variables across countries.

I.B Period 1 Goods Market Clearing Conditions

To produce I_H capital goods requires $e^{-\varepsilon_H^I} I_H + 0.5\xi(I_H)^2$ consumption goods, which enters in the period 1 goods market clearing condition. Since $I_H = 0$ in the no-shock equilibrium, this is simply equal to I_H after linearization. We can then write the period 1 good market clearing conditions as

$$\psi_{H1} AKe^{-\omega(p_{H1}-p_1^H)+c_1^H} + (1-\psi_{F1})AKe^{-\omega(p_{H1}-p_1^F)+c_1^F} + I_H = AKe^{\varepsilon_{H1}} \quad (16)$$

$$(1-\psi_{H1})AKe^{-\omega(p_{F1}-p_1^H)+c_1^H} + \psi_{F1} AKe^{-\omega(p_{F1}-p_1^F)+c_1^F} + I_F = AKe^{\varepsilon_{F1}} \quad (17)$$

Dividing by AK and log linearizing gives

$$-(1-\psi)\varepsilon_1^{\psi,D} + \psi\left(-\omega(p_{H1}-p_1^H) + c_1^H\right) + (1-\psi)\left(-\omega(p_{H1}-p_1^F) + c_1^F\right) + \frac{I_H}{AK} = \varepsilon_{H1}$$

$$(1-\psi)\varepsilon_1^{\psi,D} + (1-\psi)\left(-\omega(p_{F1}-p_1^H) + c_1^H\right) + \psi\left(-\omega(p_{F1}-p_1^F) + c_1^F\right) + \frac{I_F}{AK} = \varepsilon_{F1}$$

where $\varepsilon_1^{\psi,D} = \varepsilon_{H1}^\psi - \varepsilon_{F1}^\psi$.

Log-linearizing the price level gives

$$p_1^H = \psi p_{H1} + (1-\psi)p_{F1} \quad (18)$$

$$p_1^F = (1-\psi)p_{H1} + \psi p_{F1} \quad (19)$$

Substituting this yields

$$-(1-\psi)\varepsilon_1^{\psi,D} + \psi\left(-(1-\psi)\omega p_1^D + c_1^H\right) + (1-\psi)\left(-\omega\psi p_1^D + c_1^F\right) + \frac{I_H}{AK} = \varepsilon_{H1}$$

$$(1-\psi)\varepsilon_1^{\psi,D} + (1-\psi)\left(\omega\psi p_1^D + c_1^H\right) + \psi\left((1-\psi)\omega p_1^D + c_1^F\right) + \frac{I_F}{AK} = \varepsilon_{F1}$$

Taking the average and difference of the last two equations we have

$$c_1^A + \frac{I^A}{AK} = \varepsilon_1^A \quad (20)$$

$$-2(1-\psi)\varepsilon_1^{\psi,D} - 4\psi(1-\psi)\omega p_1^D + (2\psi-1)c_1^D + \frac{I^D}{AK} = \varepsilon_1^D \quad (21)$$

We next need expressions for c_1^A and c_1^D. We can write the expressions for $P_1^H C_1^H$ and $P_1^F C_1^F$ in log form as

$$
\begin{aligned}
AKe^{p_1^H + c_1^H} &= (A+1)K\left(1 - \frac{\beta}{1+\beta}e^{\varepsilon_H^\beta}\right) + \\
&\quad \frac{1}{1+\beta}\left((1-m)(A+1)Ke^{w_H^n} + m(1+A)Ke^{w}\right) \qquad (22)\\
AKe^{p_1^F + c_1^F} &= (A+1)K\left(1 - \frac{\beta}{1+\beta}e^{\varepsilon_F^\beta}\right) + \\
&\quad \frac{1}{1+\beta}\left((1-m)(1+A)Ke^{w_F^n} + m(1+A)Ke^{w}\right) \qquad (23)
\end{aligned}
$$

Dividing by AK and linearizing gives

$$p_1^H + c_1^H = -\beta\varepsilon_H^\beta + (1-m)w_H^n + mw \tag{24}$$
$$p_1^F + c_1^F = -\beta\varepsilon_F^\beta + (1-m)w_F^n + mw \tag{25}$$

Taking the average and difference yields

$$c_1^A = -\beta\varepsilon^{\beta,A} + (1-m)w^{n,A} + mw \tag{26}$$
$$c_1^D = -\beta\varepsilon^{\beta,D} + (1-m)w^{n,D} - (2\psi-1)p_1^D \tag{27}$$

Here we used that $p_1^A = 0$ and $p_1^H - p_1^F = (2\psi - 1)p_1^D$.

We next need to log-linearize the wealth expressions. We have

$$(1-m)(1+A)Ke^{w_H^n} = (1-m)AKe^{p_{H1}+\varepsilon_{H1}} + (1-m)Ke^{q_H} \tag{28}$$
$$(1-m)(1+A)Ke^{w_F^n} = (1-m)AKe^{p_{F1}+\varepsilon_{F1}} + (1-m)Ke^{q_F} \tag{29}$$

Linearizing gives

$$(1+A)w_H^n = Ap_{H1} + A\varepsilon_{H1} + q_H \tag{30}$$
$$(1+A)w_F^n = Ap_{F1} + A\varepsilon_{F1} + q_F \tag{31}$$

Taking the average and difference gives

$$(1+A)w^{n,A} = A\varepsilon_1^A + q^A \tag{32}$$
$$(1+A)w^{n,D} = Ap_1^D + A\varepsilon_1^D + q^D \tag{33}$$

We have analogously

$$(1+A)w = A\varepsilon_1^A + q^A \tag{34}$$

Substitution these wealth expressions in the consumption expressions gives

$$c_1^A = -\beta\varepsilon^{\beta,A} + \frac{A}{1+A}\varepsilon_1^A + \frac{1}{1+A}q^A \tag{35}$$

$$c_1^D = -\beta\varepsilon^{\beta,D} - (2\psi-1)p_1^D + (1-m)\frac{A}{1+A}(\varepsilon_1^D + p_1^D) + \frac{1-m}{1+A}q^D \tag{36}$$

For investment we have

$$I^A = \frac{K}{\xi}(q^A + \varepsilon^{I,A}) \tag{37}$$

$$I^D = \frac{K}{\xi}(q^D - p_1^D + \varepsilon^{I,D}) \tag{38}$$

Substituting these consumption and investment expressions back into (20)-(21) gives

$$-(1+\beta)\varepsilon^{\beta,A} + \left(1 + \frac{1+\beta}{\xi}\right)q^A + \frac{1+\beta}{\xi}\varepsilon^{I,A} = \varepsilon_1^A \tag{39}$$

$$\begin{aligned}
&-(2\psi-1)\beta\varepsilon^{\beta,D} - \left(4\psi(1-\psi)\omega + (2\psi-1)^2 - (1-m)\frac{A}{1+A}(2\psi-1) + \frac{\beta}{\xi}\right)p_1^D \\
&+\left(\frac{(2\psi-1)(1-m)}{1+A} + \frac{\beta}{\xi}\right)q^D + \frac{\beta}{\xi}\varepsilon^{I,D} - 2(1-\psi)\varepsilon_1^{\psi,D} = \\
&\left(1 - (2\psi-1)(1-m)\frac{A}{1+A}\right)\varepsilon_1^D
\end{aligned} \tag{40}$$

I.C Period 2 Goods Market Clearing Conditions

We finally need to impose the period 2 goods market clearing conditions, which we can write as

$$\psi_{H2}AKe^{-\omega(p_{H2}-p_2^H)+c_2^H} + (1-\psi_{F2})AKe^{-\omega(p_{H2}-p_2^F)+c_2^F} = Ae^{\varepsilon_{H2}}(K+I_H) \tag{41}$$

$$(1-\psi_{H2})AKe^{-\omega(p_{F2}-p_2^H)+c_2^H} + \psi_{F2}AKe^{-\omega(p_{F2}-p_2^F)+c_2^F} = Ae^{\varepsilon_{F2}}(K+I_F) \tag{42}$$

Dividing by AK and log linearizing gives

$$\begin{aligned}
&\psi\left(-\omega(p_{H2}-p_2^H) + c_2^H\right) + (1-\psi)\left(-\omega(p_{H2}-p_2^F) + c_2^F\right) - (1-\psi)\varepsilon_2^{\psi,D} = \\
&\varepsilon_{H2} + \frac{1}{\xi}(q_H - p_{H1} + \varepsilon_H^I)
\end{aligned} \tag{43}$$

$$\begin{aligned}
&(1-\psi)\left(-\omega(p_{F2}-p_2^H) + c_2^H\right) + \psi\left(-\omega(p_{F2}-p_2^F) + c_2^F\right) + (1-\psi)\varepsilon_2^{\psi,D} = \\
&\varepsilon_{F2} + \frac{1}{\xi}(q_F - p_{F1} + \varepsilon_F^I)
\end{aligned} \tag{44}$$

Log-linearizing the price level gives

$$p_2^H = \psi p_{H2} + (1-\psi)p_{F2} \tag{45}$$

$$p_2^F = (1-\psi)p_{H2} + \psi p_{F2} \tag{46}$$

Substituting this yields

$$\psi\left(-(1-\psi)\omega p_2^D + c_2^H\right) + (1-\psi)\left(-\omega\psi p_2^D + c_2^F\right) - (1-\psi)\varepsilon_2^{\psi,D} = \varepsilon_{H2} + \frac{1}{\xi}(q_H - p_{H1} + \varepsilon_H^I) \tag{47}$$

$$(1-\psi)\left(\omega\psi p_2^D + c_2^H\right) + \psi\left((1-\psi)\omega p_2^D + c_2^F\right) + (1-\psi)\varepsilon_2^{\psi,D} = \varepsilon_{F2} + \frac{1}{\xi}(q_F - p_{F1} + \varepsilon_F^I) \tag{48}$$

Taking the average and difference of the last two equations, we have

$$c_2^A = \varepsilon_2^A + \frac{1}{\xi}(q^A + \varepsilon^{I,A}) \tag{49}$$

$$-2(1-\psi)\varepsilon_2^{\psi,D} - 4\psi(1-\psi)\omega p_2^D + (2\psi-1)c_2^D = \varepsilon_2^D + \frac{1}{\xi}(q^D - p_1^D + \varepsilon^{I,D}) \tag{50}$$

We next need expressions for c_2^A and c_2^D. We can write the expressions for $P_2^H C_2^H$ and $P_2^F C_2^F$ in log form as

$$\begin{aligned} AKe^{p_2^H + c_2^H} &= (A+1)K\frac{\beta A}{1+\beta}e^{\varepsilon_H^\beta} + \\ &\frac{\beta A}{1+\beta}\left((1-m)(A+1)Ke^{w_H^n} + m(1+A)Ke^{w}\right) + \\ &\frac{\beta A}{1+\beta}(1-m)(1+A)Ke^{r_H} + \frac{\beta A}{1+\beta}m(1+A)Ke^{r^{p,H}} \end{aligned} \tag{51}$$

$$\begin{aligned} AKe^{p_2^F + c_2^F} &= (A+1)K\frac{\beta A}{1+\beta}e^{\varepsilon_F^\beta} + \\ &\frac{\beta A}{1+\beta}\left((1-m)(A+1)Ke^{w_F^n} + m(1+A)Ke^{w}\right) + \\ &\frac{\beta A}{1+\beta}(1-m)(1+A)Ke^{r_F} + \frac{\beta A}{1+\beta}m(1+A)Ke^{r^{p,F}} \end{aligned} \tag{52}$$

Linearizing gives

$$p_2^H + c_2^H = \varepsilon_H^\beta + (1-m)w_H^n + mw + (1-m)r_H + 0.5m(r_H + r_F) \tag{53}$$

$$p_2^F + c_2^F = \varepsilon_F^\beta + (1-m)w_F^n + mw + (1-m)r_F + 0.5m(r_H + r_F) \tag{54}$$

where we used that $r^{p,H} = r^{p,F} = 0.5r_H + 0.5r_F$ when log linearized around $z_H = z_F = 0.5$.

Taking the average and difference yields

$$c_2^A = \varepsilon^{\beta,A} + (1-m)w^{n,A} + mw + r^A \tag{55}$$

$$c_2^D = \varepsilon^{\beta,D} + (1-m)w^{n,D} - (2\psi - 1)p_2^D + (1-m)er \tag{56}$$

where $r^A = 0.5(r_H + r_F)$ and $er = r_H - r_F$. We have

$$r^A = \varepsilon_2^A - q^A \tag{57}$$

$$er = p_2^D + \varepsilon_2^D - q^D \tag{58}$$

Then

$$c_2^A = \varepsilon^{\beta,A} + (1-m)w^{n,A} + mw + \varepsilon_2^A - q^A \tag{59}$$

$$c_2^D = \varepsilon^{\beta,D} + (1-m)w^{n,D} + (2 - 2\psi - m)p_2^D + (1-m)\varepsilon_2^D - (1-m)q^D \tag{60}$$

Substituting the wealth expressions (32)-(33), we have

$$c_2^A = \varepsilon^{\beta,A} + \frac{A}{1+A}\varepsilon_1^A - \frac{A}{1+A}q^A + \varepsilon_2^A \tag{61}$$

$$\begin{aligned} c_2^D &= \varepsilon^{\beta,D} + (1-m)\frac{A}{1+A}p_1^D + (1-m)\frac{A}{1+A}\varepsilon_1^D \\ &\quad -(1-m)\frac{A}{1+A}q^D + (1-m)\varepsilon_2^D + (2-2\psi-m)p_2^D \end{aligned} \tag{62}$$

Substituting these consumption expressions back into (49)-(50) gives

$$-(1+\beta)\varepsilon^{\beta,A} + \left(1 + \frac{1+\beta}{\xi}\right) q^A + \frac{1+\beta}{\xi}\varepsilon^{I,A} = \varepsilon_1^A \tag{63}$$

$$\begin{aligned} &-2(1-\psi)\varepsilon_2^{\psi,D} + (2\psi-1)\varepsilon^{\beta,D} - \lambda p_2^D + \\ &\left((2\psi-1)\frac{1-m}{1+\beta} + \frac{1}{\xi}\right)\left(p_1^D - q^D\right) + (2\psi-1)\frac{1-m}{1+\beta}\varepsilon_1^D \\ &= \left(1 - (2\psi-1)(1-m)\right)\varepsilon_2^D + \frac{1}{\xi}\varepsilon^{I,D} \end{aligned} \tag{64}$$

where

$$\lambda = 4\psi(1-\psi)\omega + (2\psi-1)(2\psi+m-2) \tag{65}$$

I.D Portfolio Expressions

We finally need to consider the portfolio expressions. Taking the average and difference of the portfolio shares (in deviation from the no-shocks equilibrium), we have

$$z^A = \frac{E(er)}{var(er)} + 0.5\varepsilon^{z,D} \tag{66}$$

$$z^D = 2\varepsilon^{z,A} \tag{67}$$

Using that $er = p_2^D + \varepsilon_2^D - q^D$, we have

$$z^A = \frac{E(p_2^D) + E(\varepsilon_2^D) - q^D}{var(er)} + 0.5\varepsilon^{z,D} \tag{68}$$

II Net Capital Outflows, Saving Minus Investment and Current Account

In this section we will derive expressions for gross and net capital flows, saving minus investment and the current account (trade account plus net investment income). We will also show that equating net capital outflows, saving minus investment and the current account leads to equations that are the same as some of the market clearing conditions from the previous section.

II.A Capital Flows

We write outflows in period 1 as OF and inflows as IF. We have

$$OF = \frac{\beta_H}{1+\beta_H}(1-z_H)W - 0.5mKQ_F \tag{69}$$

$$IF = \frac{\beta_F}{1+\beta_F}z_F W - 0.5mKQ_H \tag{70}$$

These are equal to the asset holdings in period 1 minus the value of the holdings carried over from the previous period. Linearizing, we have

$$OF = \frac{\beta}{1+\beta}0.5m(1+A)K\varepsilon_H^{\beta} - \frac{\beta}{1+\beta}m(1+A)Kz_H + 0.5\frac{\beta}{1+\beta}m(AK\varepsilon_1^A + Kq^A) - 0.5mKq_F \tag{71}$$

In the absence of shocks we have

$$External\ Assets = 0.5\frac{\beta}{1+\beta}m(1+A)K = 0.5mK \tag{72}$$

Therefore

$$\frac{OF}{External\ Assets} = \varepsilon_H^{\beta} - 2z_H + \frac{A}{1+A}\varepsilon_1^A + \frac{1}{1+A}q^A - q_F \tag{73}$$

Here z_H (in deviation from no-shock equilibrium) is equal to

$$z_H = \frac{E(p_2^D) + E(\varepsilon_2^D) - q^D}{var(er)} + \varepsilon_H^z \tag{74}$$

We have analogously

$$\frac{IF}{External\ Assets} = \varepsilon_F^{\beta} + 2z_F + \frac{A}{1+A}\varepsilon_1^A + \frac{1}{1+A}q^A - q_H \tag{75}$$

Defining net flows as $NF = OF - IF$ and gross flows as $GF = OF + IF$, we have

$$\frac{NF}{External\ Assets} = \varepsilon^{\beta,D} - 4z^A + q^D \tag{76}$$

$$\frac{GF}{External\ Assets} = 2\varepsilon^{\beta,A} - 2z^D + \frac{2}{1+\beta}\varepsilon_1^A - \frac{2}{1+\beta}q^A \tag{77}$$

Substituting the portfolio expressions, these become

$$\frac{NF}{External\ Assets} = \varepsilon^{\beta,D} - 4\frac{E(p_2^D) + E(\varepsilon_2^D) - q^D}{var(er)} - 2\varepsilon^{z,D} + q^D \tag{78}$$

$$\frac{GF}{External\ Assets} = 2varepsilon^{\beta,A} - 4\varepsilon^{z,A} + \frac{2}{1+\beta}\varepsilon_1^A - \frac{2}{1+\beta}q^A \tag{79}$$

II.B Saving Minus Investment

We now derive an expression for saving minus investment and verify that the equality $NF = S - I$ is the same conditions as the difference between the asset market clearing conditions. Here $S - I$ is shorthand for $S_H - I_H$ as we will focus on the Home country.

The value of investment is $Q_H I_H$, which is the same as I_H after linearization. We denote saving in the Home country as S_H. Using that world saving is equal to world investment, we can write saving minus investment as

$$S_H - I_H = 0.5S^D - 0.5I^D \tag{80}$$

where $S^D = S_H - S_F$ and $I^D = I_H - I_F$.

First consider Home saving, which is equal to asset income minus consumption:

$$S_H = (1-m)P_{H1}Y_{H1} + \Pi_H + 0.5mP_{H1}Y_{H1} + 0.5mP_{F1}Y_{F1} - \frac{1}{1+\beta_H}\left(W_H^n + W\right) \quad (81)$$

Linearizing, we have

$$\begin{aligned} S_H = & \frac{\beta}{1+\beta}AK\left((1-m)(p_{H1}+\varepsilon_{H1}) + m\varepsilon_1^A\right) + \\ & \frac{\beta}{1+\beta}(1+A)K\varepsilon_H^\beta - \frac{1}{1+\beta}K\left((1-m)q_H + mq^A\right) \end{aligned} \quad (82)$$

Analogously

$$\begin{aligned} S_F = & \frac{\beta}{1+\beta}AK\left((1-m)(p_{F1}+\varepsilon_{F1}) + m\varepsilon_1^A\right) + \\ & \frac{\beta}{1+\beta}(1+A)K\varepsilon_F^\beta - \frac{1}{1+\beta}K\left((1-m)q_F + mq^A\right) \end{aligned} \quad (83)$$

Therefore

$$S^D = \frac{1}{1+\beta}K(1-m)\left(p_1^D + \varepsilon_1^D\right) + K\varepsilon^{\beta,D} - \frac{1}{1+\beta}K(1-m)q^D \quad (84)$$

We also have

$$I^D = \frac{1}{\xi}K\left(q^D - p_1^D + \varepsilon^{I,D}\right) \quad (85)$$

Therefore

$$\begin{aligned} S_H - I_H = 0.5S^D - 0.5I^D = & \; 0.5\frac{1}{1+\beta}K(1-m)\left(p_1^D + \varepsilon_1^D\right) + 0.5K\varepsilon^{\beta,D} \\ & -0.5\frac{1}{1+\beta}K(1-m)q^D - 0.5\frac{1}{\xi}K\left(q^D - p_1^D\right) - 0.5\frac{1}{\xi}K\varepsilon^{I,D} \end{aligned} \quad (86)$$

Dividing by external assets $0.5mK$, we have

$$\begin{aligned} \frac{S_H - I_H}{External\ Assets} = & \frac{1}{1+\beta}\frac{1-m}{m}\left(p_1^D + \varepsilon_1^D\right) + \frac{1}{m}\varepsilon^{\beta,D} \\ & -\frac{1}{1+\beta}\frac{1-m}{m}q^D - \frac{1}{\xi m}\left(q^D - p_1^D\right) - \frac{1}{\xi m}\varepsilon^{I,D} \end{aligned} \quad (87)$$

To check that saving minus investment is equal to net outflows, we equate (87) to (76). After multiplying by m, it is easily verified that it delivers the same equation as the difference in asset market clearing conditions, (15).

II.C Current Account

We now derive an expression for the current account CA, which is equal to the trade account plus net investment income. We show that $CA = S - I$ corresponds to the difference in the period 1 goods market clearing conditions.

The trade account is equal to the value of exports minus imports, which is

$$TA = (1-\psi_{F1})\left(\frac{p_{H1}}{p_1^F}\right)^{1-\omega}\frac{1}{1+\beta_F}(W_F^n + W) - (1-\psi_{H1})\left(\frac{p_{F1}}{p_1^H}\right)^{1-\omega}\frac{1}{1+\beta_H}(W_H^n + W) \tag{88}$$

Linearizing gives

$$\begin{aligned} TA = {} & 2\psi(1-\psi)\frac{(1+A)K}{1+\beta}(1-\omega)p_1^D + (1-\psi)\frac{(1+A)\beta K}{1+\beta}\varepsilon^{\beta,D} \\ & -(1-\psi)\frac{(1+A)(1-m)K}{1+\beta}w^{n,D} - (1-\psi)AK\varepsilon_1^{\psi,D} \end{aligned} \tag{89}$$

Dividing by external assets $0.5mK$, this is

$$\begin{aligned} \frac{TA}{External\ Assets} = {} & 4\psi(1-\psi)(1-\omega)\frac{A}{m}p_1^D + 2(1-\psi)\frac{1}{m}\varepsilon^{\beta,D} - \\ & 2(1-\psi)\frac{1-m}{m}Aw^{n,D} - 2(1-\psi)\frac{A}{m}\varepsilon_1^{\psi,D} \end{aligned} \tag{90}$$

Substituting the expression for $w^{n,D}$, this becomes

$$\begin{aligned} \frac{TA}{External\ Assets} = {} & 4\psi(1-\psi)(1-\omega)\frac{A}{m}p_1^D + 2(1-\psi)\frac{1}{m}\varepsilon^{\beta,D} \\ & -2(1-\psi)\frac{1-m}{m}\frac{A}{1+\beta}(p_1^D + \varepsilon_1^D) \\ & -2(1-\psi)\frac{1-m}{m}\frac{1}{1+\beta}q^D - 2(1-\psi)\frac{A}{m}\varepsilon_1^{\psi,D} \end{aligned} \tag{91}$$

Net investment income is equal to

$$NI = 0.5mP_{F1}Y_{F1} - 0.5mP_{H1}Y_{H1} \tag{92}$$

Linearizing and dividing by external assets $0.5mK$, we have

$$\frac{NI}{External\ Assets} = -A(p_1^D + \varepsilon_1^D) \tag{93}$$

Using $CA = TA + NI$, we then have

$$\begin{aligned} \frac{CA}{External\ Assets} = {} & 4\psi(1-\psi)(1-\omega)\frac{A}{m}p_1^D + 2(1-\psi)\frac{1}{m}\varepsilon^{\beta,D} \\ & -2(1-\psi)\frac{1-m}{m}\frac{A}{1+\beta}(p_1^D + \varepsilon_1^D) \\ & -2(1-\psi)\frac{1-m}{m}\frac{1}{1+\beta}q^D - A(p_1^D + \varepsilon_1^D) - 2(1-\psi)\frac{A}{m}\varepsilon_1^{\psi,D} \end{aligned} \tag{94}$$

It is easily verified that setting this equal to the expression (87) for saving minus investment, and then multiplying by m/A, leads to a condition that is identical to the difference between the period 1 goods market clearing conditions (40).

III Model Solution

We have eight equations: six market equilibrium conditions and two expressions for portfolio shares. Because of Walras' Law, we can remove one market equilibrium condition for each period. We remove the average first and second period goods market clearing conditions, (39) and (63). These equations are identical to the average asset market clearing condition (14). This leaves us with six equations in six variables. One of them is a trivial equation: $z^D = 2\varepsilon^{z,A}$. z^D affects gross capital flows, but does not enter in any of the other equations. After substituting the expression for z^A into the difference in period 1 asset market clearing conditions, equation (15), we have 4 equations in 4 variables: q^D, q^A, p_1^D and p_2^D.

From the average asset market clearing condition (14), we have

$$q^A = \frac{1}{1+\frac{1+\beta}{\xi}}\left(\varepsilon_1^A + (1+\beta)\varepsilon^{\beta,A} - \frac{1+\beta}{\xi}\varepsilon^{I,A}\right) \tag{95}$$

Substituting this into the expression (79) for gross capital flows, we have

$$\frac{GF}{External\ Assets} = \frac{2}{1+\beta+\xi}\left((1+\beta)\varepsilon^{\beta,A} + \varepsilon_1^A + \varepsilon^{I,A}\right) - 4\varepsilon^{z,A} \tag{96}$$

We then have 3 equations left to solve for p_1^D, p_2^D and q^D. These equations are (1) the difference in the asset market clearing conditions (same as $S-I=NF$), (2) the difference in period 1 goods market clearing conditions (same as $S-I=CA$), and (3) the difference in the period 2 goods market clearing conditions. We take the expectation of the latter as the expectation of period 2 variables affects the period 1 solution. These three equations are respectively (15), (40) and the expectation of (64):

$$\left(\frac{1-m}{1+\beta}+\frac{1}{\xi}\right)p_1^D+\frac{1-m}{1+\beta}\varepsilon_1^D+4m\frac{E(p_2^D)+E(\varepsilon_2^D)-q^D}{var(er)}+2m\varepsilon^{z,D}$$
$$+(1-m)\varepsilon^{\beta,D}=\left(\frac{A+m}{A+1}+\frac{1}{\xi}\right)q^D+\frac{1}{\xi}\varepsilon^{I,D} \tag{97}$$
$$-(2\psi-1)\beta\varepsilon^{\beta,D}-\left(4\psi(1-\psi)\omega+(2\psi-1)^2-(1-m)\frac{A}{1+A}(2\psi-1)+\frac{\beta}{\xi}\right)p_1^D$$
$$+\left(\frac{(2\psi-1)(1-m)}{1+A}+\frac{\beta}{\xi}\right)q^D+\frac{\beta}{\xi}\varepsilon^{I,D}-2(1-\psi)\varepsilon_1^{\psi,D}=$$
$$\left(1-(2\psi-1)(1-m)\frac{A}{1+A}\right)\varepsilon_1^D \tag{98}$$
$$(2\psi-1)\varepsilon^{\beta,D}-\lambda E(p_2^D)+$$
$$\left((2\psi-1)\frac{1-m}{1+\beta}+\frac{1}{\xi}\right)\left(p_1^D-q^D\right)+(2\psi-1)\frac{1-m}{1+\beta}\varepsilon_1^D-2(1-\psi)E(\varepsilon_2^{\psi,D})$$
$$=\left(1-(2\psi-1)(1-m)\right)E(\varepsilon_2^D)+\frac{1}{\xi}\varepsilon^{I,D} \tag{99}$$

We can write these three equations jointly as

$$H_1\begin{pmatrix} p_1^D \\ E(p_2^D) \\ q^D \end{pmatrix}+H_2\begin{pmatrix} \varepsilon_1^D \\ \varepsilon^{\beta,D} \\ \varepsilon^{z,D} \\ E(\varepsilon_2^D) \\ \varepsilon^{I,D} \\ \varepsilon_1^{\psi,D} \\ E(\varepsilon_2^{\psi,D}) \end{pmatrix}=0 \tag{100}$$

where

$$H_1=$$
$$\begin{pmatrix} \frac{1-m}{1+\beta}+\frac{1}{\xi} & \frac{4m}{var(er)} & -\frac{A+m}{A+1}-\frac{1}{\xi}-\frac{4m}{var(er)} \\ -4\psi(1-\psi)\omega-(2\psi-1)^2+\frac{(1-m)(2\psi-1)A}{1+A}-\frac{\beta}{\xi} & 0 & \frac{(2\psi-1)(1-m)}{1+A}+\frac{\beta}{\xi} \\ (2\psi-1)\frac{1-m}{1+\beta}+\frac{1}{\xi} & -\lambda & -(2\psi-1)\frac{1-m}{1+\beta}-\frac{1}{\xi} \end{pmatrix}$$

and

$$H_2=\begin{pmatrix} \frac{1-m}{1+\beta} & 1-m & 2m & \frac{4m}{var(er)} & -\frac{1}{\xi} & 0 & 0 \\ -1+\frac{(2\psi-1)(1-m)}{1+\beta} & -(2\psi-1)\beta & 0 & 0 & \frac{\beta}{\xi} & -2(1-\psi) & 0 \\ (2\psi-1)\frac{1-m}{1+\beta} & (2\psi-1) & 0 & -1+(2\psi-1)(1-m) & -\frac{1}{\xi} & 0 & -2(1-\psi) \end{pmatrix}$$

It follows that

$$\begin{pmatrix} p_1^D \\ E(p_2^D) \\ q^D \end{pmatrix} = -H_1^{-1}H_2 \begin{pmatrix} \varepsilon_1^D \\ \varepsilon^{\beta,D} \\ \varepsilon^{z,D} \\ E(\varepsilon_2^D) \\ \varepsilon^{I,D} \\ \varepsilon_1^{\psi,D} \\ E(\varepsilon_2^{\psi,D}) \end{pmatrix} \tag{101}$$

After substituting the solution of $E(p_2^D)$ and q^D into (78), we have a solution of net capital flows as a function of model shocks.