Online appendix to accompany

"An Energy-centric Theory of Agglomeration"

A von Thunen and Iceberg Costs of Transport

The transport cost assumptions adopted in von Thunen are subtly, but importantly, different from what we have assumed here. In short, iceberg costs require the energy costs of transportation to fall immediately and completely as energy is expended. At the practical level this rules out containers for fuel storage or combustion, residues left from incomplete combustion, and no mass of the vehicle carrying the load. In terms of the oat eating horse example both Von Thunen and Samuelson used, the horse cannot have any mass, the oats cannot remain resident in the horse, and there is of course no wagon to pull. It is fair to say that while the iceberg assumption is tractable it is also a knife edge assumption as we will show below. If even an epsilon of the mass of energy is wasted in moving containers, in moving engines or left in incompletely combusted particles then the transport process produces a result significantly different than von Thunen's but qualitatively the same as in our specification. Specifically it will lead to a formulation where there is a maximum zone of exploitation tied to the power density of energy.

To be precise, consider the case of renewable with mass discussed in section 2.1.1 on the main text. Assume there is some fixed cost associated with moving energy a distance dx. Then transportation costs over this increment are given by:

$$W^T(x) = \left(C(W_0) + \frac{\mu g d}{\Delta} W(x) \right) dx$$

Total energy remaining at distance $x+dx$ is given by $W(x+dx) = W(x) - \left(C(W_0) + \frac{\mu g d}{\Delta} W(x) \right) dx$.

Rearranging terms we can rewrite this expression as

$$\frac{W(x+dx)-W(x)}{dx}=\frac{dW(x)}{dx}=-\left(C(W_0)+\frac{\mu g d}{\Delta}W(x)\right)$$

The solution to this differential equation is:

$$W(x)=\left(W_0+\frac{\Delta}{\mu g d}C(W_0)\right)e^{-\frac{\mu g d}{\Delta}x}-\frac{\Delta}{\mu g d}C(W_0)$$

Define R as the radius for which $W(R)=0$; that is, the energy supplied to the core by any energy source further away than R is zero. The solution for R is:

$$R=\frac{\Delta}{\mu g d}\ln\left(1+\frac{\mu g d}{\Delta}\frac{W_0}{C(W_0)}\right)$$

The iceberg assumption occurs when $C(W_0)=0$ since in this case R goes to infinity. The case we consider in the text arises when $C(W_0)$ is proportional to the mass of energy transported; specifically that $C(W_0)=\frac{\mu g d}{\Delta}\frac{W_0}{e-1}$ since then we obtain $R=\frac{\Delta}{\mu g d}$.

Two observations are in order. First, there exists a finite margin of exploitation for any $C(W_0)>0$. Thus, iceberg costs represent a knife edge assumption; any value other than $C(W_0)=0$ generates a qualitatively different result. Any and all energy sources sharing the same - infinite - margin of exploitation when $C(W_0)=0$; they have finite and different margins of exploitation for any $C(W_0)>0$. Second, iceberg costs have proven tractable in general equilibrium models because they allow us to model the transportation system without introducing another economic activity complicating predictions in small dimensional models. The formulation in the body of the paper does however respect this constraint. Note the only costs of transport come from moving energy (and not containers, equipment or engines even though the result is consistent with formulations with these fixed costs), the key to our result is our assumption that the mass of energy is transported even as it is used (converted) in transport.

B Extensions

The model presented in the paper is decidedly stark and abstract. In this section of the Online Appendix we present several extensions to showcase the versatility and wider applicability of the Only Energy model. In the paper we treat power density as a primitive. While this is true in principle, investments can be made to improve the quality of the resources or to reduce the costs of transporting them to the core. In the first extension we show the incentives to expand the exploitation zone and to upgrade the quality of the resources; more importantly we show that these incentives are stronger when the intrinsic quality of the resources is higher. Another important assumption in the paper is that power density is distributed uniformly across space. In our second extension we consider the case where the distribution of resources is patchy or punctiform and we also introduce the possibility of uncertain location of resources across space. We show that all the results we find using the uniform distribution hold under these alternative resource distributions. We then introduce the case of non-renewables. Our purpose here is to demonstrate how spatial productivity is germane to both renewables and non-renewables although the details are in some cases importantly different. Perhaps more importantly we demonstrate how spatially productive environments lead to a bunching of resource extractions in calendar time.

B.1 Patchy, Punctiform and Probabilistic Resource Distributions

In the main text we assume resources are uniformly distributed, the space containing resource plays is connected, and there is no uncertainty regarding whether the energy resources in question are present. These are strong assumptions, but for some resources they seem innocuous. For example, crops and woodlands typically satisfy these constraints at least over fairly large areas. But for other resources they fit less well and it is unclear how our analysis would change under these assumptions. For example, the available locations for resource exploitation may be patchy (containing holes) because of land use restrictions, habitat conservation, or noise considerations. The siting decisions for wind and solar farms certainly fit this description. In other cases, most notably fossil fuels, there are often a few very significant deposits surrounded by areas with little if any resource potential (the space contains resource "plays" with widely different power densities). We will refer to this case as one where the resource distribution is *punctiform.* In some other cases it is not clear ex ante whether resources are present in any specific location although there maybe a well defined probability distribution over them (oil and gas deposits come to mind). We refer to this case as one where the distribution of resources is *probabilistic.* We will show that often very little of substance changes with alternative resource distributions although the calculations become more lengthy and the expressions less transparent.

To understand why these complications rarely matter, recall our discussion of energy rents which allowed us to define the extensive margin R^*, for a resource of given power density Δ. Let this reliance of the extensive margin on the power density of resources be written as $R^*(\Delta)$. Then since all resources within this margin provide positive energy rents it should be apparent that they will be exploited even if the resource base is not connected nor homogenous. And if we locate all such potential resources, identify their extensive margins, and then integrate over their relevant regions this (more complicated) sum of energy rents will equal the energy supply just as before. Apart from mathematical complications, patchy and punctiform resource distributions pose no special problem. Alternatively if we assume resources are present in specific locations with given probabilities, we can again identify $R^*(\Delta)$ and integrate over this space to find what would now be expected energy supply. And if the space defined by $R^*(\Delta)$ can be divided into many resource plays with identical and independent success distributions, then a law of large numbers result could be invoked to render expected energy supply equal to ex post energy supply. At bottom the reason why these complications do not matter much is the constant returns built into transport costs by the physics of the underlying problem. Moving an object twice as far is twice the work; moving an object with twice the mass is twice the work; and if movement is output and work (energy) is the input, this production function is CRS. The CRS feature of the problem allows us to aggregate easily, define boundaries simply, and replace patchy, punctiform and probabilistic resource distributions with much simpler connected and homogenous ones in many cases.

To see exactly how to incorporate complicated resource distributions, we construct two examples.

B.1.1 Patchy and Punctiform

It may be clear from the description above that the key complication is locating the various resources in space. To make the analysis tractable and transparent, we construct discrete resource distributions. Consider a division of the space surrounding the core into concentric circles that are then divided further into wedges created by extending rays from the core. The result, shown in Figure 1, is a sequence of land parcels we will refer to as resource plays.

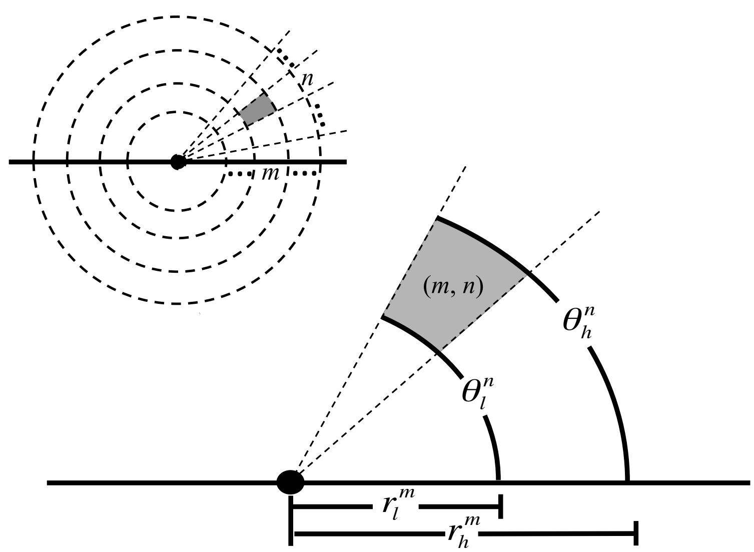


Figure 1: Uneven distribution of power density into parcels

Let there be $n = 1, ..., N$ rays and $m = 1, ..., M$ circles, then there are $N \times M$ resource plays each uniquely identified by the duple (m, n). Suppose each play has an associated power density Δ_{mn} with geometrical shape characterized by its width $r^m = r_h^m - r_l^m$ and the angle of the wedge $\theta^n = \theta_h^n - \theta_l^n$. Where h and l refer to both the higher and lower radius bounds defining the play; and the higher and lower angles (measured in radians) that define its location in the plane. We can write the (maximum) energy supplied by any given resource play:

$$W_{mn} = \int_{\theta_l^n}^{\theta_h^n} \int_{r_l^m}^{r_h^m} v \left(\Delta_{mn} - cv\right) dv d\varphi$$

$$W_{mn} = \frac{1}{2}\left(\theta_h^n - \theta_l^n\right)\left((r_h^m)^2 - (r_l^m)^2\right)\left(\Delta_{mn} - \frac{2}{3}c\frac{(r_h^m)^3 - (r_l^m)^3}{(r_h^m)^2 - (r_l^m)^2}\right) \tag{B.1}$$

Since an energy supplier with play (m, n) supplies energy if the play provides positive energy rents, we need to account for this complication by noting that each density Δ_{mn} has an associated energy margin $\bar{R}_{mn} = \frac{\Delta_{mn}}{c}$. This implies the actual energy supplied to the core by any resource play must be such that:

$$W_{mn}^S = \begin{cases} W_{mn} & \text{if } r_h^m \leq \bar{R}_{mn} \\ \bar{W}_{mn} & \text{if } r_l^m \leq \bar{R}_{mn} < r_h^m \\ 0 & \text{if } \bar{R}_{mn} < r_l^m < r_h^m \end{cases} \tag{B.2}$$

where $\bar{W}_{mn}$ has the same form as equation (B.1) but where r_h^m is replaced by $\bar{R}_{mn}$.

To find the aggregate energy supplied we add the n plays of each annuli m and then add all the annuli. Without further restrictions, the possibilities are very numerous. Therefore consider the case where each play within an annuli m has the same power density Δ_m. As well, order the power densities from lowest to highest such that $\Delta_0 = 0 < \Delta_1 < ... < \Delta_m < ... < \Delta_M$ so distant resources are the most power dense, and in order to eliminate potential gaps in our distribution we assume the width of each annuli is determined by the energy margins of the neighboring annuli. That is assume $r_l^m = \bar{R}_{m-1}$ and $r_h^m = \bar{R}_m$. Alternate assumptions are readily investigated. Using these assumptions we can now replace the definition of $\bar{R}_m$ back in equation (B.1) to find the energy supplied to the core:

$$W_{mn} = \frac{1}{2}\frac{(\theta_h^n - \theta_l^n)}{c^2}\left(\frac{\Delta_m^3}{3} - \Delta_m\Delta_{m-1}^2 + \frac{2}{3}\Delta_{m-1}^3\right)$$

Add over all wedges in the annuli m and over all the annuli M to find

$$W^S = \frac{\pi}{3c^2}\sum_{m=1}^{M}\Delta_m^3\left(1 - 3\frac{\Delta_{m-1}^2}{\Delta_m^2} + 2\frac{\Delta_{m-1}^3}{\Delta_m^3}\right)$$

Two observations are in order. First, since the summation is over primitive determinants of the model, we could just as well replace this complicated sum with $\tilde{\Delta}$, where $\tilde{\Delta}$ is the power density of a hypothetical connected and uniformly distributed resource base yielding the same energy supply. $\tilde{\Delta} > 0$ by virtue of our ordering of power densities, and we can write it simply as:

$$\tilde{\Delta} = \left[\sum_{m=1}^{M}\Delta_m^3\left(1 - 3\frac{\Delta_{m-1}^2}{\Delta_m^2} + 2\frac{\Delta_{m-1}^3}{\Delta_m^3}\right)\right]^{1/3}$$

Second, if we alter the power density of our hypothetical resource base, $\tilde{\Delta}$, by $\lambda > 0$ this is equivalent to uniform scaling by λ of all power densities in the heterogenous resource zone. A moment's reflection will show that energy supply is homogenous of degree three in all power densities taken together. Therefore, for many purposes we can simply write

$$W^S = \frac{\pi\tilde{\Delta}^3}{3c^2}$$

and ignore the fact that the exploitation zone in question is both patchy and punctiform.

B.1.2 Probabilistic

Here we assume the power density of the resource is uniform across space and it is given by Δ_o. This implies all the resources found inside the margin of extraction given by $R_o = \Delta_o/c$ are going to be exploited. Divide this space as we did before using N rays and M circles to identify $N \times M$ resource plays but now assume each play has a probability q of having a resource with power density Δ_o in place and a probability $1-q$ of being empty. Given Δ_o is uniform and constant, our previous assumptions imply the width of each annuli is equal to R_o/M. Thus, the geometrical shape of the parcel (m, n) is characterized by its boundaries set by $r_m^h = (m+1)\frac{R_o}{M}$ and $r_m^l = m\frac{R_o}{M}$ and the angle of the wedge $\theta^n = \theta_h^n - \theta_l^n$. In the case where parcel (m, n) is not empty, we can calculate the same double integral we calculated for the case of patchy distributions and replace the values for r_m^l and r_m^h to find

$$W_{mn} = \frac{(\theta_h^n - \theta_l^n)}{2}\left(\frac{R_o}{M}\right)^2 \Delta_o\left((m+1)^2 - m^2\right)\left(1 - \frac{2}{3}\frac{1}{M}\frac{((m+1)^3 - m^3)}{((m+1)^2 - m^2)}\right) \tag{B.3}$$

Replacing the definition for R_o and noting the wedges are of equal size given by $\theta^n = 2\pi/N$ we find:

$$W_{mn} = \frac{\pi}{NM^2}\frac{\Delta_o^3}{c^2}\left((m+1)^2 - m^2\right)\left(1 - \frac{2}{3}\frac{1}{M}\frac{((m+1)^3 - m^3)}{((m+1)^2 - m^2)}\right) \tag{B.4}$$

As we mentioned above, the power collected from parcel (m, n) is W_{mn} with probability q and it is zero with probability $1 - q$. Therefore, the expected value of energy provided by parcel (m, n) is:

$$E[W_{mn}] = q \cdot W_{mn} + (1 - q) \cdot 0 \tag{B.5}$$

We can now aggregate across parcels and use the linearity of the expected value operator to find:

$$E[W^S] = q \cdot \bar{M}\frac{\pi\Delta_o^3}{3c^2} \tag{B.6}$$

where $\bar{M}$ is a constant given by:

$$\bar{M} = \sum_{m=0}^{M}\frac{((m+1)^2 - m^2)}{M^2}\left(3 - 2\frac{1}{M}\frac{((m+1)^3 - m^3)}{((m+1)^2 - m^2)}\right) \tag{B.7}$$

In this more complicated case very little seems to change. Power density again enters as a cubic as before since now the area of *exploration* rises with the square of the extensive margin and success is proportional to this exploration zone. As well, as mentioned earlier if the number of plays were large a variety of assumptions are available on the joint distribution across the plays that would render a law of large numbers result. The simplest case being the one employed above where each play is treated as an independent and identically distributed Bernoulli random variable.

B.2 Non-Renewables: Oil, Gas and Coal

Extending our framework to non-renewables presents several challenges. First, since using non-renewable energy today precludes you from using it tomorrow the exploitation zone must change over time as the resource stock is depleted. This is true because with non-renewables, energy flows come from depleting the resource stock and not from harvesting the perpetual yield from a renewing resource. One simple and natural way to address depletion is to assume ongoing extractions hollow out the exploitation zone as the resource is extracted.[1]

Second, while we can for the most part ignore the potential impact current energy collection has on the future productivity of renewables (harvesting solar power today does not affect the likelihood of sunshine tomorrow), this is not possible with non-renewables. To see why, use the approach discussed above and assume all energy resources up to r have already been extracted. Then, the remaining non-renewable energy that could be supplied to the core is given by:

$$W^S = 2\pi \int_r^{R^*} (\Delta - cv)\, v dv = \frac{\pi\Delta^3}{3c^2} - \pi\left(\Delta r^2 - \frac{2}{3} c r^3\right) \tag{B.8}$$

where $R^* = \Delta/c$ as before. The intuition is clear. The economy loses the energy it would have been able to collect over the area already mined — this is, $\pi\Delta r^2$—net of the energy it would have expended to bring this energy to the core, $(2/3)\pi c r^3$. Previous extractions raise the cost of current extractions, and the key economic problem is to determine the rate

[1] There is a small literature examining least cost paths for depletion in situations with multiple deposits or resources. This literature, started by Herfindahl (1967), examines when, and under what conditions, a least cost order of extraction path will be optimal. Chakravorty and Kruice (1994) contains relevant references, some discussion, and a neat result showing the typical least cost path prediction does not hold up when the resources in question are not perfect substitutes in use. This possibility is ruled out in our one energy source set up, but would be relevant in any extension with two, less than perfectly substitutable, resource types.

we wish to use these resources over time. To address this problem we will assume a time separable CRRA utility function that maps delivered energy into instantaneous utility flows, and maximize the discounted sum of these flows subject to resource availability and costs.

B.2.1 A Solow-Wan Reformulation

In order to solve our intertemporal energy supply problem we start by recognizing that our spatial model with reserves differentiated by location, can be rewritten as a standard problem where there is a fixed and given resource stock exploited subject to rising marginal extraction costs. A similar reformulation was first suggested by Solow and Wan (1976) in an environment where resources were differentiated by their grade, and it proves useful to do so here.[2]

To reformulate the problem along Solow-Wan lines, we first recognize that the exploitation zone has radius $R^* = \Delta/c$, and this exploitation zone implies a corresponding limit on recoverable reserves which we denote $\bar{X}$. These recoverable reserves are simply equal to $\bar{X} = \pi\Delta^3/c^2$ which again reflects our scaling law. But if the current resource frontier is $r(t) < R^*$, then the remaining recoverable reserves at t, which we denote $X(t)$ must be equal to

$$X(t) = 2\pi \int_{r(t)}^{R^*} \Delta\iota d\iota = \bar{X} - \Delta\pi r(t)^2 \tag{B.9}$$

where $r(0) = 0$ since no resources have been extracted at the start of time. Cumulative extractions at t, are simply $\Delta\pi r(t)^2$.

Differentiating with respect to time, we find the needed link between remaining recoverable reserves and today's rate of extractions:

$$\dot{X} = -2\pi r(t)\Delta\dot{r}(t) = -W(t) \tag{B.10}$$

The intuition is simple. As extraction proceeds, reserves are drawn down and the resource

[2]Solow and Wan (1976) suggested this reformulation in a short footnote; for a more illuminating treatment see section 2 of Swierzbinski and Mendelsohn (1989).

frontier expands. The frontier expands at rate $\dot{r}(t)$ as resources with power density Δ are reaped from a ring with density $2\pi r(t)$ per unit time. The last equality in (B.10) follows because the instantaneous change in the stock must equal $W(t)$ – the flow of energy extracted at t measured in Watts. This completes the first step of the reformulation.

The second step in the reformulation is to find the associated cost function for extractions. When $W(t)$ is extracted, it is divided between deliveries to the core $W^S(t)$ and energy used in transport which we denote $W^T(t)$. We refer to these costs as extraction costs. Because at any t there is a unique $r(t)$, $W^T(t)$ must equal $r(t)[c/\Delta]W(t)$ and hence $r(t)[c/\Delta]$ represents unit extraction costs. While this is useful, we need to eliminate $r(t)$ and purge the problem of all spatial elements. To do so use (B.9) to substitute for $r(t)$ as a function of remaining reserves $X(t)$ and total reserves $\bar{X}$. With some simplification, we can now write the relationship between energy supplied to the core, $W^S(t)$, current extractions, $W(t)$, remaining reserves $X(t)$, and recoverable reserves $\bar{X}$, as:

$$W^S(t) = [1 - C(X(t))]W(t) \tag{B.11}$$

$$C(X) = \left(1 - \frac{X}{\bar{X}}\right)^{1/2} \tag{B.12}$$

where we now interpret $C(X(t))W(t)$ as the cost of extracting $W(t)$ units of energy from a homogenous pool of recoverable reserves $\bar{X}$, when remaining reserves equal X. $C(X(t))$ is therefore the unit extraction cost function (where we have suppressed its reliance on recoverable reserves, $\bar{X}$). With this machinery in place, we solve for the optimal extraction path.

A social planner maximizes the welfare of a representative consumer who values the energy services available for consumption in the core. By choosing service units appropriately, utility is defined over net energy supplied. The planner has a CRRA instantaneous utility function with coefficient of relative risk aversion equal to $\sigma > 0$. Social welfare is:

$$\max_{W(t)} \int_0^\infty e^{-\rho t} U(W^S(t))dt \quad where \quad U(W^S) = \frac{(W^S)^{1-\sigma} - 1}{1-\sigma} \tag{B.13}$$

The planner maximizes (B.13) subject to the constraints (B.10) and (B.11). We write the current value Hamiltonian as

$$\mathscr{H} = U\left[(1 - C(X(t)))\, W(t)\right] - \lambda(t) W(t) \tag{B.14}$$

where $\lambda(t)$ is the co-state variable associated with the stock of resources. The optimality conditions are given by:

$$\frac{\partial \mathscr{H}}{\partial W(t)} = U'(W^S(t))(1 - C(X(t))) - \lambda(t) = 0 \tag{B.15}$$

$$\frac{\partial \mathscr{H}}{\partial X(t)} = -U'(W^S(t))C'(X)W = \rho\lambda(t) - \dot{\lambda}(t) \tag{B.16}$$

with transversality condition $\lim_{t\to\infty} e^{\rho t}\lambda(t)X(t) = 0$.

Using the definition of the utility function in (B.13) and taking the time derivative of (B.15), substituting in (B.16), and rearranging we find one differential equation linking the current rate of extractions to cumulative extractions:

$$\frac{\dot{W}(t)}{W(t)} = -\frac{\rho}{\sigma} - \frac{C'(X)}{1 - C(X)} W(t) \tag{B.17}$$

A second differential equation is provided by (B.10) while one initial condition and the transversality condition close the system.

The behavior of the dynamic system is presented in Figure 2. The $\dot{W}(t) = 0$ isocline is depicted by the solid curve in Figure 2. This curve is positive for values of $X(t) \in [0, \bar{X}]$ and has a maximum value when cumulative extraction is one quarter of total reserves; i.e. with remaining reserves $X(t) = 3\bar{X}/4$ and cumulative extraction $\bar{X} - X(t) = \bar{X}/4$. The

$\dot{X}(t) = 0$ isocline is coincident with the horizontal axis. At all points above the $\dot{X}(t) = 0$ isocline, movement must be rightwards to extract all reserves, giving arrows of motion in the positive direction parallel to the horizontal axis. At points above the $\dot{W}(t) = 0$ isocline, extractions must be increasing since costs are currently too low; below the isocline just the opposite is true. This information is captured by the arrows of motion shown.

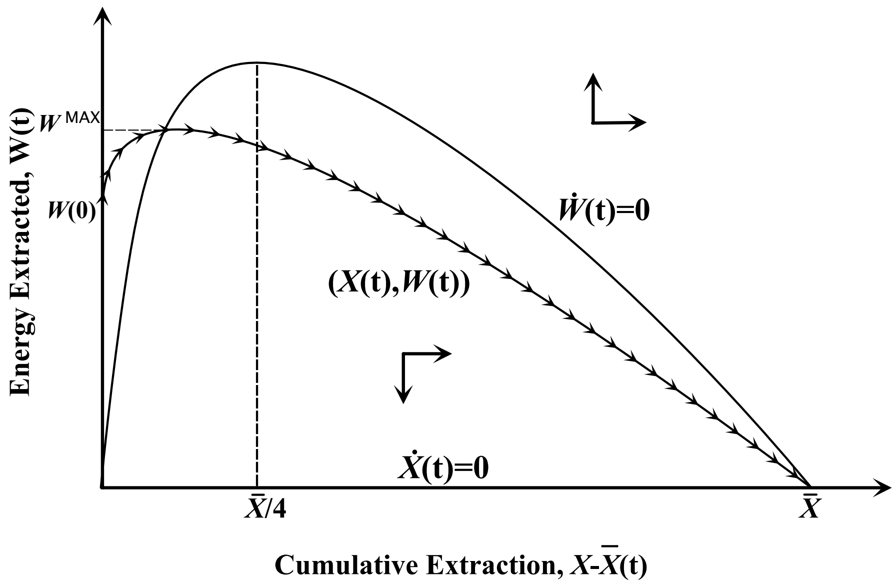


Figure 2: Optimal Extraction Rule

Assume we start with a new resource and hence cumulative extractions are zero. Since the arrows of motion near the origin imply all movement must be upwards and to the right, we know the system must move immediately to an initial extraction point like that shown by $W(0) = W_0$. From this initial point, the arrows of motion indicate we move upwards and to the right and cut the $\dot{W}(t) = 0$ isocline at zero slope. Once we cross this isocline, the arrows of motion tell us the extraction path must turn downwards and the transversality condition requires the path slowly approach $\bar{X}$ on the horizontal axis. Working backwards it is now apparent the transversality condition chooses the initial W_0 and this choice has to feature less extraction than that given by the peak of the $\dot{W}(t)$ isocline.

Somewhat surprisingly, extractions must at first boom and then (optimally) bust. This is true despite the uniform distribution of resources across space; despite the very conventional form of intertemporal utility and absence of demand shocks; and despite the absence of learning by doing, technological change or exploration activity. Moreover, the peak in extractions is greater the more power dense the underlying resource.

At bottom the cause is the scaling law linked to the spatial structure of the model but to understand why this is true, we need to understand the two quite different motivations governing optimal depletion. First, and not surprisingly, there are the standard Hotelling motives arising from the finiteness of the resource stock and the impatience of our planner. For example, if the costs of bringing energy to the core was zero (but there remained a finite resource stock available for use), then the shadow value of the resource *in situ* rises at the rate of time preference. Energy extracted would equal energy supplied to the core and the time profile for extractions would be given by

$$\frac{\dot{W}(t)}{W(t)} = -\frac{\rho}{\sigma} < 0 \tag{B.18}$$

where ρ is the discount rate, and σ the elasticity of marginal utility from the CRRA specification. Since optimality requires the value of marginal utilities discounted to time zero to be equalized across all periods, this is achieved by energy consumption falling at a rate proportional to time preference and the elasticity of marginal utility. This motivation follows from the finiteness of the reserves; it predicts a declining path for extractions; and, it reflects the forces identified in Hotelling's classic work (Hotelling 1931).

Second, and less familiar, are what we could call Ricardian motives. These motives come from the fact that reserves differ in their Ricardian rents: energy resources very proximate to the core have large rents and are very scarce; while very distant ones have very little rent but are abundant. Once we translate this feature of our spatial structure - via the Solow-Wan reformulation - into an implication on extraction costs, it implies that differences in Ricardian rent across reserves are now reflected in extraction costs that rise rapidly with cumulative extraction. Since any unit extracted today raises the cost of all future extractions, all else equal, it pays to shift these extraction costs into later periods. These Ricardian motives argue for a delay in extractions or what is the same, a rising path of extractions over time. Ignoring the Hotelling term given above, an extraction path that reflects only Ricardian considerations is given by

$$\frac{\dot{W}(t)}{W(t)} = -W(t)\frac{C'(X)}{1 - C(X)} > 0 \tag{B.19}$$

Equation (B.17) shows optimal extractions is the simple sum of the right hand sides of (B.18) and (B.19), and hence the interplay of these two forces produce a boom and bust path for energy production.

Descriptively, the result follows because the Ricardian motivations initially dominate Hotelling considerations. Analytically, it follows because at the very first instant of time, energy consumption must be positive $W(0) > 0$, and $C'(X(0)) = -\infty$ implying $\dot{W}(t) > 0$ at least initially. And as extraction proceeds $W(t)$ must approach zero (the resource is finite) and $C'(X(t))/(1 - C(X))$ increases. Therefore, the Ricardian forces fall over time and are eventually dominated by the Hotelling ones.

More deeply, the impact of using up the very first unit of resources on subsequent extraction costs is so costly, $C'(X(0)) = -\infty$, not because these initial energy resources have the greatest rents (which they do) but because they are so scarce in relation to the resource pool whose extraction costs are now raised. Scarcity drives the result and high rent resources are so scarce because of our scaling law. To see why, recall that energy rents fall linearly with distance, but the quantity of reserves rises with the square of distance. This implies low rent resources are abundant, and high rent resources are scarce. Increasing the power density of the energy source raises rents everywhere, but also brings in play new low rent resources at the margin of exploitation. Consequently, the motivation for pushing extractions into the future is strengthened and the peak of extractions rises.

While it is well known that the typical Hotelling's prediction can be overturned in a variety of settings, the boom and bust in extractions is a necessity in our framework and not a possibility.[3] Moreover, it follows from our scaling law which has the dual cost implications reflected in (B.12). The spatial setting provides us with a neat analytical representation for a well known empirical fact: high rent resources are scarce and low rent resources abundant - and then suggests that the logical implication of this fact is that both Ricardian and Hotelling motives drive optimal extractions. Ignoring spatial elements and their attendant impact on rents removes a key force driving optimal extractions; and taking them into account suggests an interesting parallel. Non-renewable resource extraction should be concentrated or bunched in time, just as renewables energy extractions should be bunched across space. We summarize this result in the following proposition.

Proposition B.1 *Assume intertemporal utility is of the CRRA form, then the optimal depletion path has extractions rising to a peak and then declining. Peak extractions are rising in the power density of the energy resource.*

Proof. *In text.* ■

The proof of the second part of Proposition B.1 is as follows. For any $Z = \bar{X} - X$ constant, W^{MAX} in Figure 2 is increasing in $\bar{X}$. Setting $dW/dX = 0$ we find $\frac{\partial W}{\partial \bar{X}} = 2\frac{\rho}{\sigma}Z\frac{(\bar{X}^{1/2}-Z^{1/2})-\frac{1}{2}\bar{X}^{1/2}}{(\bar{X}^{1/2}-Z^{1/2})^2}$. Therefore, $\frac{\partial W}{\partial \bar{X}} > 0$ if $\frac{\bar{X}}{4} > (\bar{X} - X)$ which always holds as the peak in the extraction always occurs to the left of the peak in the $\dot{W} = 0$ locus.

C Endogenous Transport Costs

We solve the energy producer's problem in two stages. In the first stage transportation costs are minimized by choosing how much distance to cover by land and how much distance to cover by river. In the second stage energy rents are maximized. The cost minimization problem is given by:

$$\min_{r_1,r_2} cr_1 + \rho cr_2, \text{ subject to } r^2 = r_1^2 - r_2^2 + 2rr_2\cos\theta \tag{C.1}$$

[3] The first observation that extractions may boom and bust is often credited to Livernois and Uhler (1987).

where r_1 is the distance travelled by land and r_2 is the distance travelled by road. The constraint follows directly from the law of triangles with r_1 being opposite to the angle θ.We can replace the constraint in the objective function to find the optimal distances travelled by land and by road:

$$r_1^* = \frac{\sin\theta}{(1-\rho^2)^{1/2}} * r \text{ and } r_2^* = \cos\theta - \frac{\rho\sin\theta}{(1-\rho^2)^{1/2}} * r. \tag{C.2}$$

If the distance r_2^* is strictly positive, the supplier deviates to the road, otherwise the supplier goes straight to the core. We can solve for the critical value of θ that separates the suppliers that go straight to the core from those who deviate to the road:

$$r_2^* > 0 \text{ if and only if } \theta \leq \cos^{-1}\rho \equiv \bar{\theta} \tag{C.3}$$

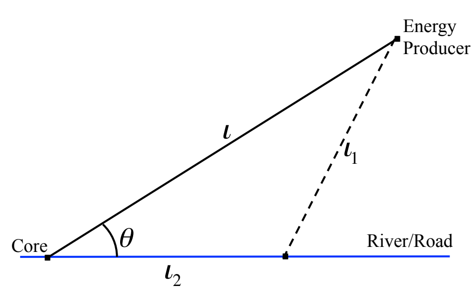
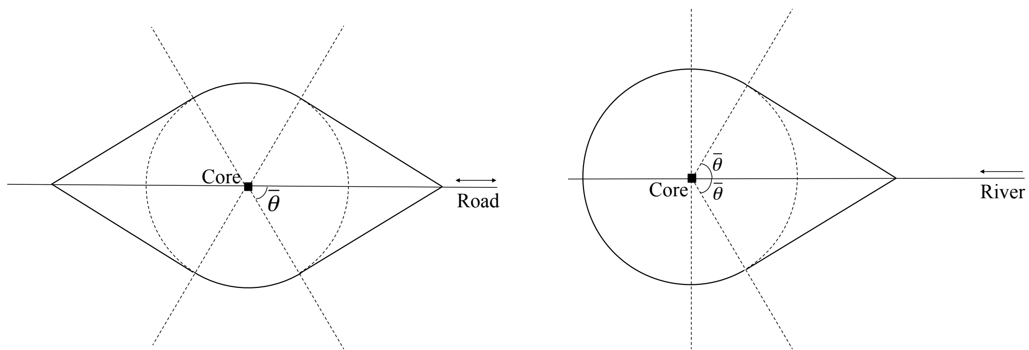


Figure 3: Transport via Road or River

Energy suppliers located at any angle $\theta < \bar{\theta}$ are "close" to the low cost alternative and choose to use it. Since $\rho = \cos(\bar{\theta})$, we know that as $\rho \to 0$ everyone deviates, since it is so cost effective. Alternatively, as $\rho \to 1$, the road offers no advantage and no one uses it.

The second part of the energy producer's problem is to decide whether or not to take its energy to the core. An energy producer situated a distance r from the core and forming an angle θ with the road will go to the core if the net energy supplied to the core is positive; i.e., if there are positive energy rents at this location. Energy supplied by this producer is given by $W^S = \Delta - c(r_1^* + \rho r_2^*)$ Replacing equations (C.2) in the previous equation makes energy rents a function we find $W^S = \Delta - c(\theta)r$ where $c(\theta)$ given by (**??**).

We depict the exploitation zone in the river and road case in the two panels of Figure 4 assuming $\rho < 1$.



(a) Area of Exploitation with a Road (b) Area of Exploitation with a River

Figure 4: Rivers and Roads

Just as before, the total energy supplied to the core is found by "adding up" all energy rents.

$$W^S = 4 \times \left[\int_0^{\bar{\theta}} \int_0^{r^*} v\left(\Delta - c(\theta)v\right) dv d\theta + \int_{\bar{\theta}}^{\pi/2} \int_0^{r^*} v\left(\Delta - cv\right) dv d\theta \right]$$

The first integral represents energy coming from suppliers who are close enough to the road to use it in transport. The second integral represents the energy coming from those who travel directly to the core. We have multiplied the integrals by 4 since we are adding up over the quarter circles of $\pi/2$ radians.

Integrating and simplifying gives us a net energy supply much like that we had before:

$$W^S = \frac{1}{3} \frac{\Delta^3}{c^2} g(\rho) \tag{C.4}$$

$$g(\rho) = \pi + 2(\tan(\bar{\theta}) - \bar{\theta}) \geqslant 0 \tag{C.5}$$

where the function $g(\rho)$ is positive and monotonic $g'(\rho) < 0$, approaches infinity as ρ goes to zero and approaches π as ρ goes to 1.

References for Online Appendix

Chakravorty, U. and D.L. Kruice. 1994.“Heterogenous Demand and Order of Resource Extraction.” *Econometrica*, 62(6): 1445-1452.

Gipe P. 2004. “Wind Power: Renewable Energy for Home, Farm, and Business.” Chelsea Green Publishing Company ISBN: 978-1-931498-14-2.

Herfindahl, O.C. 1967. “Depletion in Economic Theory.” In *Extractive Resources and Taxation,* edited by Mason Gaffney, 63-90. Madison, Wisconsin: University of Wisconsin Press.

Hotelling, H. “The Economics of Exhaustible Resources.” 1931. *The Journal of Political Economy,* 39(2): 137-175.

Livernois JR and R Uhler. 1987.“Extraction Costs and the Economics of Nonrenewable Resources” *Journal of Political Economy* 95(1): 195-203

Solow, R.M. and F.Y. Wan. 1976. “Extraction Costs in the Theory of Exhaustible Resources.” *The Bell Journal of Economics*, 7(2): 359-370.

Swierzbinski, J.E. and Mendelsohn, R. 1989. “Exploration and Exhaustible Resources: The Microfoundations of Aggregate Models.” *International Economic Review*, 30(1): 175-186.