Online Appendix for "Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy"

(Not for Publication)

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This online appendix consists of Section A, "Empirical Appendix", and Section B, "Model Appendix."

A   Empirical Appendix

A.1   Currency names and codes

Table A.1 lists the country name, currency name, and the three-letter currency code for our sample countries.

Table A.1: Currency names and codes

Developed markets			Emerging markets		
Country	Currency	Code	Country	Currency	Code
Australia	Australian dollar	AUD	Brazil	Brazilian real	BRL
Canada	Canadian dollar	CAD	Chile	Chilean peso	CLP
Denmark	Danish krone	DKK	Colombia	Colombian peso	COP
Germany	Euro	EUR	Czech Republic	Czech koruna	CZK
Japan	Japanese yen	JPY	Hungary	Hungarian forint	HUF
New Zealand	New Zealand dollar	NZD	Indonesia	Indonesian rupiah	IDR
Norway	Norwegian krone	NOK	Israel	Israeli shekel	ILS
Sweden	Swedish krona	SEK	Malaysia	Malaysian ringgit	MYR
Switzerland	Swiss franc	CHF	Mexico	Mexican peso	MXN
United Kingdom	British pound	GBP	Peru	Peruvian nuevo sol	PEN
United States	US dollar	USD	Philippines	Philippine peso	PHP
			Poland	Polish zloty	PLN
			Singapore	Singapore dollar	SGD
			South Africa	South African rand	ZAR
			South Korea	South Korean won	KRW
			Thailand	Thai baht	THB
			Turkey	Turkish lira	TRY

A.2   Comparing External Debt Sources

Figure A.1: External LC Debt Share in Global Mutual Funds and US TIC, 2015
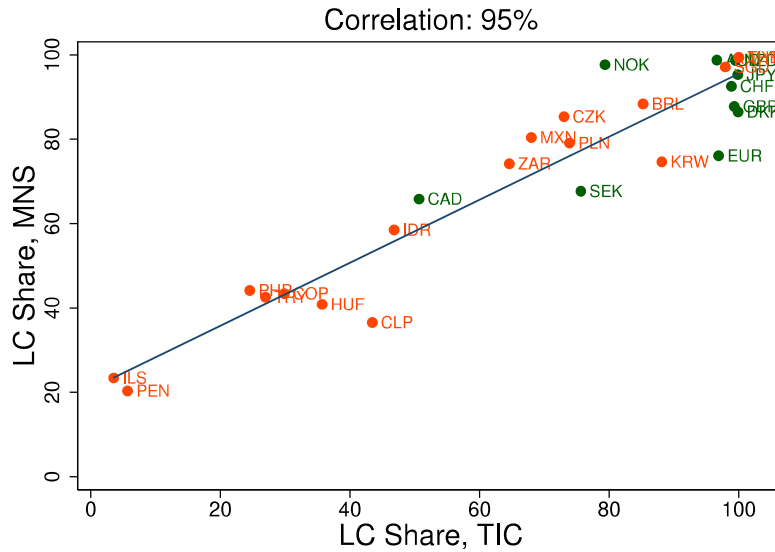
Note: This figure plots the percentage of external government debt denominated in each country's local currency using data from global mutual funds in Maggiori et al. (2019) (MNS) and the US external position in the Treasury International Capital (TIC) data. MNS data uses data for the entire European Monetary Union (EMU). TIC data uses Germany for the Euro area. All data are for end of year 2015.

A.3 Standard errors and t-statistics of LC bond-stock betas

Table A.2 reports regression estimates for the LC bond-local stock betas by country, $\beta(bond_i, stock_i)$. It shows the point estimates from estimating Eqn. (1) for each country, together with Hansen-Hodrick and bootstrap standard errors. Both asymptotic Hansen-Hodrick standard errors and bootstrap standard errors account for serial correlation due to using overlapping return observations. For details of the bootstrap standard errors see Appendix A.6. Table A.3 presents summary statistics for the t-statistic of three regression betas: LC bond-local stock beta, $\beta(bond_i, stock_i)$, local stock-US stock return beta, $\beta(stock_i, stock_{US})$, and realized inflation-output beta, $\beta(\pi_i, IP_i)$. The betas based on daily overlapping bond and stock returns, $\beta(bond_i, stock_i)$ and $\beta(stock_i, stock_{US})$, are more precisely estimated than the betas based on monthly macroeconomic data, $\beta(\pi_i, IP_i)$. For the majority of countries $\beta(bond_i, stock_i)$ is statistically different from zero and $\beta(\pi_i, IP_i)$ is statistically different from zero for 11 out of 28 countries.

Table A.2: Regression estimates of LC bond-local stock beta

	Beta	Hansen-Hodrick SE	Bootstrap SE	N	R^2
AUD	-0.185***	(0.0356)	(0.0365)	2,453	0.406
BRL	0.149**	(0.0591)	(0.0536)	2,044	0.117
CAD	-0.0936***	(0.0211)	(0.0254)	2,421	0.207
CHF	-0.0757***	(0.0243)	(0.0244)	2,421	0.185
CLP	-0.0448	(0.0375)	(0.0382)	2,189	0.025
COP	0.166***	(0.0444)	(0.0434)	2,291	0.215
CZK	0.00405	(0.0171)	(0.0193)	2,420	0.001
DKK	-0.0581***	(0.0185)	(0.0213)	2,395	0.108
EUR	-0.107***	(0.0195)	(0.0199)	2,484	0.302
GBP	-0.120***	(0.0265)	(0.0299)	2,455	0.175
HUF	0.218***	(0.0257)	(0.0301)	2,393	0.387
IDR	0.292***	(0.0361)	(0.0489)	2,291	0.340
ILS	-0.0367	(0.0195)	(0.0301)	1,701	0.033
JPY	-0.0303***	(0.00684)	(0.0072)	2,315	0.185
KRW	-0.0612**	(0.0245)	(0.0247)	2,361	0.102
MXN	0.0601**	(0.0235)	(0.0276)	2,428	0.048
MYR	0.00862	(0.0280)	(0.0292)	2,338	0.002
NOK	-0.0745***	(0.00992)	(0.0128)	2,422	0.268
NZD	-0.0894**	(0.0360)	(0.0340)	2,427	0.080
PEN	0.0869***	(0.0157)	(0.0192)	2,124	0.267
PHP	0.157**	(0.0615)	(0.0587)	2,289	0.138
PLN	0.00857	(0.0151)	(0.0165)	2,402	0.002
SEK	-0.101***	(0.0269)	(0.0275)	2,423	0.220
SGD	-0.0390*	(0.0173)	(0.0209)	2,423	0.071
THB	-0.0696**	(0.0302)	(0.0344)	2,283	0.088
TRY	0.316***	(0.0713)	(0.0694)	2,248	0.296
USD	-0.134***	(0.0170)	(0.0344)	2,427	0.269
ZAR	-0.0478	(0.0366)	(0.0428)	2,394	0.021

Note: This table shows the regression estimates of the LC bond-local stock beta, $\beta(bond_i, stock_i)$, based on Eqn. (1) by country. The regressions are estimated using daily observations on overlapping one-quarter holding returns from 2005 to 2014. Hansen-Hodrick standard errors are used with 120-day lags to adjust for overlapping holding periods of returns. Bootstrap standard errors are computed as the standard deviation of bond-stock betas estimated on bootstrapped data, $\hat{\beta}^{boot}(bond_i, stock_i)$, where the standard deviation is taken across 500 independent bootstraps. The bootstrap procedure adjusts for serial correlation and heteroskedasticity in bond and stock returns and is described in detail in Appendix A.6. Statistical significance is based on the larger standard error between Hansen-Hodrick and bootstrap standard errors, with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.

Table A.3: Summary Statistics of the t-statistic for Various Betas

| | (1) | (2) | (3) | (4) | (5) | (6) |
	Mean	Median	Min	Max	# of countries with $|t| > 1.96$	N
Panel (A) Developed Markets						
$\beta(bond_i, stock_i)$	4.72	4.44	2.48	7.87	11	11
$\beta(stock_i, stock_{US})$	12.94	12.95	8.94	22.25	10	10
$\beta(\pi_i, IP_i)$	3.66	2.90	0.59	9.65	6	11
Panel (B) Emerging Markets						
$\beta(bond_i, stock_i)$	2.97	2.50	0.24	8.47	11	17
$\beta(stock_i, stock_{US})$	9.50	9.71	5.70	13.83	17	17
$\beta(\pi_i, IP_i)$	1.91	1.45	0.38	4.49	5	17
Panel (C) Full Sample						
$\beta(bond_i, stock_i)$	3.66	3.13	0.24	8.47	22	28
$\beta(stock_i, stock_{US})$	10.77	9.98	5.70	22.25	27	27
$\beta(\pi_i, IP_i)$	2.74	2.20	0.38	9.65	11	28

Note: This table presents summary statistics for the absolute value of the t-statistic of three regression betas: LC bond-local stock beta, $\beta(bond_i, stock_i)$, local stock-US stock return beta, $\beta(stock_i, stock_{US})$, and realized inflation-output beta, $\beta(\pi_i, IP_i)$. Hansen-Hodrick standard errors with 120-day lags are used for $\beta(bond_i, stock_i)$ and $\beta(stock_i, stock_{US})$, which are estimated using daily regressions. Hansen-Hodrick standard errors with 12-month lags are used for $\beta(\pi_i, IP_i)$, which is estimated using monthly regressions. We do not report the t-statistic of $\beta(stock_i, stock_{US})$ for $i = US$ because it is equal to 1 by definition. Panel (A) shows results for developed markets. Panel (B) shows results for emerging markets. Panel (C) shows results for the full sample.

A.4 Robustness Checks for the Main Empirical Results

A.4.1 Long-Term Debt

The cross-sectional relationship between LC bond-stock betas and LC debt shares is robust to measuring the LC debt share only in long-term debt, as shown in Figure A.2. We obtain face values and issuance dates for all historical individual sovereign bond issuances from Bloomberg for 14 emerging markets and estimate the long-term LC debt share as the outstanding amount of LC debt with five or more years remaining to maturity relative to all outstanding debt with five or more years remaining to maturity.

Figure A.2: LC Debt Share in Long-Term Debt versus Bond-Stock Beta
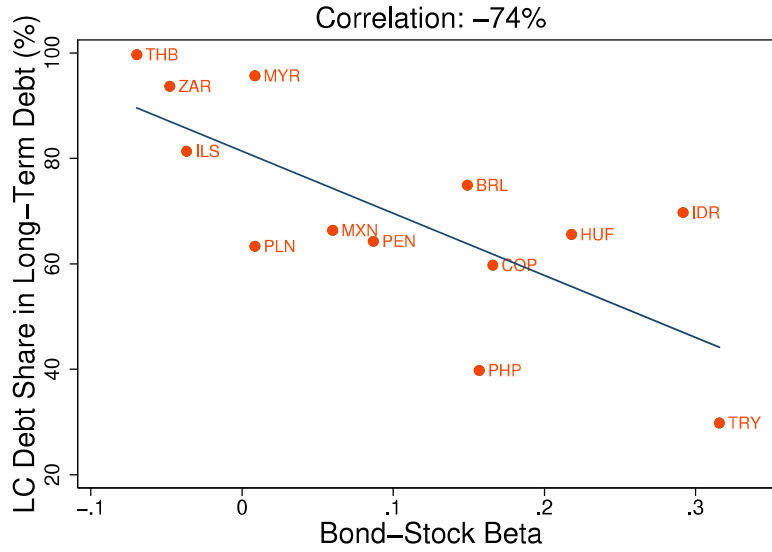
Note: This figure plots the bond-stock beta on the x-axis and the share of LC debt in all outstanding long-term debt on the y-axis. Long-term debt is defined as having a remaining time to maturity of five or more years. The share of LC debt in long-term debt is estimated from individual bond issuance data from Bloomberg.

A.4.2 Excluding the Financial Crisis

One important period in the middle of our sample is the financial crisis of 2008—2009. While this period marked an important recession for the US and many other countries, we show in this section that our main empirical results are not driven by the financial crisis. Figure A.3 shows our baseline LC bond-stock beta on the y-axis against a LC bond-stock beta excluding the financial crisis period on the x-axis. We see that the bond-stock betas are extremely similar when excluding the financial crisis, indicating that our key bond cyclicality measure is not driven by a small number of observations. Figure A.4 shows that our main stylized fact in Figure 2 remains unchanged if we exclude the crisis period in our construction of LC bond betas.

Figure A.3: Local Currency Bond Betas Excluding 2008—2009
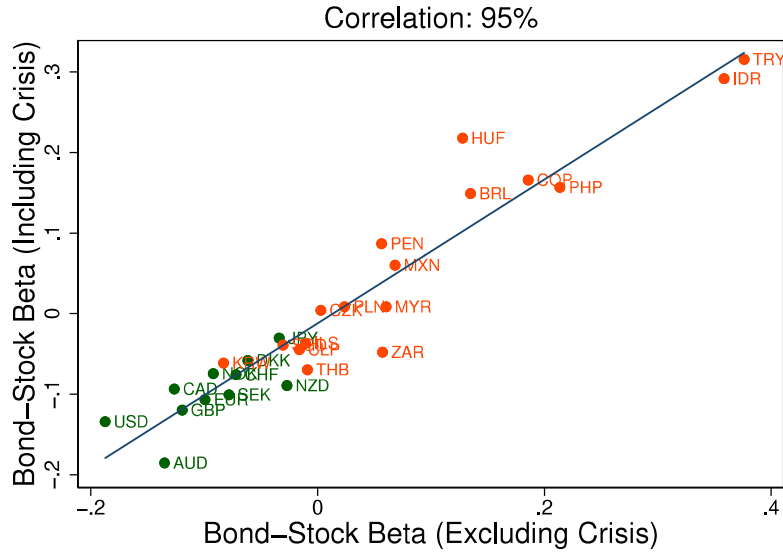
Note: This figure shows LC bond-stock betas excluding the period 2008–2009 on the x-axis and LC bond-stock betas for the full sample (including 2008—2009) on the y-axis.

Figure A.4: Local Currency Debt Shares and Bond Betas Excluding 2008—2009
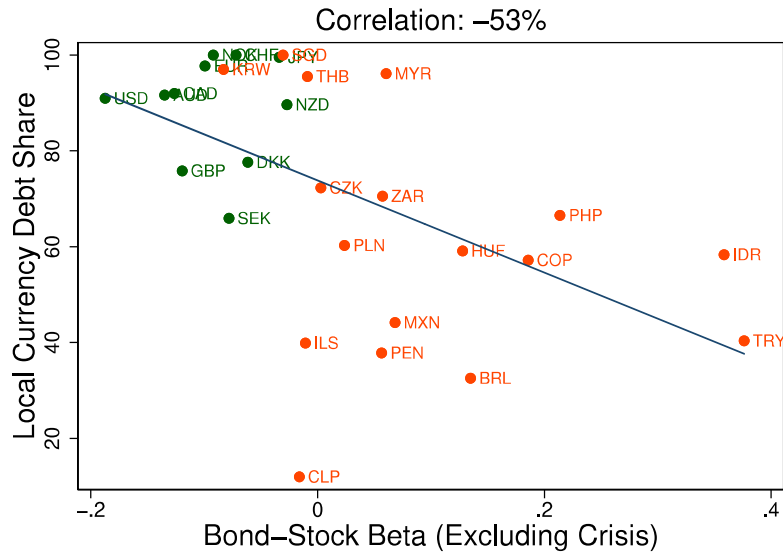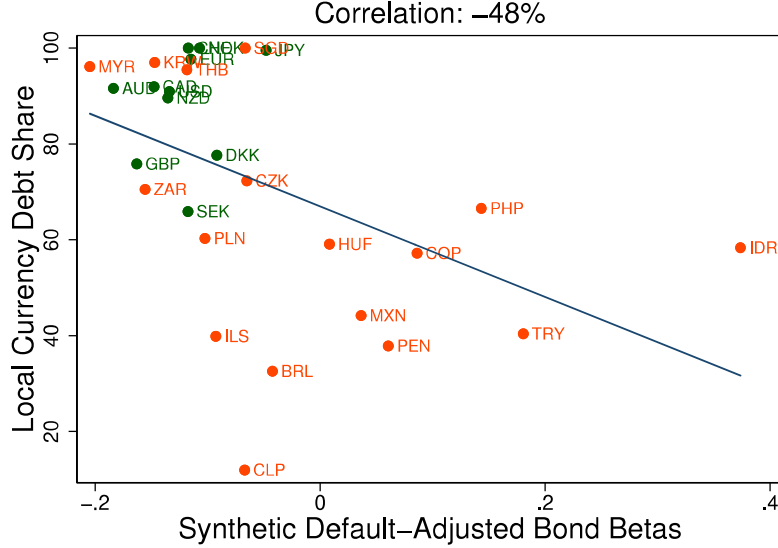

Note: This figure differs from Figure 2 only in that it excludes 2008—2009 from the computation of LC bond betas on the x-axis.

A.4.3 Default-Adjusted Bond Risk Premia

To adjust for default risk, we construct a synthetic default-free nominal bond yield. We follow Du and Schreger (2016a) by combining a US Treasury bond with a fixed-for-fixed cross-currency

swap to create a synthetic default-free local bond. Figure A.5 plots the LC debt share against default-adjusted bond-stock betas, which are computed by replacing LC bond yields by synthetic default-free LC bond yields in the computation of LC bond returns. The strong similarity to Figure 2 shows that our main empirical finding is robust to adjusting for the default component of LC bond returns.

Figure A.5: Local Currency Debt Shares and Default-Adjusted Bond Betas

Note: This figure differs from Figure 2 only in that it uses synthetic default-adjusted LC bond log excess returns in Eqn. (1) to estimate bond-stock betas.

A.4.4 Adjusting for FX hedging errors

In Section 2.1.1, we calculated the LC bond excess return over the local T-bill rate in local currency units. We discussed that from the dollar investor's perspective, these excess returns approximately hedge the LC fluctuation against the US dollar for the holding period between quarter t and $t+1$. In this section, we re-calculate the bond-stock beta after adjusting for these FX hedging errors for the USD investor.

In particular, suppose that the USD investor invests \$1 in the LC bond at t and funds the position by shorting \$1 of the LC T-bill. At $t+1$, the gross USD return on the LC bond is

$$\frac{P^{LC}_{i,n-1,t+1}}{P^{LC}_{i,n,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \exp[\tau_{i,n,t}y^{LC}_{i,n,t} - (\tau_{i,n,t}-1)y^{LC}_{i,n-1,t+1}]\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}},$$

where $P^{LC}_{i,n,t}$ denotes the price of the n-quarter LC bond at time t in country i, and $\mathcal{E}_{i,t}$ denotes the LC exchange rate defined as USD per LC units, so an increase in $\mathcal{E}_{i,t}$ corresponds to a LC appreciation against the USD. Recall that $\tau_{i,n,t}$ is equal to 5 years. The USD cost of shorting the LC T-bill from time t to time $t+1$ is:

$$\frac{1}{P^{LC}_{i,1,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \exp(-y^{LC}_{i,1,t}/4)\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}.$$

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So the exact USD excess return of going long the LC bond and shorting the LC T-bill becomes:

$$\tilde{x}r^{LC}_{i,n,t+1} = \frac{P^{LC}_{i,n-1,t+1}}{P^{LC}_{i,n,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} - \frac{1}{P^{LC}_{i,1,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}\left[\exp[\tau_{i,n,t}y^{LC}_{i,n,t} - (\tau_{i,n,t}-1)y^{LC}_{i,n-1,t+1}] - \exp(-y^{LC}_{i,1,t}/4)\right].$$

Similarly, for a USD investor, the USD excess return of going long in the LC equity and shorting the LC T-bill is:

$$\tilde{x}r^{m}_{i,t+1} = \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}[P^{m}_{t+1}/P^{m}_t - \exp(-y^{LC}_{i,1,t}/4)].$$

We estimate the bond-stock betas adjusted for FX hedging errors by running the regression:

$$\tilde{x}r^{LC}_{i,n,t} = a_i + \tilde{\beta}(bond_i, stock_i) \times \tilde{x}r^{m}_{i,t} + \epsilon_{i,t}.$$

Figure A.6 shows that adjusting these FX hedging errors has no effect on the estimated bond-stock betas. The correlation between the bond-stock beta in local currency units (y-axis) and the bond-stock beta after adjusting for the FX hedging errors (x-axis) is 99.8%.

Figure A.6: Bond-Stock Beta Adjusting for FX Hedging Errors
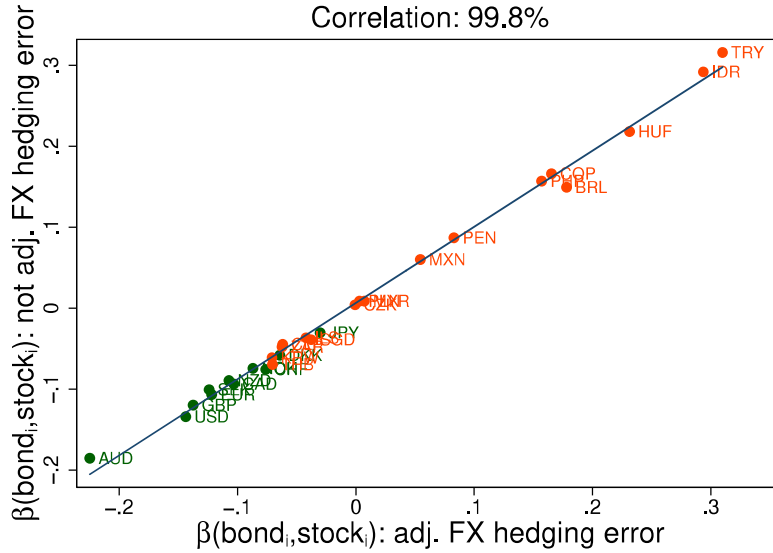
Note: On the horizontal axis, we plot the bond-stock beta using the bond and stock dollar excess returns after adjusting FX hedging errors, as described in Section A.4.4. On the vertical axis, we plot our baseline bond-stock beta in local currency units.

A.4.5 Controlling for Debt/GDP ratios and using LC Debt/GDP ratio as the dependent variable

We first add the Debt/GDP ratio as an additional control into our benchmark regression. The regression results are shown in Table A.4. The coefficient on $\beta(bond_i, stock_i)$ remains significant and similar in magnitude compared to the benchmark regression results in Tables 3 and 4.

We then repeat our benchmark empirical specification using the LC debt/GDP ratio as the dependent variable. Table A.5 presents the regression results. In column (1), we regress the total LC debt/GDP ratio on the bond-stock beta, after controlling the local-US stock return beta,

$\beta(stock_i, stock_{US})$, log GDP, the FX regime, the commodity share, and the total debt/GDP ratio. The coefficient on $\beta(bond_i, stock_i)$ is negative and significant. In columns (2)-(4), we regress the external LC debt/GDP ratio estimated from the TIC, Morningstar, and the BIS data, respectively, on the bond-stock beta, the same set of macroeconomic controls, and the external debt/GDP ratios. The coefficients on $\beta(bond_i, stock_i)$ remain negative and significant.

Table A.4: Controlling for Debt/GDP Ratios

	(1) s^{TOT}	(2) s^{TIC}	(3) s^{MNS}	(4) s^{BIS}
$\beta(bond_i, stock_i)$	-99.97**	-119.2**	-92.08**	-149.0**
	(35.17)	(49.47)	(33.38)	(58.82)
$\beta(stock_i, stock_{US})$	5.018	-7.431	8.524	-3.487
	(24.97)	(21.64)	(22.73)	(30.82)
log(GDP)	1.635	3.698	2.379	-4.400
	(4.674)	(5.843)	(4.208)	(7.821)
FX Regime	-0.560	3.554	4.387	-0.815
	(3.557)	(3.328)	(3.716)	(8.700)
Commodity Share	-0.230	-0.441**	-0.350	-0.391
	(0.265)	(0.205)	(0.226)	(0.282)
Total government debt/GDP	0.140			
	(0.0878)			
External government debt/GDP		0.394	-0.474	-0.177
		(0.334)	(0.318)	(0.665)
Constant	53.11	36.56	46.29	114.5
	(53.42)	(58.60)	(51.50)	(66.37)
Observations	26	26	26	26
R-squared	0.467	0.652	0.530	0.301

Note: This table shows the cross-country regression results of the LC debt shares on measures of bond-stock betas and other macroeconomic controls. In column (1), the dependent variable is the share of LC in total government debt. In column (2), the dependent variable is the LC debt share estimated using the TIC data. In column (3), the dependent variable is the LC debt share estimated using the Morningstar data. In column (4), the dependent variable is the LC debt share estimated using the BIS Locational Banking Statistics. The independent variables are the bond-stock betas $\beta(bond_i, stock_i)$ and the local stock-US stock betas $\beta(stock_i, stock_{US})$. We control for the total Debt/GDP ratio in column (1) and the external Debt/GDP ratio in columns (2) through (4). We also control for average log per capita GDP between 2005 and 2014, the average exchange rate classification used in Reinhart and Rogoff (2004), and the commodity share of exports. The commodity share of exports is defined as the sum of "Ores and Metals" and "Fuel" exports as a percentage of total merchandise exports from World Bank World Development Indicators. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.

Table A.5: LC Debt/GDP ratios and LC Bond Cyclicality

	(1) LC debt/GDP Total	(2) LC debt/GDP External-TIC	(3) LC debt/GDP External-MNS	(4) LC debt/GDP External-BIS
$\beta(bond_i, stock_i)$	-51.30***	-16.37***	-13.85**	-25.57*
	(17.42)	(5.569)	(6.594)	(13.33)
$\beta(stock_i, stock_{US})$	-3.296	-3.661	-0.607	-3.638
	(5.788)	(2.656)	(3.390)	(3.672)
log(GDP)	-0.404	0.991	0.789	-0.439
	(1.771)	(0.796)	(0.705)	(1.157)
FX Regime	-0.972	0.166	0.453	-1.254
	(1.129)	(0.648)	(0.873)	(2.037)
Commodity Share	0.0276	-0.0432*	-0.0201	-0.0248
	(0.0619)	(0.0242)	(0.0359)	(0.0363)
Total government debt/GDP	1.007***			
	(0.0767)			
External government debt/GDP		0.835***	0.616***	0.537**
		(0.0715)	(0.118)	(0.217)
Constant	-1.508	-7.165	-6.277	11.77
	(17.03)	(8.212)	(8.332)	(11.24)
Observations	26	26	26	26
R-squared	0.943	0.916	0.802	0.529

Note: This table shows the cross-country regression results of the LC debt/GDP ratios on bond-stock betas and other macroeconomic controls. In column (1), the dependent variable is the total LC debt/GDP ratio, including domestic and external government debt. In column (2), the dependent variable is the LC external/GDP ratio, estimated using the TIC data. In column (3), the dependent variable is the LC debt/GDP ratio, estimated using the Morningstar data. In column (4), the dependent variable is the LC debt/GDP ratio, estimated using the BIS Locational Banking Statistics. The independent variables are the bond-stock beta $\beta(bond_i, stock_i)$ and the local stock-US stock beta $\beta(stock_i, stock_{US})$. We control for the total Debt/GDP ratio in column (1) and the external Debt/GDP ratio in columns (2) through (4). We also control for average log per capita GDP between 2005 and 2014, the average exchange rate classification used in Reinhart and Rogoff (2004), and the commodity share of exports. The commodity share of exports is defined as the sum of "Ores and Metals" and "Fuel" exports as a percentage of total merchandise exports from World Bank World Development Indicators. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.

A.4.6 Weight the benchmark regression by per capita GDP

We show in Table A.6 that weighting the benchmark regression presented in Table 3 by per capita GDP does not change the results.

Table A.6: LC Debt Shares in Total Government Debt onto LC Bond Cyclicality (Weighted by per capita GDP)

Local Currency Debt Share	(1) s^{TOT}	(2) s^{TOT}	(3) s^{TOT}	(4) s^{TOT}
$\beta(bond_i, stock_i)$	-110.1***			-92.24**
	(19.53)			(36.40)
$\beta(\tilde{\pi}, \widetilde{gdp})$		69.74***		
		(15.58)		
$\beta(\pi, IP)$			74.20**	
			(34.85)	
$\beta(stock_i, stock_{US})$				13.06
				(24.19)
log(GDP)				4.029
				(4.741)
FX Regime				0.643
				(3.803)
Commodity Share				-0.198
				(0.244)
Constant	72.86***	53.07***	72.97***	23.74
	(3.935)	(6.817)	(4.594)	(53.73)
Observations	28	22	28	28
R-squared	0.309	0.335	0.063	0.384

Note: This table differs from Table 3 in that observations are weighted by per capital GDP. Robust standard errors are used in all regressions with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.

A.4.7 Larger sample using the inflation-GDP beta

Our main sample in the paper is constrained by the availability of long-term LC bond yields to estimate bond-stock betas. We can extend our sample to over 100 countries by measuring the realized inflation-GDP beta. To obtain standardized data across as many countries as possible, we use the inflation and GDP data from the World Bank World Development Indicator (WDI), which are available at the annual frequency. In order to obtain more precise estimates, we use a longer sample from 1980 to 2017. We require at least 20 observations for a country to be included in the sample, which leaves us with 107 sample countries.

Similar to the realized inflation beta with respect to industrial production as estimated by Eqn. (3), we can estimate the realized inflation beta with respect to GDP by running the following regression.

$$\Delta \pi_{i,t} = a_i + \beta(\pi_i, GDP_i)\Delta GDP_{i,t} + \epsilon_t, \tag{A.1}$$

where $\Delta \pi_{i,t}$ is the yearly change in the year-over-year inflation rate, and $\Delta GDP_{i,t}$ is the annual change in the GDP growth rate. The coefficient $\beta(\pi_i, GDP_i)$ measures the realized inflation cyclicality with respect to GDP for country i. Before estimating the inflation-output relation for each

country, we winsorize the top and bottom 1 percent of the inflation rate and the GDP growth rate across countries to remove extreme outliers. Having estimated inflation-output betas for each country, we do not do any further winsorization.

Figure A.7 is a binscatter plot showing a positive relationship between the LC debt share in external debt based on the TIC data and the realized inflation-GDP beta. Regression results are reported in Table A.7. We use bootstrap standard errors that account for correlated estimation error in $\beta\left(\pi_i, GDP_i\right)$, as described in Appendix A.6. We can see that the coefficient on the realized inflation-GDP is positive and significant. However, the magnitude of the coefficient is notably smaller compared to the regression coefficients in Table 3, likely reflecting the fact that the inflation-GDP beta is less precisely estimated than our main cyclicality measures used in the paper, leading to classical measurement error and attenuation bias.

Figure A.7: Binscatter plot of LC debt share vs. realized inflation-GDP beta
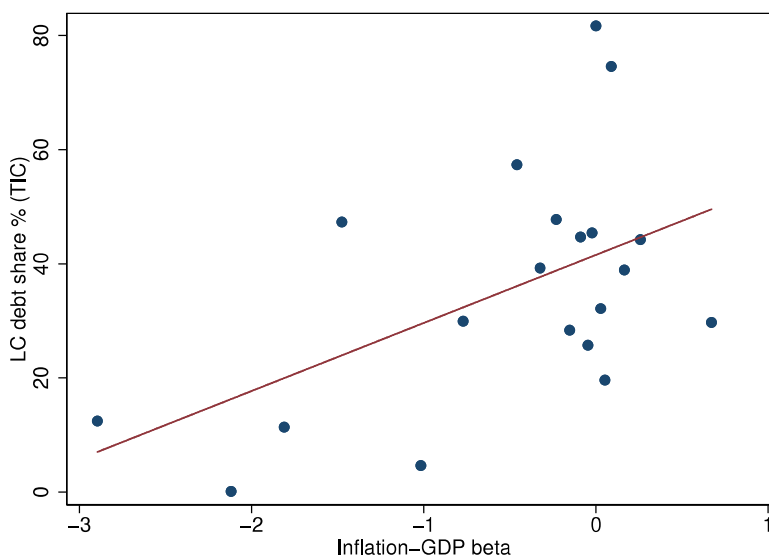
Note: On the horizontal axis, we plot the realized inflation-GDP beta. On the vertical axis, we plot the share of LC debt in total external debt, measured by TIC. The binscatter is plotted with 20 bins.

Table A.7: LC Debt Shares in External Debt and Realized Inflation-GDP beta

Local Currency Debt Share	(1) s^{TIC}	(2) s^{TIC}
$\beta(\pi_i, GDP_i)$	11.92***	8.70***
	(3.00)	(3.20)
log(GDP)		14.03***
		(2.76)
FX Regime		-0.29**
		(0.14)
Commodity Share		7.24**
		(3.10)
Constant	41.56***	-93.79***
	(4.69)	(27.16)
Observations	107	107
R-squared	0.08	0.36

Note: This table shows the regression results of the LC debt share in external debt based on TIC on the realized inflation-GDP beta. Column (1) shows the univariate specification without controls. Column (2) shows the specification with controls. Standard errors used in all regressions are the larger ones between Huber-White robust standard errors and bootstrap standard errors. We use a wild bootstrap (Davison and Hinkley (1997)) to account for heteroskedasticity in regression residual. We account for estimation error in $\beta(\pi_i, GDP_i)$ by bootstrapping inflation and GDP with a moving block bootstrap (Maddala (2001)) with lag length 4 years taking into account cross-country correlations. For details of the bootstrap procedure see Appendix A.6. Significance levels indicated by *** p<0.01, ** p<0.05, * p<0.1.

A.4.8 Time-varying betas and LC debt shares

We next show that our results are stable across different time periods. To start, we show that individual countries' bond-stock betas are stable over time. We estimate time-varying LC bond-stock betas, $\beta_t(bond_i, stock_i)$, using five-year rolling windows between $t-5$ and t. Panel (A) of Figure A.8 shows the average bond-stock beta for developed and emerging markets. The average beta for developed countries fluctuated between -0.15 and 0, and the average beta for emerging market fluctuated between 0 and 0.1. Panel (B) of Figure A.8 plots the cross-country rankings of the bond-stock betas between 2008 and 2014. We can see that the cross-sectional ranking is very persistent. The average pairwise rank correlation between 2008 and 2014 is 92%.

We next run the cross-sectional regressions of the LC debt share at time t on $\beta_t(bond_i, stock_i)$ for every year in our sample. The regression results are shown in Table A.8. The coefficient on $\beta(bond_i, stock_i)_t$ is negative and statistically significant for all sample years.

Figure A.8: Time variations in the bond-stock beta

(A) Rolling bond-stock betas
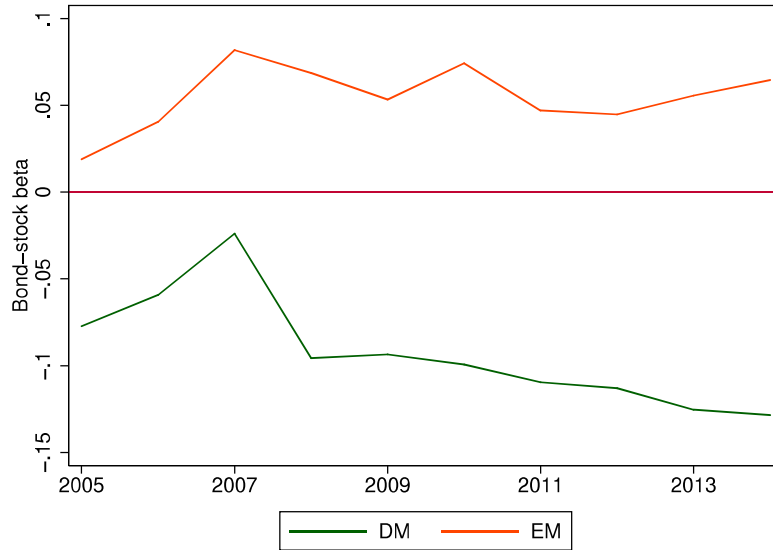
(B) Ranking of rolling betas
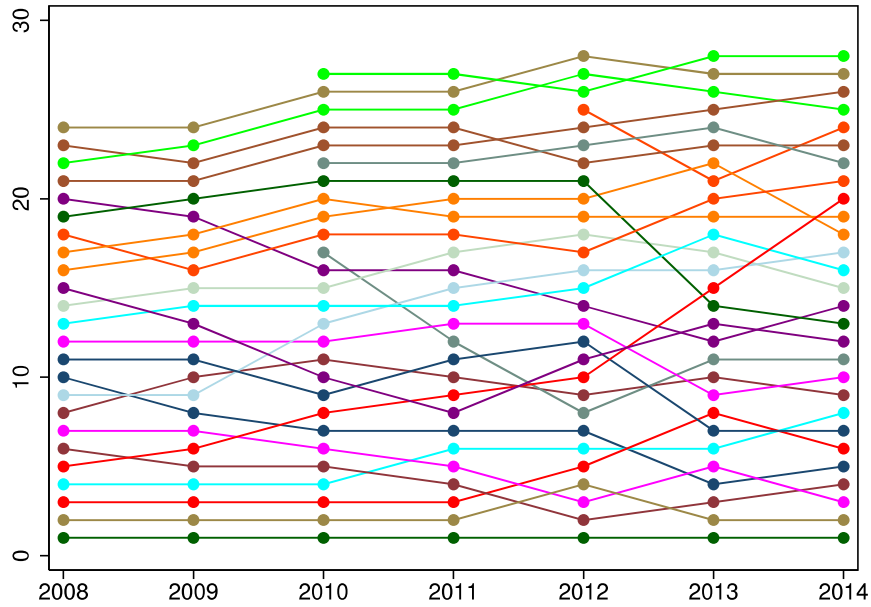
Note: Panel (A) plots the average rolling LC bond-local stock beta over time for developed markets (G10) and emerging markets (EM). The bond-stock beta at time t is calculated using a five-year rolling window between $t-5$ and t. Panel (B) plots the cross-country ranking of the five-year rolling bond-stock betas over time, with each color indicating a sample country.

Table A.8: Regression of the LC debt share by year

	(1) 2005	(2) 2006	(3) 2007	(4) 2008	(5) 2009	(6) 2010	(7) 2011	(8) 2012	(9) 2013	(10) 2014
$\beta_t(bond_i, stock_i)$	-124.1***	-115.4***	-123.4***	-95.46***	-96.42***	-96.70***	-105.6***	-90.73***	-74.53***	-65.63***
	(22.25)	(20.93)	(33.12)	(13.86)	(15.37)	(17.91)	(22.87)	(23.10)	(14.62)	(18.24)
Constant	79.00***	81.45***	85.50***	78.24***	77.82***	73.98***	70.58***	71.04***	71.34***	71.18***
	(2.627)	(2.628)	(2.762)	(3.213)	(3.217)	(3.959)	(4.012)	(4.064)	(4.143)	(4.262)
Observations	20	20	22	24	24	26	27	27	27	27
R-squared	0.634	0.564	0.468	0.381	0.362	0.321	0.281	0.225	0.194	0.157

Note: This table shows the yearly regression results of the LC debt share in year t on the bond-stock beta estimated using a five-year rolling window between $t-5$ and t.

A.5 LC Bond Return Comovement with US Stock Returns

We now show empirically that the LC bonds with the best hedging value for the domestic government are risky for international lenders. In this analysis, we proxy for domestic agents' marginal utility of consumption with the local log excess stock return and for international lenders' SDF with the US log excess stock return. We decompose the local log excess stock return into a global and an idiosyncratic component according to:

$$xr_{i,t}^m = a_i + \beta(stock_i, stock_{US}) \times xr_{US,t}^m + xr_{i,t}^{idio}. \tag{A.2}$$

We define the systematic global component of local stock returns as the fitted value of Eqn. (A.2):

$$xr_{i,t}^G = \beta(stock_i, stock_{US}) \times xr_{US,t}^m.$$

It is conceivable that LC bond returns co-move with domestic stock returns only through the idiosyncratic component, $xr_{i,t}^{idio}$, that is orthogonal to US stock returns. In this case, LC bonds would have zero covariance with US stock returns and present no systematic risk to international lenders, and our main channel would not be operative.

To alleviate this concern, we show in two ways that the LC bonds with the best hedging benefit for the domestic borrower are indeed risky for international lenders. First, we directly estimate the beta of LC bond returns with respect to US stock returns from a regression:

$$xr_{i,n,t}^{LC} = a_i + \beta(bond_i, stock_{US}) \times xr_{US,t}^m + \epsilon_{i,t}. \tag{A.3}$$

Panel (A) of Figure A.9 shows that $\beta(bond_i, stock_{US})$ is highly correlated with our baseline measure of bonds' hedging value for the domestic borrower, $\beta(bond_i, stock_i)$, estimated in Eqn. (1). The cross-country correlation of these two different bond betas equals 89%, clearly supporting a link between the domestic borrower's hedging value and international lenders' risk of holding LC bonds.

Second, we estimate LC bond excess return loadings on the systematic global component of domestic stock returns using the regression:

$$xr_{i,n,t}^{LC} = a_i + \beta(bond_i, stock_{i,US}^G) \times xr_{i,t}^G + \epsilon_{i,t}. \tag{A.4}$$

Panel (B) of Figure A.9 shows that $\beta(bond_i, stock_{i,US}^G)$ is 89% correlated with our baseline measure of bonds' hedging value for the domestic borrower $\beta(bond_i, stock_i)$. In other words, LC bond returns co-move with the global component of local LC stock returns.

Figure A.9: Local and Global Risks of LC Bonds

(A) Beta onto US Stock Returns
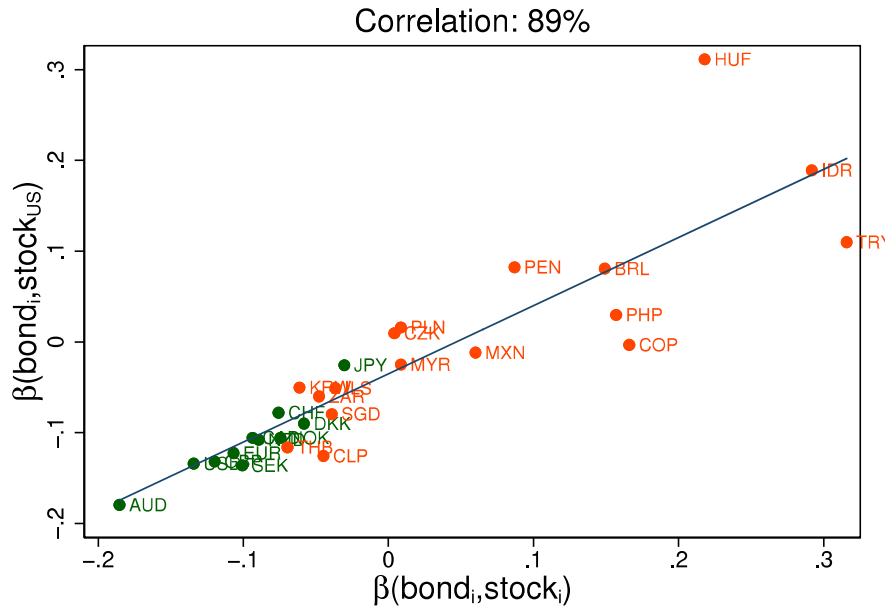
(B) Beta onto Global Component of Local Stock Returns
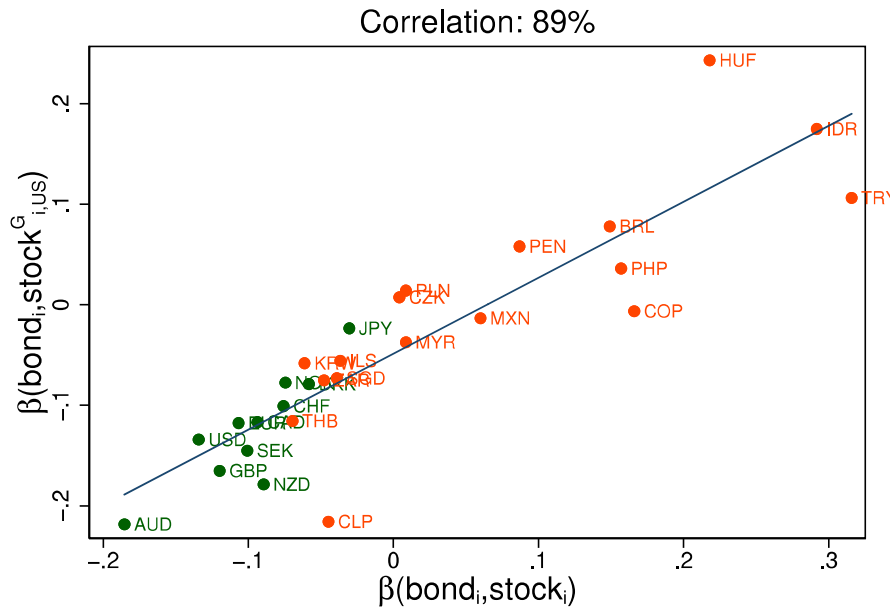
Note: Panel (A) plots on the y-axis the regression beta of LC bond excess returns on US S&P stock excess returns, $\beta(bond_i, stock_{US})$, estimated from Eqn. (A.3). Panel (B) plots on the y-axis the regression beta of LC bond excess returns on the global component of local LC bond returns, $\beta(bond_i, stock_{i,US}^G)$, estimated from Eqn. (A.4). Our baseline one-factor bond-stock beta with respect to the local stock market, estimated from Eqn. (1), is shown on the x-axis in both panels. The bivariate correlation across countries is shown in the figure title.

A.6   Details: Bootstrap Standard Errors

We now give the implementation details for the bootstrap standard errors shown in Tables 3 and 4. We first describe the bootstrap used in Table 3, column (1), because this is illustrative of our methodology overall. We bootstrap both the LC debt shares and bond and stock returns. On the bootstrapped data, we then re-estimate bond-stock betas according to Eqn. (1) in the main paper, and then regress LC debt shares on these estimated bond-stock betas. By re-estimating bond-stock betas on the bootstrapped data this bootstrap procedure accounts for estimation error in bond-stock betas.

We generate bootstrapped LC debt shares according to a wild bootstrap that accounts for heteroskedasticity (Davison and Hinkley (1997)). Let b_0 and b_1 denote the point estimates from regressing the LC debt share on bond-stock betas in actual data:

$$s_i^{TOT} = b_0 + b_1 \beta \left(bond_i, stock_i \right) + \varepsilon_i, \tag{A.5}$$

that is b_0 and b_1 are the estimated constant and coefficient shown in Table 3, column (1). We use ε_i to denote the residual for country i estimated on actual data. The bootstrapped LC debt share $s_i^{TOT,boot}$ is then defined as:

$$s_i^{TOT,boot} = b_0 + b_1 \beta \left(bond_i, stock_i \right) + X_i \varepsilon_i, \tag{A.6}$$

where X_1, X_2,..., X_N are random variables that we draw independently from a standard normal distribution with with mean zero and variance one. The conditional mean of $s_i^{TOT,boot}$ is therefore $b_0 + b_1 \beta(bond_i, stock_i)$ as in a standard parametric bootstrap and the conditional variance of the bootstrap residual is $\mathbb{V}\left(X_i \varepsilon_i \right) = \varepsilon_i^2$. The wild bootstrap preserves the volatility of the residual and it is appropriate when one is concerned about heteroskedasticity, similar to situations when Huber-White heteroskedasticity-robust standard errors would be used.

Figure A.10: Bootstrap Bond and Stock Returns

(A) Defining Overlapping Blocks on Original Data

		Bond Returns			Stock Returns		
Day		Country 1 ...		Country N	Country 1 ...		Country N
1	block 1	$xr^{LC}_{1,n,1}$	...	$xr^{LC}_{N,n,1}$	$xr^{m}_{1,1}$	...	$xr^{m}_{N,1}$
2	block 2	$xr^{LC}_{1,n,2}$	...	$xr^{LC}_{N,n,2}$	$xr^{m}_{1,2}$	...	$xr^{m}_{N,2}$
.	block 3	.		.	.		.
.		.		.	.		.
120		$xr^{LC}_{1,n,120}$	...	$xr^{LC}_{N,n,120}$	$xr^{m}_{1,120}$	...	$xr^{m}_{N,120}$
121		$xr^{LC}_{1,n,121}$	...	$xr^{LC}_{N,n,121}$	$xr^{m}_{1,121}$	...	$xr^{m}_{N,121}$
.		.		.	.		.
.		.		.	.		.
.		.		.	.		.
T		$xr^{LC}_{1,n,T}$	...	$xr^{LC}_{N,n,T}$	$xr^{m}_{1,T}$	...	$xr^{m}_{N,T}$

(B) Defining Bootstrap Bond and Stock Returns

		Bond Returns			Stock Returns		
Day		Country 1	...	Country N	Country 1	...	Country N
1	block rand1	$xr^{LC}_{1,n,rand1}$	...	$xr^{LC}_{N,n,rand1}$	$xr^{m}_{1,rand1}$	...	$xr^{m}_{N,rand1}$
2		$xr^{LC}_{1,n,rand1+1}$	...	$xr^{LC}_{N,n,rand1+1}$	$xr^{m}_{1,rand1+1}$	...	$xr^{m}_{N,rand1+1}$
		.		.	.		.
		.		.	.		.
		$xr^{LC}_{1,n,rand1+120}$	...	$xr^{LC}_{N,n,rand1+120}$	$xr^{m}_{1,rand1+120}$	...	$xr^{m}_{N,rand1+120}$
	block rand2	$xr^{LC}_{1,n,rand2}$	...	$xr^{LC}_{N,n,rand2}$	$xr^{m}_{1,rand2}$	...	$xr^{m}_{N,rand2}$
		$xr^{LC}_{1,n,rand2+1}$	...	$xr^{LC}_{N,n,rand2+1}$	$xr^{m}_{1,rand2+1}$	...	$xr^{m}_{N,rand2+1}$
		.		.	.		.
		.		.	.		.
		$xr^{LC}_{1,n,rand2+120}$	...	$xr^{LC}_{N,n,rand2+120}$	$xr^{m}_{1,rand2+120}$	...	$xr^{m}_{N,rand2+120}$
		.		.	.		.
		.		.	.		.
	block randK	$xr^{LC}_{1,n,randK}$	.	$xr^{LC}_{N,n,randK}$	$xr^{m}_{1,randK}$	.	$xr^{m}_{N,randK}$
		.		.	.		.
T		$xr^{LC}_{1,n,randK+120}$	.	$xr^{LC}_{N,n,randK+120}$	$xr^{m}_{1,randK+120}$	.	$xr^{m}_{N,randK+120}$

Note: This figure illustrates the moving block bootstrap to generate bootstrapped LC bond and local stock log excess returns (Maddala (2001)). $xr^{LC}_{i,n,t}$ denotes the log excess return over the 91 calendar day period ending on day t for the country i LC bond with remaining time to maturity n. $xr^{m}_{i,t}$ denotes the log excess return over the 91 calendar day period ending on day t on the local equity benchmark in excess of a 3-month T-bill. As illustrated, we preserve the correlation structure across stocks and bonds and across countries by choosing the same blocks for stocks and bonds and all countries. $rand1$, $rand2$, ... , $randK$ are iid random variables drawn uniformly from the integers between 1 and $T-120$. We then define bootstrap returns as the sequence of block $rand1$, followed by block $rand2$,... up to block $randK$.

We generate bootstrapped bond and stock returns while accounting for serial correlation and heteroskedasticity in bond and stock returns. We use a moving block bootstrap, which Maddala (2001) and Lahiri (1999) argue has superior properties to account for time series correlation. Let N be the number of countries and T be the length of the time series. We define $T-120$ overlapping blocks of length 120 days, where we use the same blocks for both bond returns $xr^{LC}_{i,n,t}$ and stock returns $xr^{m}_{i,t}$ and we use the same blocks for all countries. Figure A.10, Panel (A) illustrates how we define overlapping blocks on the actual data. Define $K = \lfloor T/120 \rfloor$, such that a combination of K blocks will generate a bootstrap sample of length $K \times 120 \approx T$. Because T is not generally a multiple of 120, we round K down to be conservative. We then generate bootstrap samples

$xr_{i,n,t}^{LC,boot}$ and $xr_{i,t}^{m,boot}$ by randomly drawing K blocks and concatenating them. Formally, we draw iid random variables $rand1, rand2, \dots , randK$ uniformly from the integers between 1 and $T - 120$ and define the bootstrap returns as the sequence of blocks $rand1, rand2, \dots, randK$. Figure A.10, Panel (B) illustrates the construction of the bootstrapped bond and stock returns. Because we use the same blocks across all countries, the bootstrap sample preserves the correlation of bond and stock returns across countries. We choose a block length of 120 trading days as a trade-off between capturing the serial correlation of overlapping returns (which are defined using 91 calendar days) and having a sufficient number of blocks to generate plausible variation across the bootstrapped samples.

Having generated a bootstrap sample, we follow the same estimation procedure as in the actual data. We re-estimate Eqn. (1) country-by-country on the bootstrapped data:

$$xr_{i,n,t}^{LC,boot} \quad = \quad a_i + \beta^{boot} (bond_i, stock_i) \times xr_{i,t}^{m,boot} + \epsilon_{i,t}^{boot}. \tag{A.7}$$

We then run a cross-sectional regression of $s_i^{TOT,boot}$ onto the bond-stock betas estimated on the bootstrapped sample, $\hat{\beta}^{boot} (bond_i, stock_i)$, where we use a hat to emphasize that we use betas that were estimated on bootstrapped data:

$$s_i^{TOT,boot} \quad = \quad b_0^{boot} + b_1^{boot} \hat{\beta}^{boot} (bond_i, stock_i) + \varepsilon_i^{boot}. \tag{A.8}$$

The estimated slope coefficient b_1^{boot} is the coefficient of interest in the bootstrapped data. The bootstrap standard error reported in Table 3 is the standard deviation of b_1^{boot} across 500 independent bootstrap samples.

We verify that our bootstrap procedure captures plausible volatility for measurement errors in bond-stock betas, and also how these measurement errors are correlated across countries. Table A.2 reports the standard deviation of $\hat{\beta}^{boot} (bond_i, stock_i)$ across 500 independent bootstrap samples alongside with asymptotic Hansen-Hodrick standard errors for $\beta (bond_i, stock_i)$. Hansen-Hodrick standard errors with lag length 120 days explicitly account for heteroskedasticity and serial correlation induced by using daily data on overlapping returns. Comparing across columns in Table A.2 confirms that the block bootstrap volatility in the measurement error of bond-stock betas is similar to asymptotic Hansen-Hodrick standard errors, so these very different approaches to adjust for heteroskedasticity and overlapping return observations generate consistent results. Table A.9 similarly shows summary statistics for the bootstrap t-statistics. Each t-statistic in this table is defined as the point estimate of $\beta (bond_i, stock_i)$ divided by the standard deviation of $\hat{\beta}^{boot} (bond_i, stock_i)$ across 500 independent bootstrap samples. Table A.9 is analogous to Table A.3, which reports the properties of t-statistics based on asymptotic standard errors. Because the point estimates are the same in both tables, any differences in t-statistics are due to differences between bootstrap and asymptotic standard errors. The comparison again reveals that the significance of bootstrap t-statistics is similar to Hansen-Hodrick t-statistics.

Table A.9: Bootstrap t-Statistics for Bond-Stock Betas

	(1)	(2)	(3)	(4)	(5)	(6)
	Mean	Median	Min	Max	# of countries with $\lvert t \rvert > 1.96$	N
Panel (A) Developed Markets Bootstrap t-Statistics						
$\beta(bond_i, stock_i)$	4.12	4.00	2.63	5.83	11	11
$\beta(stock_i, stock_{US})$	10.85	10.91	8.29	14.87	10	10
$\beta(\pi_i, IP_i)$	1.90	1.35	0.64	4.50	4	11
Panel (B) Emerging Markets Bootstrap t-Statistics						
$\beta(bond_i, stock_i)$	2.64	2.17	0.21	7.25	10	17
$\beta(stock_i, stock_{US})$	8.10	8.62	4.74	12.52	17	17
$\beta(\pi_i, IP_i)$	1.25	0.96	0.06	3.89	3	17
Panel (C) Full Sample Bootstrap t-Statistics						
$\beta(bond_i, stock_i)$	3.22	2.94	0.21	7.25	21	28
$\beta(stock_i, stock_{US})$	9.12	8.74	4.74	14.87	27	27
$\beta(\pi_i, IP_i)$	1.50	1.01	0.06	4.50	7	28

Note: This table is analogous to Table A.3 except that t-statistics are based on the standard deviation across independent bootstrap samples. Bootstrap samples for bond returns, stock returns, inflation, and industrial production are constructed as described in Appendix A.6. This table presents summary statistics for the absolute value of the t-statistic based on bootstrap standard errors of three regression betas: LC bond-local stock beta, $\beta(bond_i, stock_i)$, local stock-US stock return beta, $\beta(stock_i, stock_{US})$, and realized inflation-output beta, $\beta(\pi_i, IP_i)$. The standard deviation of bootstrapped betas across 500 independent bootstrap samples are used for the standard errors.

Because our bootstrap procedure preserves the correlation structure of returns across countries, we expect bootstrapped bond-stock betas to be correlated across countries. We verify that the cross-country correlations of bootstrapped bond-stock betas are intuitive. Across 500 independent simulations the bootstrapped US bond-stock beta, $\hat{\beta}^{boot}(bond_{US}, stock_{US})$, is slightly positively correlated with $\hat{\beta}^{boot}(bond_i, stock_i)$. Taking the average across all countries except the US, the average cross-country correlation of bootstrapped bond-stock betas equals

$$\frac{1}{N-1} \sum_{i \neq US} corr\left(\hat{\beta}^{boot}(bond_{US}, stock_{US}), \hat{\beta}^{boot}(bond_i, stock_i)\right) = 0.17.$$

As one might expect, US bootstrapped bond-stock betas have a higher correlation with developed markets (0.28) than with emerging markets (0.11). The highest pairwise correlation of $\hat{\beta}^{boot}(bond_{US}, stock_{US})$ with any country is 0.60 for Canada, which is intuitive because of the especially close economic and financial linkages between the US and Canada. While these correlations are positive they are on average quantitatively small, consistent with their effect on bootstrapped standard errors being modest.

Next, we show how the different components of the bootstrap procedure affect the resulting bootstrap standard errors in our main results in Table 3. For this, we switch on the different components of the bootstrap one-by-one for the benchmark regression in Table 3, column (1). Table 3, Panel (A) reports results with the total LC debt share on the left-hand side and Table 3, Panel (B) reports results with the external LC debt share from TIC on the left-hand side. We look at these two different left-hand-side variables to show that bootstrap standard errors can be

either smaller or larger than Huber-White robust standard errors. Each column in Table A.10 uses a different methodology to compute standard errors. For both panels, column (1) of Table A.10 reports asymptotic Huber-White heteroskedasticity-robust standard errors for comparison. The standard error in column (1) is therefore not a bootstrap standard error and it treats $\beta(bond_i, stock_i)$ as observed. However, it is a robust standard error that accounts for the fact that LC debt shares may have heteroskedasticity. Moving to column (2), we run a bootstrap that bootstraps LC debt shares according to Eqn. (A.6) but does not bootstrap bond and stock returns, therefore treating $\beta(bond_i, stock_i)$ as observed without estimation error. The point of this is to verify that the wild bootstrap procedure for the LC debt share works and gives similar statistical significance as the robust standard error in column (1). We see that the standard error in column (2) is very similar to the one in column (1) in both cases, though slightly smaller. A look at the associated t-statistics reveals that the magnitude of the difference between column (1) and (2) standard errors is tiny and has no bearing whatsoever on statistical significance at any conventional significance levels. This reassures us that the bootstrapping procedure for the LC debt share is reasonable and accounts for conditional heteroskedasticity similarly to robust Huber-White asymptotic standard errors. Next, column (3) shows bootstrap standard errors that account for estimation error in $\hat{\beta}^{boot}(bond_i, stock_i)$ but do not allow these estimation errors to be correlated across countries. Formally, column (3) bootstraps the LC debt share according to Eqn. (A.6) and it bootstraps bond and stock returns for each country independently, i.e. it uses a block bootstrap as depicted in Figure A.10 but it runs the block bootstrap separately for each country. We thereby generate the same amount of volatility in $\hat{\beta}^{boot}(bond_i, stock_i)$ across bootstrap simulations as in the full bootstrap, but we switch off the cross-country correlation in $\hat{\beta}^{boot}(bond_i, stock_i)$. We can see that the standard errors in column (3) are again extremely similar to the standard errors in columns (1) and (2) with virtually identical t-statistics in both Panels (A) and (B). Finally, column (4) runs the full bootstrap procedure described above, re-estimating $\hat{\beta}(bond_i, stock_i)$ on bootstrapped data for bond and stock returns. Column (4) accounts for cross-country correlations in bootstrapped bond and stock returns by using the same blocks across countries as shown in Figure A.10. We see that the resulting standard errors and t-statistics are again virtually indistinguishable from the ones reported in columns (1) through (3). To be conservative, in the main text we report the maximum of the Huber-White robust standard error and the bootstrap standard error from the full bootstrap procedure throughout Tables 3 and 4 and Appendix Table A.7. In Table A.10 Panel (A), the column (1) robust standard error is slightly larger than the column (4) bootstrap standard error, so we report the robust standard error in column (1) of Table 3 in the main text. Conversely, in Table A.10 Panel (B) the column (4) bootstrap standard error is slightly larger than the column (1) robust standard error, so we report the bootstrap standard error in column (1) of Table 4 in the main text.

Having explained the baseline bootstrap, we now briefly outline how we modify this procedure to account for different estimated betas on the right-hand side of the regressions in Table 3. In column (4) and in the second set of columns in Table 4, we replace Eqn. (A.5) with a regression that includes additional controls and use that to bootstrap LC debt shares. We additionally generate a bootstrap sample of US stock returns and re-estimate the local-US stock return on the bootstrapped sample. We use the same blocks for US stock returns as for bond and stock returns in all other countries, thereby preserving the correlation between US and local stock returns.

The bootstrap in column (3) of Table 3 is analogous to the baseline bootstrap described above. However, we choose a different block length for the moving block bootstrap because our data for $\Delta\pi_{i,t}$ and $\Delta IP_{i,t}$ is monthly, whereas our data for overlapping bond and stock returns is daily. We use a moving block bootstrap with block length 12 months to generate bootstrap samples for $\Delta\pi_{i,t}$ and $\Delta IP_{i,t}$.

Due to data limitations, we choose a simpler bootstrap procedure for column (2) of Table 3. We no longer have access to the raw survey data and therefore cannot generate a bootstrap sample of survey inflation and output expectations. Instead, we account for measurement error in $\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right)$ by re-sampling the betas directly from the asymptotic distribution estimated with Newey-West standard errors with lag length 3 months. Concretely, we draw $\beta^{boot}\left(\pi_i^{Survey}, gdp_i^{Survey}\right) \sim N\left(\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right), SE\left(\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right)\right)\right)$ independently across countries, where $SE\left(\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right)\right)$ denotes the standard error for $\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right)$ estimated with Newey-West and lag length of 3 months. The bootstrap sample for the LC debt share is generated as in our baseline bootstrap. This simpler bootstrap accounts for measurement error in $\beta\left(\pi_i^{Survey}, gdp_i^{Survey}\right)$, but unlike our baseline bootstrap it cannot account for correlations in this measurement error across countries. We think that these bootstrap standard errors are still informative, because we have seen in Table A.10 that allowing for cross-country correlations in the estimation error has only negligible effects on the standard error.

Table A.10: Isolating Different Components of Bootstrap

(A) Total Government Debt

Local Currency Debt Share	(1) s^{TOT}	(2) s^{TOT}	(3) s^{TOT}	(4) s^{TOT}
Standard Error	Huber-White	bootstrap	bootstrap	bootstrap
		LC debt share only	no cross-country correlation	with cross-country correlation
$\beta(bond_i, stock_i)$	-106.64***	-106.64***	-106.64***	-106.64***
SE	(20.37)	(19.30)	(20.90)	(20.27)
t-statistic	5.24	5.53	5.10	5.26
Constant	72.54***	72.54***	72.54***	72.54***
SE	(4.04)	(3.87)	(3.94)	(4.29)
t-statistic	17.96	18.74	18.41	16.91
Observations	28	28	28	28
R-squared	0.30	0.30	0.30	0.30

(B) External Government Debt

Local Currency Debt Share	s^{TIC}	s^{TIC}	s^{TIC}	s^{TIC}
Standard Error	Huber-White	bootstrap	bootstrap	bootstrap
		LC debt share only	no cross-country correlation	with cross-country correlation
$\beta(bond_i, stock_i)$	-136.36***	-136.36***	-136.36***	-136.36***
SE	(25.07)	(24.57)	(24.57)	(25.37)
t-statistic	5.44	5.55	5.55	5.37
Constant	68.85***	68.85***	68.85***	68.85***
SE	(4.56)	(4.26)	(4.65)	(4.84)
t-statistic	15.10	16.16	14.81	14.23
Observations	28	28	28	28
R-squared	0.36	0.36	0.36	0.36

Note: Panel A compares asymptotic and bootstrap standard errors for the cross-country regression results of the LC debt share in total central government debt, s^{TOT} (between 0 and 1), on bond-stock return betas. Panel (B) uses the LC debt share in externally-held debt but is otherwise analogous to Panel (A). The point estimates in Panel (A) are identical to column (1) of Table 4. In both panels, column (1) shows asymptotic heteroskedasticity-robust Huber-White standard errors that treat bond-stock betas as observed (i.e. no bootstrap). Column (2) bootstraps the LC debt share according to Eqn. (A.6) but continues to treat $\beta(bond_i, stock_i)$ as observed. Column (3) bootstraps both the LC debt share and bond and stock returns. It bootstraps bond and stock returns independently for each country and re-estimates $\hat{\beta}^{boot}(bond_i, stock_i)$ on the bootstrapped returns, thereby treating $\hat{\beta}^{boot}(bond_i, stock_i)$ as having estimation error that is uncorrelated across countries. Column (4) bootstraps both the LC debt share and bond and stock returns according to the full bootstrap procedure described in Appendix A.6. The bootstrap in column (4) preserves the correlation structure of bond and stock returns across countries, as illustrated in Figure A.10, thereby treating $\hat{\beta}^{boot}(bond_i, stock_i)$ as having estimation error that is correlated across countries. The standard error in column (4) is reported in Table 3 in the main text. The significance levels indicated by *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

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A.7 Testing the CAPM

A.7.1 GRS Test of the CAPM

The paper treats stock market betas as proxies for expected excess returns. We now estimate a standard Gibbons et al. (1989) (GRS) test for the CAPM, with the US stock market as a proxy for total wealth.

We start by sorting our countries into five equal-sized portfolios, sorted by their LC bond betas with respect to the US stock market. We obtain quarterly bond excess returns (not overlapping) for these five portfolios. Due to our short sample period, it is unsurprising that average excess returns are noisy.

We test the CAPM with the GRS statistic, which Campbell (2017) shows can be written as:

$$GRS \quad = \quad \frac{T - N - 1}{N} \frac{(Sharpe^{LC,tangency})^2 - (Sharpe^m_{US})^2}{1 + (Sharpe^m_{US})^2}. \tag{A.9}$$

Here, $Sharpe^{LC,tangency}$ is the Sharpe ratio of the tangency portfolio of the LC bond portfolios, $Sharpe^m_{US}$ is the Sharpe ratio of the US equity market, $T = 42$ is the number of quarterly returns, and $N = 5$ is the number of portfolios. The GRS statistic, hence, increases in the distance between the Sharpe ratios for the tangency portfolio and the US equity market.

We estimate the tangency portfolio Sharpe ratio from the portfolio returns as in Campbell (2017) Chapter 2.2.3. This gives a tangency Sharpe ratio of $Sharpe^{LC,tangency} = 0.52$, compared to a US equity market Sharpe ratio of $Sharpe^m_{US} = 0.17$, over our sample period 2004-2015. The Sharpe ratio for the LC bond tangency portfolio hence exceeds the equity Sharpe ratio over our short sample period. However, the tangency Sharpe ratio is very close to the US equity Sharpe ratio of 0.56 reported in Campbell (2003) for a longer sample that is conventionally used to obtain a more precise estimate of average US equity excess returns. The proximity between the tangency Sharpe ratio and the US equity Sharpe ratio from this longer sample is an intuitive indication that the difference between tangency and US equity Sharpe ratios over the shorter sample is not statistically significantly different.

Substituting the values for $Sharpe^{LC,tangency}$, $Sharpe^m_{US}$, T, and N into (A.9) gives a value for the GRS statistic of $GRS = 1.72$. Comparing this value to the critical values of a $F_{N,T-N-1}$ distribution gives a p-value of 0.16, showing formally that we cannot reject CAPM at any conventional significance level.

A.7.2 GMM Risk Premium Estimation

We next make use of the fact that our assets of interest are bonds and that we can use quoted bond yields to construct ex ante measures of LC bond risk premia. Ex ante bond risk premia may be more precisely measured than the ex post average returns over a limited sample used for the GRS test. We find that ex ante LC bond risk premia have a statistically and quantitatively significant relationship with US stock market betas across countries. This estimation is similar to the GRS test in Section A.7.1, because we seek to estimate whether investors require a higher risk premium for LC bonds that comove more with the US stock market. Further, we want to understand whether this price of risk is statistically distinguishable from the average US equity risk premium.

A concrete example makes clear the advantage of ex ante risk premia measures based on bond yields, whereas realized bond returns are noisy measures of ex ante expected risk premia over our short sample. For instance, the US had extremely low government bond yields throughout our sample, indicating that investors required low risk premia for holding US Treasuries. However,

US Treasury yields dropped even lower during our sample and, in particular, during the financial crisis, an event that would have been very hard to predict ex ante. As a result, looking at US excess returns, it would appear as if the US had a high risk premium, whereas clearly markets price a very low risk premium into US Treasuries.

We estimate a regression of ex ante average expected risk premia onto the beta of LC bond returns with respect to the US stock market, while accounting for the fact that the betas on the right-hand side of this regression are not known but instead must be estimated.

For comparison and to set the stage, we first estimate this relationship in two steps without accounting for generated regressors. As a first-step, we estimate country-by-country regressions:

$$xr_{i,n,t}^{LC} = \alpha_i + \beta_i xr_{US,t}^m + \epsilon_{i,t}, \tag{A.10}$$

using daily data on overlapping 1-quarter holding returns. Because we use daily overlapping returns, the average number of return observations per country is high at 2513. For comparison, the maximum number of return observations is 2608, so our data is close to a balanced panel. Let $\overline{RP}_{i,n}$ denote the average ex ante risk premium estimated for country i. In a second step, we then estimate the regression:

$$\overline{RP}_{i,n} = \mu + \kappa\beta_i + u_i. \tag{A.11}$$

The coefficient, κ, estimates the cost of exposure to the US stock market and is the coefficient of interest.

To estimate α_i, β_i, μ, and κ in a single step while accounting for estimation error in the first stage, we define the following Generalized Method of Moments (GMM) moments, which we expect to have a population mean of zero:

$$g_{i,t} = \begin{cases} \overline{RP}_{i,n} - \mu - \kappa\beta_i & \text{for} \quad 1 \le i \le N \\ \left(\overline{RP}_{i,n} - \mu - \kappa\beta_i\right)\beta_i & \text{for} \quad N+1 \le i \le 2N \\ xr_t^{LC} - \alpha_i - \beta_i xr_{US,t}^m & \text{for} \quad 2N+1 \le i \le 3N \\ \left(xr_{i,t}^{LC} - \alpha_i - \beta_i xr_{US,t}^m\right) xr_{US,t}^m & \text{for} \quad 3N+1 \le i \le 4N \end{cases} \tag{A.12}$$

Here, N denotes the number of countries in the sample and the parameter vector to be estimated is:

$$\begin{aligned} b &= [\mu, \kappa, \boldsymbol{\alpha}, \boldsymbol{\beta}]', \\ \boldsymbol{\alpha} &= [\alpha_1, \alpha_2, ..., \alpha_N], \\ \boldsymbol{\beta} &= [\beta_1, \beta_2, ..., \beta_N]. \end{aligned}$$

The first $2N$ moment conditions in (A.12) are for the cross-sectional regression in the second stage. Moment conditions $2N+1$ through $4N$ are for the first-stage regressions. In sample, the $4N$ moments (A.12) cannot all simultaneously be set to zero, because we only have $2N+2$ parameters. The GMM estimator $\hat{b}$ is defined by setting:

$$A \times \frac{1}{T} \sum_{t=1}^{T} g_t\left(\hat{b}\right) = 0, \tag{A.13}$$

where A is a weighting matrix of size $(2N+2) \times 4N$ that has full rank. It is a standard result for

GMM that the estimated parameter vector $\hat{b}$ has asymptotic distribution

$$\hat{b} \quad \sim \quad \mathcal{N}\left(b_0, V\right) \tag{A.14}$$

$$V \quad = \quad T^{-1}\left(AD\right)^{-1} A S A'\left(AD\right)^{-1'}, \tag{A.15}$$

where b_0 is the true underlying parameter value, $D = E\left[\frac{\partial g}{\partial b'}\right]$ is the sample average of the derivative of g with respect to the parameter vector, b, and S is the spectral density matrix of g_t at frequency zero.

We implement GMM with weighting matrix $A = \left[(2N+2) \times 4N\right]$ that ensures that the GMM estimates for μ and κ agree with the point estimates from the two-step procedure. This requirement pins down the weighting matrix:

$$A \quad = \quad \left[\begin{array}{ccc} 1_{1\times N} & 0_{1\times N} & 0_{1\times 2N} \\ 0_{1\times N} & 1_{1\times N} & 0_{1\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & \mathcal{I}_{2N} \end{array}\right]. \tag{A.16}$$

Here $0_{M\times P}$ and $1_{M\times P}$ define block matrices of all zeros and ones with size $[M \times P]$, respectively. We use $\mathcal{I}_{2N}$ to denote the identity matrix of size $2N$. For our application, we use the consistent estimator for D:

$$\hat{D} \quad = \quad \left[\begin{array}{cccc} -1_{N\times 1} & -\boldsymbol{\beta} & 0_{N\times N} & -\kappa I_N \\ -\boldsymbol{\beta} & -\boldsymbol{\beta}^2 & 0_{N\times N} & -2\kappa \times diag(\boldsymbol{\beta}) \\ 0_{N\times 1} & 0_{N\times 1} & -\mathcal{I}_N & -\mathcal{I}_N \sum_{t=1}^{T} xr_{US,t}^m T^{-1} \\ 0_{N\times 1} & 0_{N\times 1} & -\mathcal{I}_N \sum_{t=1}^{T} xr_{US,t}^m T^{-1} & -\mathcal{I}_N \sum_{t=1}^{T} \left(xr_{US,t}^m\right)^2 T^{-1} \end{array}\right], \tag{A.17}$$

where $diag(\boldsymbol{\beta})$ denotes the matrix with the elements of $\boldsymbol{\beta}$ along the diagonal. We estimate the upper left $[2N \times 2N]$ submatrix of S from the cross-section of countries, with the assumption that (β_i, u_i) are independent but not necessarily identically distributed. We also assume that $g_{i,t}, 1 \le i \le 2N$ are independent of $g_{j,t}, 2N < j < 4N$, so we can set the upper-right $2N \times 2N$ and the lower-left $2N \times 2N$ block matrices of the spectral density matrix, S, to zero. We cannot estimate the upper-right $2N \times 2N$ and the lower-left $2N \times 2N$ block matrices of the spectral density matrix, S, because $\overline{RP}_{i,n}$ is constant over time for each country. The spectral density for moments $2N+1$ through $4N$ is estimated from the time series with a Newey-West kernel with m lags to account for serial correlation and overlapping return observations:

$$\hat{S} \quad = \quad \left[\begin{array}{ccc} \mathcal{I}_N \hat{s}_1 & \mathcal{I}_N \hat{s}_{12} & 0_{N\times 2N} \\ \mathcal{I}_N \hat{s}_{12} & \mathcal{I}_N \hat{s}_2 & 0_{N\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & T^{-1} \sum_{t=1}^{T} \left(\tilde{g}_t \tilde{g}_t' + \sum_{i=1}^{m} \left(1 - \frac{i}{m+1}\right)\left[\tilde{g}_t \tilde{g}_{t-i}' + \tilde{g}_{t+i} \tilde{g}_t'\right]\right) \end{array}\right]. \tag{A.18}$$

Here,

$$\hat{s}_1 \quad = \quad \frac{1}{N-2} \sum_{t=1}^{T} \sum_{i=1}^{N} g_{i,t}^2, \tag{A.19}$$

$$\hat{s}_2 \quad = \quad \frac{1}{N-2} \sum_{t=1}^{T} \sum_{i=N+1}^{2N} g_{i,t}^2, \tag{A.20}$$

$$\hat{s}_{12} \quad = \quad \frac{1}{N-2} \sum_{t=1}^{T} \sum_{i=1}^{N} g_{i,t} g_{i+N,t}, \tag{A.21}$$

and $\tilde{g}_t$ refers to the vector containing elements $g_{2N+1,t}$ through $g_{4N,t}$. We choose a lag length of $m = 120$ days to account for the length of overlapping observations of approximately 60 trading days. A lag length of $m = 120$ days is sufficiently small relative to our overall sample length of 2608 trading days that standard asymptotic standard errors apply.

We then compute the GMM standard errors for μ and κ as follows:

$$SE(\hat{\mu}) \quad = \quad \sqrt{V(1,1)}, \tag{A.22}$$

$$SE(\hat{\kappa}) \quad = \quad \sqrt{V(2,2)}. \tag{A.23}$$

Table A.11 column (1) starts by reporting the estimated regression Eqn. (A.11) without accounting for generated regressors. We note that the bond-US stock beta enters with a strongly positive coefficient that is also statistically significant. The results suggest that the price of US stock market risk is 8.96%, i.e. an asset with a unit beta with respect to the US stock market has a risk premium of 8.96%. This number is very close to and not statistically significantly different from the equity premium of 8.1% reported in Campbell (2003). Column (2) in Table A.11 reports results from the GMM procedure, which accounts for generated regressors. The point estimates are identical to column (1) and the standard errors are only slightly larger without affecting statistical significance, as one would expect if the vector of bond betas, $\boldsymbol{\beta}$, is precisely estimated.

Table A.11: GMM: Bond Risk Premia onto Bond-US Stock Betas

LC Bond Risk Premium	(1) OLS	(2) GMM
β_i	8.96***	8.96***
	(2.69)	(3.31)
Constant	2.80***	2.80***
	(0.33)	(0.39)
Observations	28	28

Note: This table estimates the regression (A.11), where LC bond-US stock return betas are estimated via (A.10). The specification in column (1) does not account for generated regressors. Column (2) accounts for generated regressors by using the GMM procedure described in Appendix A.7.2. Significance levels are indicated by *** p<0.01, ** p<0.05, * p<0.1.

B   Model Appendix

The Model Appendix is structured as follows:

- Appendix B.1 microfounds real exchange rate shocks.

- Appendix B.2 shows that under the assumptions of lump-sum taxes and a representative domestic consumer, inflating away domestically-held LC debt has no effect on domestic real consumption. Inflating away LC debt is only an aggregate transfer of resources to domestic consumers when the debt is owned by international lenders. This insight allows us to focus on externally-held debt throughout the main paper.

- Appendix B.3 derives the first-order conditions.

- Appendix B.4 proves Proposition 1.

- Appendix B.5 describes the numerical solution.

- Appendix B.7 shows that the quantitative results are robust to reasonable variation in parameter values and in particular to allowing separate exchange rate processes for emerging and developed markets.

B.1   Microfounding the Real Exchange Rate

This section describes the goods and preferences microfounding the real exchange rate.

B.1.1   International Consumers

Following Gabaix and Maggiori (2015), we assume that international consumers consume a consumption basket:

$$C_t^* \quad = \quad (A_t^*)^{\mathcal{E}_t} (O_t^*)^{1-\mathcal{E}_t}, \tag{B.1}$$

where $\mathcal{E}_t$ is a non-negative, potentially stochastic preference parameter.[32] A_t^* denotes the number of apples and O_t^* the number of oranges consumed by international consumers in periods $t = 1, 2$. We normalize the preference shock in period 1 to one. The period 2 preference shock is log-normally distributed according to Eqns. (20) and (21). To summarize, the distribution of the preference shock is:

$$\mathcal{E}_1 \quad = \quad 1,$$
$$\mathcal{E}_2 \quad = \quad exp\left(\varepsilon_2 - \frac{1}{2}\sigma_\varepsilon^2\right), \tag{B.2}$$
$$\varepsilon_2 \quad = \quad \lambda^{\varepsilon,x^*} x_2^* + e_2, \tag{B.3}$$

where e_2 is distributed according to:

$$e_2 \quad \sim \quad N(0, \sigma_e^2),$$

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[32]Pavlova and Rigobon (2007) also consider a similar foundation for real exchange rate fluctuations based on preference shocks.

independently of x_2 and x_2^*. International consumers' welfare function is given by:

$$U^* = E \sum_{t=1}^{2} (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}. \tag{B.4}$$

We assume that the international economy is endowed with an equal amount of apples and oranges in each period. Furthermore, the international economy's endowment of apples and oranges equals $A_1^* = O_1^* = X_1^* = 1$ in period 1 and it equals $A_2^* = O_2^* = X_2^*$ in period 2, where X_2^* follows the distribution described in the main paper. Since the domestic economy is assumed to be small, the effect of domestic bond payoffs on international consumers' consumption is negligible. The international consumers' consumption bundle then equals:

$$C_1^* = A_1^* = O_1^* = 1,$$
$$C_2^* = A_2^* = O_2^* = X_2^*.$$

B.1.2 Domestic Economy

Domestic consumers have preferences over the real domestic consumption bundle and domestic log inflation:

$$U(C_2, \pi_2) = \frac{C_2^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_2^2. \tag{B.5}$$

The domestic consumption bundle consists entirely of apples:

$$C_2 = A_2. \tag{B.6}$$

The amount of apples consumed in Eqn. (B.6) is endogenous, and depends on the exogenous endowment net of real debt repayments, as specified in Eqn. (9) We define the consumption-weighted real exchange rate as the price that international consumers are willing to pay for apples, where the numeraire is one unit of the international consumers' consumption bundle. With (B.1), (B.4), (B.5), and (B.6), the real exchange rate equals:

$$\frac{\frac{dU^*}{dA_t^*}}{\frac{dU^*}{dC_t^*}} = \mathcal{E}_t, \tag{B.7}$$

showing that the real exchange rate indeed follows the process described in the main paper.

B.2 Domestic Debt Extension

We now present an extension of the model with domestically-held LC debt. That is, the government can borrow from its own domestic consumers with LC debt in addition to borrowing from international lenders. We show that under the assumptions that the government has access to lump-sum taxes and a representative consumer, inflating away domestically-held LC debt leaves domestic real consumption unchanged. Inflating away LC debt only generates an aggregate transfer of resources to domestic consumers if that debt is held by international lenders. This observation motivates our focus on internationally-held debt throughout the paper.

We assume that the government borrows face value $D^{LC,dom}$ of LC debt from domestic consumers and face value D^{LC} of LC from international lenders at prices $Q^{LC,dom}$ and Q^{LC}. Note that we allow for potentially different bond prices paid by domestic consumers and international lenders. We continue to assume that the government needs to raise external financing $\bar{D}/R^*$ in period 1. To leave period 1 consumption normalized at 1, we assume that proceeds from domestic bond sales are rebated to domestic consumers.

The real amount of domestic goods needed to repay the government debt in period 2 becomes:

$$D_2 = \frac{D^{FC}}{\mathcal{E}_2} + \left(D^{LC} + D^{LC,dom}\right) exp(-\pi_2). \tag{B.8}$$

Because the government has access to lump-sum taxes, real period 2 domestic consumption equals the domestic endowment minus real resources needed to repay government debt plus the payoff on the domestically-held LC bond portfolio:

$$C_2 = X_2 - D_2 + D^{LC,dom} exp(-\pi_2). \tag{B.9}$$

Substituting (B.8) into (B.9) shows that domestic real consumption depends on D^{FC} and D^{LC} but is independent of domestically-held debt $D^{LC,dom}$:

$$C_2 = X_2 - \left(\frac{D^{FC}}{\mathcal{E}_2} + D^{LC} exp(-\pi_2)\right). \tag{B.10}$$

Intuitively, surprise inflation reduces domestic consumers' returns on their LC bond portfolio. However, surprise inflation also reduces the taxes required to repay debt. With lump sum taxes these two effects exactly cancel and domestic consumption is independent of the return on domestically-held debt. The finding that real domestic consumption is independent of domestically-held LC debt makes clear that externally-held debt is the key variable for the equilibrium inflation policy and bond risks.

B.3   First-Order Conditions

Proof of Inflation First-Order Condition with Commitment

We now prove the commitment government's first-order condition characterized by Eqns. (26) and (27). To simplify the derivation, we assume that there is a discrete number of states $j = 1, ..., N$ that are realized with probability f_j. With a discrete number of states f_j takes the role of the probability density function $f(X_2)$ in the main text. We use x_j, π_j etc. to denote the values for log real domestic output and log inflation if state j is realized in period 2. In this section we omit the superscript ^{c} and time period $_2$ subscript and reserve subscripts to indicate the state that has been realized. For simplicity, we first prove Eqn. (26) with the two additional simplifying assumptions that there is only one output shock ($x_j^* = x_j \forall j$) and there is no real exchange rate shock ($\varepsilon_j = 0 \forall j$), before proving the general case. In this simplified special case, the commitment government's problem is to choose the vector $\pi_1, \pi_2, ..., \pi_N$ to maximize:

$$\mathbb{E}U = \sum_{j=1}^{N} f_j \left(\frac{C_j^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_j^2\right), \tag{B.11}$$

where consumption in state i is given by:

$$C_i = \bar{X} exp\left(x_i/\bar{X}\right) - \bar{D}\left(\frac{s}{R^*}\frac{exp(-\pi_i)}{Q^{LC}} + (1-s)\right), \tag{B.12}$$

and the LC bond price equals:

$$Q^{LC} = \sum_{j=1}^{N} f_j M_j^* exp\left(-\pi_j\right). \tag{B.13}$$

Here, the international lenders' SDF in state j follows from Eqn. (14) and equals:

$$M_j^* = \delta^* exp\left(-\gamma^* x_j\right). \tag{B.14}$$

The international real risk-free rate satisfies:

$$\frac{1}{R^*} = \sum_{j=1}^{N} f_j M_j^*. \tag{B.15}$$

The commitment government chooses the inflation rate in state i such that the marginal benefit of raising inflation in that state equals the marginal cost. The derivative of ex-ante expected utility with respect to log inflation in state i, $\frac{d\mathbb{E}U}{d\pi_i}$, equals:

$$\frac{d\mathbb{E}U}{d\pi_i} = f_i U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \frac{\partial}{\partial Q^{LC}}\left(\sum_{j=1}^{N} f_j \frac{C_j^{1-\gamma}}{1-\gamma}\right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i, \tag{B.16}$$

$$= f_i U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \left(\sum_{j=1}^{N} f_j U'(C_j)\frac{\partial C_j}{\partial Q^{LC}}\right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i, \tag{B.17}$$

where we use the notation $U'(C_j) = \frac{\partial U}{\partial C_j}(C_j, \pi_j) = C_j^{-\gamma}$. Dividing by the probability f_i and setting $\frac{d\mathbb{E}U}{d\pi_i} = 0$ gives the first-order condition:

$$\alpha\pi_i = U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \frac{1}{f_i}\frac{dQ^{LC}}{d\pi_i}\mathbb{E}\left[U'(C_i)\frac{\partial C_i}{\partial Q^{LC}}\right]. \tag{B.18}$$

Differentiaing Eqn. (B.13) with respect to π_i shows that:

$$\frac{dQ^{LC}}{d\pi_i} = -f_i M_i^* exp\left(-\pi_i\right). \tag{B.19}$$

When the domestic output X_2 follows a continuous probability distribution we need to replace f_i by the density $f(X_2)$, π_i by $\pi_2^c(X_2)$, C_i by $C_2^c(X_2)$, and M_i^* by $M_2^*(X_2)$. For brevity, we omit the

arguments of π_2^c, C_2^c, and M_2^*, so Eqns. (B.18) and (B.19) become:[33]

$$\alpha \pi_2^c \quad = \quad U'(C_2) \frac{\partial C_2}{\partial \pi_2^c} + \frac{1}{f(X_2)} \frac{dQ^{LC}}{d\pi_2^c} \mathbb{E}\left[U'(C_2^c) \frac{\partial C_2^c}{\partial Q^{LC}}\right], \tag{B.20}$$

$$\frac{1}{f(X_2)} \frac{dQ^{LC}}{d\pi_2^c} \quad = \quad -M_2^* exp\left(-\pi_2^c\right). \tag{B.21}$$

This proves Eqns. (26) and (27) for the special case where X_2 is the only shock in the model.

Next, we extend the proof to the case with exchange rate shocks and separate international and domestic endowment shocks. Let f_{jk} denote the probability that domestic real output state X_j and real exchange rate $\mathcal{E}_k$ are realized. Note that we allow domestic output and the real exchange rate to be correlated. We write the probability that output state j is realized as:

$$f_j \quad = \quad \sum_k f_{jk}, \tag{B.22}$$

so f_j continues to be the analogue of the probability density $f(X_2)$ when X_2 follows a continuous distribution. When domestic output takes a discrete set of N values the government's problem simplifies to choosing $\pi_1, \pi_2, ..., \pi_N$ to maximize:

$$\mathbb{E}U \quad = \quad \sum_{j,k} f_{jk} \left(\frac{C_{jk}^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_j^2\right), \tag{B.23}$$

where consumption in state (j, k) is given by

$$C_{jk} \quad = \quad \bar{X} exp(x_j/\bar{X}) - \bar{D}\left(\frac{s}{R^*} \frac{exp(-\pi_j)}{Q^{LC}} + (1-s)\frac{1}{\mathcal{E}_k}\right) \tag{B.24}$$

and the LC bond price is given by:

$$Q^{LC} \quad = \quad \mathbb{E}\left[M_2^* exp\left(-\pi_2^c\right)\mathcal{E}_2\right], \tag{B.25}$$

$$= \quad \mathbb{E}\left[\mathbb{E}\left[M_2^*\mathcal{E}_2 \,|X_2\right] exp\left(-\pi_2^c\right)\right], \tag{B.26}$$

$$= \quad \sum_{j,k} f_{jk}\mathbb{E}\left[M_2^*\mathcal{E}_2 \,|X_2 = X_j\right] exp(-\pi_j), \tag{B.27}$$

$$= \quad \sum_j f_j\mathbb{E}\left[M_2^*\mathcal{E}_2 \,|X_2 = X_j\right] exp(-\pi_j). \tag{B.28}$$

Here, we have used the law of iterated expectations and the definition of f_j in Eqn. (B.22). Taking the derivative of expected domestic consumer utility, $\frac{d\mathbb{E}U}{d\pi_i}$, with respect to the optimal

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[33]Formally, the proof with a continous probability density relies on the Calculus of Variations but is otherwise analogous to the discrete probability case.

inflation rate in state i gives:

$$\frac{d\mathbb{E}U}{d\pi_i} = \sum_k f_{ik} U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} + \frac{\partial}{\partial Q^{LC}} \left(\sum_{j,k} f_{jk} \frac{C_{jk}^{1-\gamma}}{1-\gamma} \right) \frac{dQ^{LC}}{d\pi_i} - f_i \alpha \pi_i \quad (B.29)$$

$$= \sum_k f_{ik} U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} + \left(\sum_{j,k} f_{jk} U'(C_{jk}) \frac{\partial C_{jk}}{\partial Q^{LC}} \right) \frac{dQ^{LC}}{d\pi_i} - f_i \alpha \pi_i. \quad (B.30)$$

Setting $\frac{d\mathbb{E}U}{d\pi_i} = 0$ and dividing by the probability f_i gives the inflation first-order condition:

$$\alpha \pi_i = \sum_k \frac{f_{ik}}{f_i} U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} + \frac{1}{f_i} \frac{dQ^{LC}}{d\pi_i} \left(\sum_{j,k} f_{jk} U'(C_{jk}) \frac{\partial C_{jk}}{\partial Q^{LC}} \right), \quad (B.31)$$

$$= \mathbb{E} \left[U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} \middle| X_2 = X_i \right] + \frac{1}{f_i} \frac{dQ^{LC}}{d\pi_i} \mathbb{E} \left[U'(C_{jk}) \frac{\partial C_{jk}}{\partial Q^{LC}} \right]. \quad (B.32)$$

Taking the derivative of expression (B.28) with respect to π_i shows that:

$$\frac{1}{f_i} \frac{dQ^{LC}}{d\pi_i} = -exp(-\pi_i) \mathbb{E} \left[M_2^* \mathcal{E}_2 \middle| X_2 = X_i \right]. \quad (B.33)$$

If X_2 and $\mathcal{E}_2$ follow continuous distributions we need to replace f_i by $f(X_2)$, and π_i by $\pi_2^c(X_2)$, and C_{ik} by $C_2^c(X_2, \mathcal{E}_2)$ in Eqns. (B.32) and (B.33). For brevity, we omit the arguments of π_2^c and C_2^c in the main text. This proves Eqns. (26) and (27) in the main text.[34]

Proof of Inflation First-Order Condition without Commitment

We next prove the no-commitment government's inflation first-order condition Eqn. (25). Without commitment, the government's problem is simply to maximize (B.23) subject to (B.24) but taking the bond price Q^{LC} as given. If X_2 and $\mathcal{E}_2$ follow discrete probability distributions the first-order-condition becomes:

$$\alpha \pi_i = \sum_k \frac{f_{ik}}{f_i} U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i}. \quad (B.34)$$

$$= \mathbb{E} \left[U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} \middle| X_2 = X_i \right] \quad (B.35)$$

If X_2 and $\mathcal{E}_2$ follow continuous probability distributions, we need to replace π_i by $\pi_2^{nc}(X_2)$ and C_{ik} by $C_2^{nc}(X_2, \mathcal{E}_2)$. We again omit the arguments of π_2^{nc} and C_2^{nc}, giving:

$$\alpha \pi_2^{nc} = \mathbb{E} \left[U'(C_2^{nc}) \frac{\partial C_2^{nc}}{\partial \pi_2^{nc}} | X_2 \right]. \quad (B.36)$$

Proof of Eqn. (28)

---

[34]A formal proof with X_2 and $\mathcal{E}_2$ continuous again requires the Calculus of Variations but is otherwise analogous to the discrete case.

We now prove Eqn. (28) in the main paper. First,

$$\frac{d\mathbb{E}U}{ds} = \frac{d}{ds}\mathbb{E}\left[\frac{C_2^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_2^2\right], \tag{B.37}$$

$$= -\alpha\mathbb{E}\left[\pi_2\frac{d\pi_2}{ds}\right] + \mathbb{E}\left[U'(C_2)\frac{dC_2}{ds}\right]. \tag{B.38}$$

Now, recall from Eqns. (9) and (10) that we can write real domestic consumption as:

$$C_2 = X_2 - \frac{\bar{D}}{R^*}\left(sR^{LC} + (1-s)R^{FC}\right). \tag{B.39}$$

Because $R^{FC} = R^*$ is independent of s it follows that:

$$\frac{dC_2}{ds} = -\frac{\bar{D}}{R^*}\left(R^{LC} - R^{FC}\right) - \frac{s\bar{D}}{R^*}\left(\frac{dR^{LC}}{ds}\right). \tag{B.40}$$

Combining Eqns. (B.38) and (B.40) proves Eqn. (28) in the main paper. Because the government faces a constrained optimization problem of choosing s from the interval $[0,1]$, a necessary condition for an equilibrium is complementary slackness, that is either $\frac{d\mathbb{E}U}{ds} = 0$ and s is at an interior solution, or $s = 1$ and $\frac{d\mathbb{E}U}{ds} > 0$, or $s = 0$ and $\frac{d\mathbb{E}U}{ds} < 0$.

B.4   Proof of Proposition 1

Government without Commitment

We start by log-linearizing the first-order condition for the no-commitment government around $c_2 = 0$ and $\pi_2 = 0$. Recall that the first-order condition for the inflation problem of a government without commitment is given by:

$$\alpha\pi_2 = \mathbb{E}\left[U'(C_2)\frac{\partial C_2}{\partial \pi_2}\bigg| X_2\right]. \tag{B.41}$$

Before log-linearizing, we note that the following expressions hold exactly:

$$C_2 = \bar{X}\exp\left(x_2/\bar{X}\right) - \bar{D}\left((1-s) + s\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right)\right), \tag{B.42}$$

$$\frac{\partial C_2}{\partial \pi_2} = s\bar{D}\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right). \tag{B.43}$$

Eqn. (B.42) follows from combining Eqns. (9), (10), (11), and (12) and using that in the simplified special case $\mathcal{E}_2$ is constant and equal to one. Eqn. (B.43) is the partial derivative of Eqn. (B.42) with respect to π_2.

We start the log-linearization by noting that:

$$U'(C_2) = C_2^{-\gamma},$$
$$= \exp(-\gamma c_2)$$
$$\approx 1 - \gamma c_2. \tag{B.44}$$

We can approximately write period 2 consumption as a log-linear function of domestic output and inflation as follows:

$$
\begin{aligned}
C_2 &= \bar{X} \exp\left(x_2/\bar{X}\right) - \bar{D}\left((1-s) + s\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right)\right) & \text{(B.45)} \\
&\approx \bar{X} + x_2 - \bar{D}\left((1-s) + s\left(1 - (\pi_2 - \mathbb{E}\pi_2)\right)\right) \\
&= 1 + x_2 + s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right).
\end{aligned}
$$

Note that we have used the definition that $\bar{X} = \bar{D} + 1$, which ensures that C_2 equals one when all shocks are equal to zero. We have also dropped second- and higher-order terms. It then follows that log consumption approximately equals:

$$
\begin{aligned}
c_2 &\approx C_2 - 1 & \text{(B.46)} \\
&\approx x_2 + s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right). & \text{(B.47)}
\end{aligned}
$$

Substituting Eqn. (B.47) into Eqn. (B.44) shows that we can write domestic marginal consumption utility as an approximately log-linear function of domestic output and inflation:

$$
U'(C_2) \approx 1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right). \tag{B.48}
$$

Also, we have the log-linear approximation

$$
\begin{aligned}
\frac{\partial C_2}{\partial \pi_2} &= s\bar{D}\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right) & \text{(B.49)} \\
&\approx s\bar{D}\exp\left(-\left(\pi_2 - \mathbb{E}\pi_2\right)\right) \\
&\approx s\bar{D}\left(1 - \left(\pi_2 - \mathbb{E}\pi_2\right)\right).
\end{aligned}
$$

In the special case with no real exchange rate shocks ($\varepsilon_2 = 0$) and only one global output shock ($x_2 = x_2^*$), the conditional expectation on the right-hand side of Eqn. (B.41) is trivial and the first-order condition for the no-commitment government has the following log-linear approximation:

$$
\begin{aligned}
\alpha\pi_2 &= U'(C_2)\frac{\partial C_2}{\partial \pi_2}, \\
\alpha\pi_2 &\approx s\bar{D}\left(1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right)\right)\left(1 - \left(\pi_2 - \mathbb{E}\pi_2\right)\right), \\
&\approx s\bar{D}\left(1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right) - \left(\pi_2 - \mathbb{E}\pi_2\right)\right), & \text{(B.50)}
\end{aligned}
$$

where in the last row we have dropped quadratic terms in x_2 and π_2. Solving for π_2 gives the optimal no-commitment inflation policy:

$$
\left(\alpha + \gamma\left(s\bar{D}\right)^2 + s\bar{D}\right)\pi_2 = s\bar{D}\left(1 - \gamma x_2\right) + \left(s\bar{D} + \gamma\left(s\bar{D}\right)^2\right)\mathbb{E}\pi_2. \tag{B.51}
$$

Because lenders' expectations are rational (B.51) implies that $\mathbb{E}\pi_2 = \frac{s\bar{D}}{\alpha}$ and therefore that:

$$
\pi_2 = \frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2 + s\bar{D}}\left(1 - \gamma x_2\right) + \frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2 + s\bar{D}}\frac{s\bar{D} + \gamma\left(s\bar{D}\right)^2}{\alpha}. \tag{B.52}
$$

We keep only the lowest-order terms in the debt-to-GDP ratio $\bar{D}$ in the expression (B.52).

Using the first-order Taylor approximations

$$\frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2 + s\bar{D}} \approx \frac{s\bar{D}}{\alpha} + \mathcal{O}\left(\bar{D}^2\right) \tag{B.53}$$

$$\frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2 + s\bar{D}}\frac{s\bar{D} + \gamma\left(s\bar{D}\right)^2}{\alpha} \approx 0 + \mathcal{O}\left(\bar{D}^2\right) \tag{B.54}$$

shows that up to second- and higher-order terms in $\bar{D}$ the inflation policy function (B.52) has the following simple form:

$$\pi_2^{nc} \approx \frac{s\bar{D}}{\alpha} - \gamma\frac{s\bar{D}}{\alpha}x_2. \tag{B.55}$$

Government with Commitment

We log-linearize the first-order condition for the commitment government around $c_2 = 0$ and $\pi_2 = 0$. Substituting in for $\frac{\partial C_2}{\partial \pi_2}$ from Eqn. (B.43) and $\frac{1}{f(X_2)}\frac{dQ^{LC}}{d\pi_2(X_2)} = -\exp(-\pi_2^c(X_2))\mathbb{E}\left[M_2^*\mathcal{E}_2\middle| X_2\right]$ from Eqn. (27), the first-order condition for the inflation problem of a government with commitment is given by:

$$\alpha\pi_2 = \frac{s\bar{D}}{R^*Q^{LC}} \quad \mathbb{E}\left[U'\left(C_2\right)\exp\left(-\pi_2\right)\middle| X_2\right] - \mathbb{E}\left[U'\left(C_2\right)\frac{\partial C_2}{\partial Q^{LC}}\right]\exp(-\pi_2)\mathbb{E}\left[M_2^*\mathcal{E}_2\middle| X_2\right] \tag{B.56}$$

Note that Eqn. (26) is exact and is the starting point for our log-linearization. In the special case with no real exchange rate shocks ($\varepsilon_2 = 0$) and only one output shock ($x_2 = x_2^*$), Eqn. (14) can be written as:

$$\mathbb{E}\left[M_2^*\mathcal{E}_2\middle| X_2\right] = \delta^* \exp(-\gamma^* x_2), \tag{B.57}$$

$$= \frac{1}{R^*}\exp\left(-\gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2\right). \tag{B.58}$$

Taking the partial derivative of Eqn. (B.42) with respect to the LC bond price gives the exact expression:

$$\frac{\partial C_2}{\partial Q^{LC}} = \frac{s\bar{D}}{R^*}\frac{\exp(-\pi_2)}{\left(Q^{LC}\right)^2}. \tag{B.59}$$

Substituting Eqn. (B.58) and Eqn. (B.59) into Eqn. (B.56), the commitment government's first-order condition can be written as:

$$\begin{aligned}
\alpha\pi_2 = {} & \frac{s\bar{D}}{R^*Q^{LC}}U'\left(C_2\right)\exp\left(-\pi_2\right) \\
& -\frac{s\bar{D}}{\left(R^*Q^{LC}\right)^2}\mathbb{E}\left[U'\left(C_2\right)\exp(-\pi_2)\right]\exp\left(-\pi_2 - \gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2\right).
\end{aligned} \tag{B.60}$$

The conditional expectations drop out of Eqn. (B.60) because we are considering the special case with only one shock. We again use the log-linear expression Eqn. (B.48) and log-linearize the last

term in Eqn. (B.60) to obtain:

$$-\frac{s\bar{D}}{\left(R^{*}Q^{LC}\right)^{2}}\mathbb{E}\left[U'\left(C_{2}\right)\exp\left(-\pi_{2}\right)\right]\exp\left(-\pi_{2}-\gamma^{*}x_{2}-\frac{1}{2}\left(\gamma^{*}\right)^{2}\sigma_{x}^{2}\right),$$

$$\approx\;\;-s\bar{D}\mathbb{E}\left[\left(1-\gamma x_{2}-\gamma s\bar{D}\left(\pi_{2}-\mathbb{E}\pi_{2}\right)\right)\exp\left(-\pi_{2}\right)\right]\exp(2\mathbb{E}\pi_{2}-\pi_{2}-\gamma^{*}x_{2}-\frac{1}{2}\left(\gamma^{*}\right)^{2}\sigma_{x}^{2}),$$

$$\approx\;\;-s\bar{D}\mathbb{E}\left[1-\gamma x_{2}-\gamma s\bar{D}\left(\pi_{2}-\mathbb{E}\pi_{2}\right)-\pi_{2}\right]\exp(2\mathbb{E}\pi_{2}-\pi_{2}-\gamma^{*}x_{2}-\frac{1}{2}\left(\gamma^{*}\right)^{2}\sigma_{x}^{2}),$$

$$\approx\;\;-s\bar{D}\left(1-\gamma^{*}x_{2}-\left(\pi_{2}-\mathbb{E}\pi_{2}\right)\right).$$

The remaining terms in the commitment government's first-order condition (B.60) are identical to the no-commitment case, so the log-linear approximation to Eqn. (B.60) (and hence Eqn. (26)) is given by:

$$\begin{aligned}\alpha\pi_{2}\;\;=\;\;&s\bar{D}\left(1-\gamma x_{2}-\gamma s\bar{D}\left(\pi_{2}-\mathbb{E}\pi_{2}\right)-\left(\pi_{2}-\mathbb{E}\pi_{2}\right)\right)\\&-s\bar{D}\left(1-\gamma^{*}x_{2}-\left(\pi_{2}-\mathbb{E}\pi_{2}\right)\right).\end{aligned} \tag{B.61}$$

Taking expectations of the left-hand side and right-hand side of Eqn. (B.61) and imposing that lenders' expectations are rational shows that $\mathbb{E}\pi_{2}=0$, so optimal log inflation for a government with commitment equals:

$$\pi_{2}\;\;=\;\;\frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^{2}}\left(\gamma^{*}-\gamma\right)x_{2}. \tag{B.62}$$

Using the first-order Taylor approximation

$$\frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^{2}}\;\;=\;\;\frac{s\bar{D}}{\alpha}+\mathcal{O}\left(\bar{D}^{2}\right) \tag{B.63}$$

shows that up to second- and higher-order terms in $\bar{D}$ the inflation policy function (B.62) has the following simple form:

$$\pi_{2}\;\;\approx\;\;\frac{s\bar{D}}{\alpha}\left(\gamma^{*}-\gamma\right)x_{2} \tag{B.64}$$

B.5   Numerical Solution

We solve the model numerically using global projection methods. Our strategy for the numerical solution uses the following strategy:

No-Commitment

1. For any given LC debt share s, we choose the no-commitment inflation function $\pi_{2}^{nc}\left(x_{2}\right)$ to minimize the error in the government's inflation first-order condition while holding constant the LC debt share using the MATLAB function fminsearch.

2. In an outer loop, we maximize expected domestic consumer utility with respect to the LC debt share, s. For this step, we use the MATLAB function fminbnd over the interval $[0,1.001]$.

The maximization is over optimal expected domestic consumer utility conditional on the LC debt share, s, which we obtain by repeating step 1. above for every value of s.

Commitment

1. For any given LC debt share s, we choose the commitment policy function $\pi_2^c(x_2)$ to maximize expected domestic utility conditional on the LC debt share using the MATLAB function fminsearch.

2. In an outer loop, we maximize expected domestic consumer utility with respect to the LC debt share, s. For this step, we use the MATLAB function fminbnd over the interval $[0, 1.001]$. The maximization is over optimal expected domestic consumer utility conditional on the LC debt share, s, which we obtain by repeating step 1. above for every value of s.

B.5.1 Functional Form

Our numerical procedure considers inflation functions that can be written as a third-order polynomial in x_2:

$$\pi_2^{nc}(x_2) = b_1(s) + b_2(s)x_2 + b_3(s)x_2^2 + b_4(s)x_2^3, \tag{B.65}$$
$$\pi_2^c(x_2) = c_1(s) + c_2(s)x_2 + c_3(s)x_2^2 + c_4(s)x_2^3. \tag{B.66}$$

All coefficients may depend on the LC debt share, s. We use the following vectors as the starting point for our optimization routine:

$$b = [0.0183, -0.5363, 7.9462, -60] \tag{B.67}$$
$$c = [0.0028, 0.2061, -5.8417, 20]. \tag{B.68}$$

B.5.2 Bond Pricing Function

For any given inflation function, we need to solve for bond prices numerically. To facilitate numerical integration, we first project all exogenous random variables onto x_2 and a shock that is orthogonal to x_2 but is correlated with real exchange rates. We re-write international log real consumption as a component correlated with domestic output plus and an independent shock:

$$x_2^* = \lambda^* x_2 + \eta_2^*, \tag{B.69}$$

where we define:

$$\lambda^* = \lambda^{x,x^*} \frac{(\sigma^*)^2}{\sigma_x^2}, \tag{B.70}$$

$$\eta_2^* \perp x_2, \tag{B.71}$$

$$\left(\sigma_\eta^*\right)^2 = (\sigma^*)^2 - (\lambda^*)^2 \sigma_x^2. \tag{B.72}$$

Note that writing the relation between domestic and international endowments as (B.69) is consistent with assumptions (18) through (19) in the main paper. That η_2^* is uncorrelated with x_2 is not a new assumption and indeed follows from (18), (19), and the definition λ^*.

For the numerical solution, we use the notation $\rho^* = \lambda^{\varepsilon,x^*}$, so with Eqn. (B.69) the log real exchange rate can be written as:

$$\varepsilon_2 = \rho^* x_2^* + e_2 \tag{B.73}$$
$$\sigma_e^2 = \sigma_\varepsilon^2 - (\rho^* \lambda^*)^2 \sigma_x^2 - (\rho^*)^2 \left(\sigma_\eta^*\right)^2, \tag{B.74}$$

where σ_ε is the standard deviation of the real exchange rate and e_2 is uncorrelated with x_2^* and x_2. We can then write the real exchange rate as a component correlated with log domestic log output plus a shock, e_2^*, that is uncorrelated with domestic output:

$$\varepsilon_2 = (\rho^* \lambda^*) x_2 + e_2^*, \tag{B.75}$$
$$e_2^* = e_2 + \rho^* \eta_2^*,$$
$$(\sigma_e^*)^2 = \sigma_\varepsilon^2 - (\rho^* \lambda^*)^2 \sigma_x^2.$$

We next use that $1/R^* = \delta^* exp\left(\frac{1}{2}(\gamma^* \sigma^*)^2\right)$. The ratio of LC bond prices to $1/R^*$ then equals:

$$
\begin{aligned}
\frac{Q_1^{LC}}{1/R^*} &= \mathbb{E}_{x_2, e_2^*, \xi_2, \eta_2^*} \left[\exp\left(-\gamma^* x_2^* - \frac{1}{2}(\gamma^* \sigma^*)^2 - \pi_2 + \rho^* x_2^* + e_2 - \frac{1}{2}\sigma_\varepsilon^2\right) \right] \\
&= \mathbb{E}_{x_2, e_2^*, \xi_2, \eta_2^*} \left[\exp\left(-\gamma^* x_2^* - \frac{1}{2}(\gamma^* \sigma^*)^2 - \pi_2 + \rho^* x_2^* - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right) \right] \tag{B.76} \\
&= \mathbb{E}_{x_2, e_2^*, \xi_2} \left[\exp\left(-(\theta^* - \rho^* \lambda^*) x_2 - (\gamma^* - \rho^*) \eta_2^* - \pi_2 - \frac{1}{2}(\gamma^* \sigma^*)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right) \right] \tag{B.77} \\
&= \mathbb{E}_{x_2, e_2^*, \xi_2} \left[\exp\left(-(\theta^* - \rho^* \lambda^*) x_2 - \pi_2 + \frac{1}{2}(\gamma^* - \rho^*)^2 \left(\sigma_\eta^*\right)^2 - \frac{1}{2}(\gamma^* \sigma^*)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right) \right],
\end{aligned}
$$

where we define the international lenders' effective risk aversion over domestic output as:

$$\theta^* = \gamma^* \lambda^*. \tag{B.78}$$

For any given inflation policy $\pi_2(x_2)$ it is then relatively convenient to evaluate the following ratio numerically:

$$
\frac{Q_1^{LC}}{1/R^* \exp\left(\frac{1}{2}(\gamma^* - \rho^*)^2 \left(\sigma_\eta^*\right)^2 - \frac{1}{2}(\gamma^* \sigma^*)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right)} = \mathbb{E}_{x_2} \left[\exp\left(-(\theta^* - \rho^* \lambda^*) x_2 - \pi_2\right) \right] \tag{B.79}
$$

We evaluate the expectation (B.79) numerically using Gauss-Legendre quadrature with 30 node points, truncating the interval at -6 and +6 standard deviations of x_2.

B.5.3 No-Commitment Policy Function

For a given LC debt share s, we choose the coefficients (b_1, b_2, b_3, b_4) to set the government's inflation first-order condition as close as possible to zero. For any set of coefficients, we evaluate the first-order condition error:

$$Error(x_2) = \mathbb{E}_{e_2^*} \left[\left(-\alpha \pi_2^{nc}(x_2) + (C_2^{nc})^{-\gamma} \frac{dC_2^{nc}}{d\pi_2^{nc}} \right) \bigg| x_2 \right]. \tag{B.80}$$

The expectation (B.80) is averaged over e_2^* but conditional on domestic output x_2. At any value

of x_2 and e_2^*, no-commitment consumption is evaluated via:

$$C_2^{nc} = \bar{X} \exp(x_2/\bar{X}) - \bar{D} \left((1-s) \exp \left(-\varepsilon_2 + \frac{1}{2}\sigma_\varepsilon^2 \right) + s \frac{1/R^*}{Q_1^{LC}} \exp(-\pi_2^{nc}(x_2)) \right). \quad (B.81)$$

$$= \bar{X} \exp(x_2/\bar{X}) - \bar{D} \left((1-s) \exp \left(-(\rho^*\lambda^*) x_2 - e_2^* + \frac{1}{2}\sigma_\varepsilon^2 \right) + s \frac{1/R^*}{Q_1^{LC}} \exp(-\pi_2^{nc}(x_2)) \right),$$

and the partial derivative of no-commitment consumption with respect to no-commitment inflation is evaluated via:

$$\frac{dC_2^{nc}}{d\pi_2^{nc}} = \bar{D} s \frac{1/R^*}{Q^{LC}} \exp(-\pi_2^{nc}). \quad (B.82)$$

Because lenders have rational expectations the LC bond price is evaluated via Eqn. (B.79). We evaluate the expectation in Eqn. (B.80) over e_2^* numerically using Gauss-Legendre quadrature with 30 nodes and truncation at -6 and +6 standard deviations. We choose the vector of coefficients (b_1, b_2, b_3, b_4) to minimize the expected squared Euler equation error averaged over possible realizations of x_2, that is we minimize $\mathbb{E}_{x_2} \left[Error(x_2)^2 \right]$. That is, we minimize the weighted average of the squared Euler equation errors, where each realization of x_2 is weighted by its probability. To take the expectation over x_2, we again use Gauss-Legendre quadrature with 30 nodes and truncation at -6 and +6 standard deviations.

In the outer loop, we then maximize expected utility $\mathbb{E}_{x_2, e_2^*} \left[\left(-\frac{\alpha}{2} (\pi_2^{nc})^2 + \frac{C_2^{nc, 1-\gamma}}{1-\gamma} \right) \right]$ in Eqn. (B.80) over s, where for any s the coefficients (b_1, b_2, b_3, b_4) are found as described above.

B.5.4 Commitment Policy Function

For a given LC debt share s, we choose the commitment inflation policy function coefficients (c_1, c_2, c_3, c_4) to maximize the expectation:

$$\mathbb{E}_{x_2, e_2^*} \left[\left(-\frac{\alpha}{2} (\pi_2^c)^2 + \frac{C_2^{c, 1-\gamma}}{1-\gamma} \right) \right], \quad (B.83)$$

where we evaluate commitment consumption numerically:

$$C_2^c = \bar{X} \exp(x_2/\bar{X}) - \bar{D} \left((1-s) \exp \left(-(\rho^*\lambda^*) x_2 - e_2^* + \frac{1}{2}\sigma_\varepsilon^2 \right) + s \frac{1/R^*}{Q_1^{LC}} \exp(-\pi_2^c(x_2)) \right). \quad (B.84)$$

and LC bond prices update with the commitment inflation policy function through (B.79). All expectations are again evaluated numerically using Gauss-Legendre quadrature using the same grid points as before.

In the outer loop, we maximize $\mathbb{E}_{x_2, e_2^*} \left[\left(-\frac{\alpha}{2} (\pi_2^c)^2 + \frac{C_2^{c, 1-\gamma}}{1-\gamma} \right) \right]$ over s, where for any s the coefficients (c_1, c_2, c_3, c_4) are found as described above.

B.5.5 Model Moments

We use Gauss-Legendre quadrature to evaluate inflation moments numerically. For both x_2 and e_2^*, we use 30 nodes and truncate the interval at -6 and +6 standard deviations. We evaluate average inflation, the bond-stock beta and the LC bond risk premium numerically as:

$$\mathbb{E}^{model}\pi_2 \quad = \quad \mathbb{E}_{x_2,e_2^*}\pi_2, \tag{B.85}$$

$$\beta^{model}(bond_i, stock_i) \quad = \quad \frac{-1}{\lambda^{m,x}}\frac{\mathbb{E}_{x_2,e_2^*}\left[(\pi_2 - \mathbb{E}\pi_2)\,x_2\right]}{\sigma_x^2}, \tag{B.86}$$

$$RP^{LC,model} \quad = \quad \log\mathbb{E}_{x_2,e_2^*}\left[\exp(-\pi_2)\right] - \log Q^{LC} - r^*. \tag{B.87}$$

B.6 Plotting the Inflation Policy Functions

Figure B.1 shows inflation as a function of period 2 log domestic output. Consistent with the intuition from Proposition 1, DM inflation decreases in the worst states of the world, thereby providing international lenders with safe assets. A government with commitment optimally adopts pro-cyclical inflation, selling insurance to international lenders and earning the risk premium. This is similar to the problem studied in Farhi and Maggiori (2018) with a risk-neutral government and risk-averse lenders. By contrast, EM inflation increases in the worst states of the world. Intuitively, EM governments cannot commit to limiting their own consumption smoothing and instead have an incentive to use inflation in the worst states of the world to smooth domestic consumption fluctuations.

B.7 Calibration Robustness

### B.7.1 Separate EM and DM Local-International Endowment Loadings

We now verify that calibration results are unchanged if we match the domestic-international endowment loadings to the data separately for emerging and developed markets. We set $\lambda^{x,x^*} = 0.87$ for EMs and $\lambda^{x,x^*} = 0.97$ for DMs to match the average slope coefficients of domestic output growth with respect to US consumption growth averaged separately for EM and DM data. All other parameter values are as listed in Table 5. Table B.1 shows that the model moments are qualitatively and quantitatively unchanged compared to Table 6 in the main paper.

Table B.1: Model Moments with Separate Local-International Endowment Loadings

	EM (no commitment)		DM (commitment)		EM-DM	
	Data	Model	Data	Model	Data	Model
Average Inflation	3.92%	2.12%	1.73%	0.00%	2.20%	2.11%
Bond-Stock Beta	0.07	0.15	-0.10	-0.06	0.17	0.21
LC Debt Share	0.55	0.38	0.90	0.96	-0.35	-0.58
LC Bond RP	3.15%	4.27%	1.53%	2.03%	1.62%	2.24%

Note: All moments are in annualized natural units. Model parameters for the EM and DM calibrations are given in Table 5, except for the local-global endowment loadings, which we set to set to $\lambda^{x,x^*} = 0.87$ for EMs and $\lambda^{x,x^*} = 0.97$ for DMs. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (39).

Figure B.1: Inflation Policy Functions
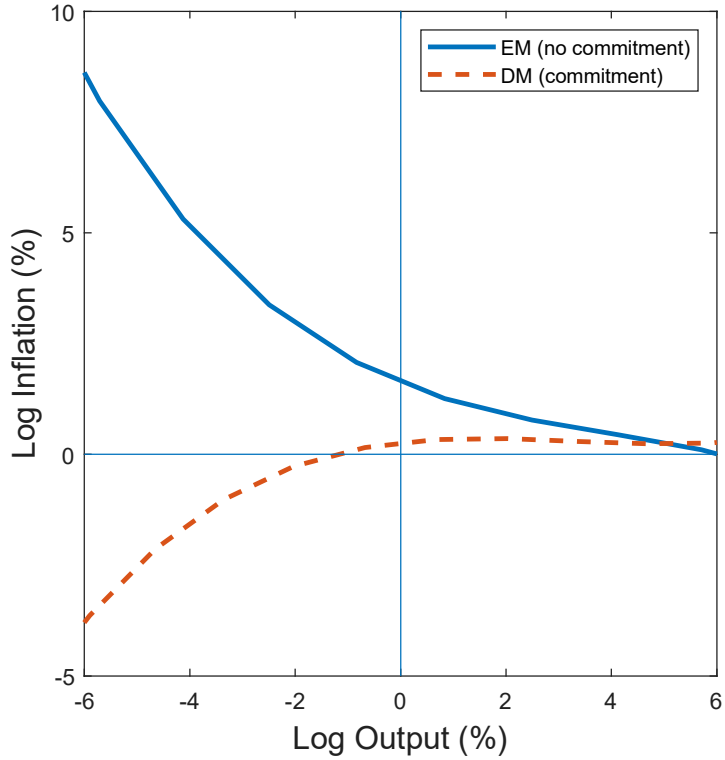
Note: This figure shows log inflation π_2 against log output x_2, defined in (17), both in annualized percent, in the calibrated model. The solid blue lines indicate the EM calibration, while the dashed red lines indicate the DM calibration.

B.7.2 Separate EM and DM Exchange Rate Processes

We now verify that the calibration results are qualitatively and quantitatively unchanged if we calibrate EM and DM real exchange rate processes separately to the data. To match the data moments averaged separately over EMs and DMs, we set $\sigma_\varepsilon = 10.4\%$ and $\lambda^{\varepsilon,x^*} = 1.33$ for the EM calibration and $\sigma_\varepsilon = 11.4\%$ and $\lambda^{\varepsilon,x^*} = 1.56$ for the DM calibration. All other parameter values are as listed in Table 5. The resulting model moments are shown in Table B.2.

Table B.2: Model Moments with Separate Exchange Rate Processes

	EM (no commitment)		DM (commitment)		EM-DM	
	Data	Model	Data	Model	Data	Model
Average Inflation	3.92%	2.07%	1.73%	0.00%	2.20%	2.06%
Bond-Stock Beta	0.07	0.15	-0.10	-0.05	0.17	0.20
LC Debt Share	0.55	0.37	0.90	0.91	-0.35	-0.54
LC Bond RP	3.15%	3.95%	1.53%	2.53%	1.62%	1.42%

Note: All moments are in annualized natural units. Model parameters for the EM and DM calibrations are given in Table 5, except for the exchange rate processes, which we calibrate separately to the data in this table. We set $\sigma_\varepsilon = 10.4\%$ and $\lambda^{\varepsilon,x^*} = 1.33$ for the EM calibration and $\sigma_\varepsilon = 11.4\%$ and $\lambda^{\varepsilon,x^*} = 1.56$ for the DM calibration. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (39).

B.7.3 Varying the DM Inflation Cost Parameter

We now verify the robustness of our calibration results to choosing different inflation cost parameters for the DM calibration. In our baseline calibration, the inflation cost parameter, α, is pinned down by the average difference in inflation between EMs and DMs in the data. In our baseline calibration, we choose the same inflation cost parameter for EMs and DMs for symmetry and to focus on the effect of credibility, which also varies across EM and DM calibrations. However, it appears plausible that the inflation cost of DMs is different from EMs. The DM inflation cost could be higher if DM policy makers assign a higher cost to inflation. Or it could be lower, if DM institutions are better able to smooth out frictions caused by inflation.

Here, we verify that the calibration results are similar for a range of values for the DM inflation cost parameter, α^{DM}. We consider a wide range of values for α^{DM}, setting it to one half and twice the baseline value of $\alpha = 4.28$. All other parameter values are set to the DM values in Table 5. The resulting model moments in Table B.3 show that DM model moments are largely insensitive to α^{DM}. Average inflation is equal to zero – the optimal level in the model – for all values of α, because a government with full commitment always chooses average inflation equal to the optimal level. The LC debt share is close to 0.90 for a wide range of inflation cost parameters, and the bond-stock beta varies within a relatively narrow range from -0.03 to -0.09.

Table B.3: Model Robustness to Different Inflation Costs

	DM Data	Baseline $\alpha = 4.28$	Low Inflation Cost $\alpha = 2.14$	High Inflation Cost $\alpha = 8.56$
Average Inflation	1.73%	0.00%	0.00%	0.00%
Bond-Stock Beta	-0.10	-0.05	-0.09	-0.03
LC Debt Share	0.90	0.91	0.92	0.90
LC Bond RP	1.53%	2.22%	1.90%	2.46%

Note: All moments are in annualized natural units. Model parameters are given by the DM calibration in Table 5, except for the inflation cost, α, which is listed in the column header. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (39).