Capital Allocation and Productivity in South Europe

Online Appendix

Gita Gopinath Sebnem Kalemli-Ozcan Loukas Karabarbounis Carolina Villegas-Sanchez

July 2015

A Data Cleaning and Summary Statistics

Our dataset combines firm-level information across different BvD products (ORBIS disk 2005, ORBIS disk 2009, ORBIS disk 2013, AMADEUS online 2010 from WRDS, and AMADEUS disk 2014). We work only with unconsolidated accounts. We clean the data in four steps. First, we clean the data of basic reporting mistakes. Second, we verify the internal consistency of balance sheet information. The first two steps are implemented at the level of the total economy. Third, we do a more specific quality control on variables of interest for firms in the manufacturing sector. Finally, we winsorize variables.

A.1 Cleaning of Basic Reporting Mistakes

We implement the following steps to correct for basic reporting mistakes:

1. We drop firm-year observations that have missing information on total assets and operating revenues and sales and employment.

2. We drop firms if total assets are negative in any year, or if employment is negative or greater than 2 millions in any year, or if sales are negative in any year, or if tangible fixed assets are negative in any year.

3. We drop firm-year observations with missing, zero, or negative values for materials, operating revenue, and total assets.

4. We drop firm-year observations with missing information regarding their industry of activity.

A.2 Internal Consistency of Balance Sheet Information

We check the internal consistency of the balance sheet data by comparing the sum of variables belonging to some aggregate to their respective aggregate. We construct the following ratios:

1. The sum of tangible fixed assets, intangible fixed assets, and other fixed assets as a ratio of total fixed assets.

2. The sum of stocks, debtors, and other current assets as a ratio of total current assets.

3. The sum of fixed assets and current assets as a ratio of total assets.

4. The sum of capital and other shareholder funds as a ratio of total shareholder funds.

5. The sum of long term debt and other non-current liabilities as a ratio of total non-current liabilities.

6. The sum of loans, creditors, and other current liabilities as a ratio of total current liabilities.

7. The sum of non-current liabilities, current liabilities, and shareholder funds as a ratio of the variable that reports the sum of shareholder funds and total liabilities.

After we construct these ratios, we estimate their distribution for each country separately. We then exclude from the analysis extreme values by dropping observations that are below the 0.1 percentile or above the 99.9 percentile of the distribution of ratios.

A.3 Further Quality Checks for Manufacturing Firms

After the implementation of the basic cleaning steps in the total economy sample we turn to examine the quality of the variables for firms in the manufacturing sector used in our analysis. At each stage, we provide the number of dropped observations for the Spanish sample. We start with 1,127,566 observations that correspond to 149,779 firms in the Spanish manufacturing sector.

1. **Age.** We construct the variable "age" of the firm as the difference between the year of the balance sheet information and the year of incorporation of the firm plus one. We drop

firms that report dates of incorporation that imply non-positive age values. This step reduces the observations in our sample by 35.

2. **Liabilities.** As opposed to listed firms, non-listed firms do not report a separate variable "Liabilities." For these firms we construct liabilities as the difference between the sum of shareholder funds and liabilities ("SHFUNDLIAB") and shareholder funds or equity ("SHFUNDS"). We drop observations with negative or zero values. This step reduces the observations in our sample by 1,374.

   We could also have computed liabilities as the sum of current liabilities and non-current liabilities. However, we find that there are more missing observations if we follow this approach. Nevertheless, for those observations with non-missing information we compare the value of liabilities constructed as the difference between SHFUNDLIAB and SHFUNDS and the value of liabilities constructed as the sum of current and non-current liabilities. We look at the ratio of the first measure relative to the second measure. Due to rounding differences the ratio is not always exactly equal to one and so we remove only firm-year observations for which this ratio is greater than 1.1 or lower than 0.9. This step reduces the observations in our sample by 1,349.

   We drop firm-year observations with negative values for current liabilities, non-current liabilities, current assets, loans, creditors, other current liabilities, and long term debt. This step reduces the observations in our sample by 40. Finally, we drop observations for which long term debt exceeds total liabilities. This step reduces the observations in our sample by 44.

3. **Net Worth.** We construct net worth as the difference between total assets ("TOTASSTS") and total liabilities. This variable should be equal to the variable SHFUNDS provided by the BvD. We drop observations that violate this identity. This step reduces the observations in our sample by 32.

4. **Wage Bill.** We drop firm-year observations with missing, zero, or negative values for the wage bill. This step reduces the observations in our sample by 20,571.

5. **Capital Stock.** We construct our measure of the capital stock as the sum of tangible fixed assets and intangible fixed assets and, therefore, we drop observations with negative values for intangible fixed assets. This step reduces the observations in our sample by 2,176. We drop observations with missing or zero values for tangible fixed assets. This step reduces the observations in our sample by 42,744. We drop firm-year observations when the ratio of tangible fixed assets to total assets is greater than one. This step reduces the observations in our sample by 4,921. We drop firm-year observations with negative depreciation values. This step reduces the observations in our sample by 1.

6. **Capital-Labor Ratio.** Next, we examine the quality of the capital to the wage bill variable. We first drop firms if in any year they have a capital to wage bill ratio in the bottom 0.1 percent of the distribution. This step reduces the observations in our sample by 5,801. After we remove the very high extreme values of this ratio there is a very positively skewed distribution of the ratio and, therefore, we drop observations with ratios higher than the 99.9 or lower than the 0.1 percentile. This step reduces the observations in our sample by 1,836.

7. **Equity.** We drop observations with negative SHFUNDS (equity or shareholders funds). This step reduces the observations in our sample by 123,208. We drop observations in the bottom 0.1 percentile in the ratio of other shareholders funds (that includes items such as reserve capital and minority interests) to TOTASSTS. This step reduces the observations in our sample by 925.

8. **Leverage Ratios.** We calculate the ratios of tangible fixed assets to shareholder funds and the ratio of total assets to shareholder funds and drop extreme values in the bottom 0.1 or top 99.9 percentile of the distribution of ratios. This step reduces the observations in our sample by 3,555.

9. **Value Added.** We construct value added as the difference between operating revenue and materials and drop negative values. This step reduces the observations in our sample by 3,966. We construct the ratio of wage bill to value added and drop extreme values in

the bottom 1 percentile or the top 99 percentile. This step reduces the observations in our sample by 18,362. In this case we choose the 1 and 99 percentiles as thresholds to drop variables because the value of the ratio at the 99 percentile exceeds 1. In addition, we drop firm-year observations if the ratio is greater than 1.1. This step reduces the observations in our sample by 11,629.

The final sample for Spain has 884,997 firm-year observations, corresponding to 124,993 firms in the manufacturing sector. This is what we call the "full sample" in our analysis. The "permanent sample" is a subset of the full sample, consisting of firms with identifiers that are observed continuously for all years between 1999 and 2012. The permanent sample has 193,452 observations, corresponding to 13,818 firms.

A.4 Winsorization

We winsorize at the 1 and the 99 percentile variables such as value added, tangible fixed assets, wage bill, operating revenue, materials, total assets, shareholder funds, fixed assets, the sum of tangible and intangible fixed assets (capital), other fixed assets, and total liabilities. We winsorize at the 1 and the 99 percentile all of our estimated firm productivity variables and the productivity residuals from an AR(1) process used to construct our uncertainty measures. Similarly, we winsorize at the 1 and the 99 percentile net worth, cash flow to total assets, and sales to total assets. In addition, we winsorize at the 0.1 and 99.9 percentile the MRPK and the MRPL before calculating our dispersion measures to make our dispersion measures less sensitive to outliers. Finally, we winsorize at the 2 and the 98 percentile the net investment to lagged capital ratio used in our regressions because this ratio has a very long right tail.

A.5 Summary Statistics

Table A.1 presents summary statistics for all countries in our dataset. Except for employment, all entries in the table are in millions of euros. Value added, wage bill, total assets, and total liabilities are deflated with gross output price indices at the two digits industry level with a base year of 2005. For France and Norway we do not have these price deflators at the two

Table A.1: Summary Statistics of Selected Variables

		Permanent Sample		**Full Sample**	
Country	Statistic	Mean	Standard Deviation	Mean	Standard Deviation
Spain	Value Added	1.23	3.00	2.16	4.08
	Employment	24.87	138.73	42.07	234.69
	Wage Bill	0.54	1.19	0.91	1.58
	Capital Stock	0.89	2.47	1.47	3.20
	Total Assets	2.66	7.10	4.52	9.32
	Total Liabilities	1.54	4.12	2.40	5.22
Italy	Value Added	2.73	5.31	4.76	6.81
	Employment	36.12	171.25	55.75	156.40
	Wage Bill	0.86	1.62	1.49	2.07
	Capital Stock	1.36	3.17	2.35	4.04
	Total Assets	5.31	11.4	9.10	14.30
	Total Liabilities	3.73	7.75	5.95	9.51
Portugal	Value Added	0.75	1.91	3.54	8.50
	Employment	22.83	71.51	39.19	152.09
	Wage Bill	0.30	0.65	1.60	3.51
	Capital Stock	0.51	1.49	1.39	4.37
	Total Assets	1.65	4.43	5.03	14.70
	Total Liabilities	1.00	2.65	3.18	9.27
Germany	Value Added	18.90	38.80	37.60	52.20
	Employment	183.47	554.55	320.62	620.60
	Wage Bill	7.39	14.40	14.40	19.20
	Capital Stock	6.41	15.80	12.80	21.60
	Total Assets	26.40	64.30	53.50	87.80
	Total Liabilities	16.40	40.80	32.40	55.80
France	Value Added	2.53	7.46	3.33	8.47
	Employment	39.51	399.07	48.43	280.92
	Wage Bill	0.95	2.52	1.26	2.85
	Capital Stock	0.66	2.40	0.85	2.73
	Total Assets	3.07	10.20	4.04	11.60
	Total Liabilities	1.80	6.04	2.24	6.71
Norway	Value Added	2.71	7.38	0.98	2.17
	Employment	29.76	122.32	28.34	76.94
	Wage Bill	1.26	3.11	0.38	0.74
	Capital Stock	1.08	3.80	0.64	1.63
	Total Assets	3.95	12.90	2.09	4.93
	Total Liabilities	2.56	8.27	1.21	2.88

digits and, therefore, we deflate with the price index for total manufacturing. The capital stock is the sum of tangible and intangible fixed assets and is deflated with the economy-wide price of investment goods. For each year, we first calculate means and standard deviations without weighting across all firms and industries. Entries in the table denote the means and standard deviations averaged across all years in each country.

B Production Function Estimates

In this appendix we discuss estimates of the production function. We estimate the production function separately for each two-digit industry s:

$$\log y_{it} = d_t(s) + \beta^{\ell}(s) \log \ell_{it} + \beta^{k}(s) \log k_{it} + \log Z_{it} + \epsilon_{it}, \tag{A.1}$$

where $d_t(s)$ is a time fixed effect, y_{it} denotes nominal value added divided by the two-digit output price deflator, ℓ_{it} denotes the wage bill divided by the same output price deflator, and k_{it} denotes the (book) value of fixed assets divided by the aggregate price of investment goods. In equation (A.1), $\beta^{\ell}(s)$ denotes the elasticity of value added with respect to labor and $\beta^{k}(s)$ denotes the elasticity of value added with respect to capital. These elasticities vary at 24 industries defined by their two-digit industry classification. Our estimation uses the methodology developed in Wooldridge (2009) and we refer the reader to his paper for details of the estimation process. Given our estimated elasticities $\hat{\beta}^{\ell}(s)$ and $\hat{\beta}^{k}(s)$, we then calculate firm (log) productivity as $\log Z_{it} = \log y_{it} - \hat{\beta}^{\ell}(s) \log \ell_{it} - \hat{\beta}^{k}(s) \log k_{it}$.

In Table A.2 we present summary statistics for the sum of the elasticities $\hat{\beta}^{l}(s) + \hat{\beta}^{k}(s)$ estimated from regression (A.1) separately in each country. Our estimates look reasonable as the sum of elasticities is close to 0.80. Because we do not observe prices at the firm level, these elasticities are more appropriately defined as revenue elasticities. In the presence of markups, these estimates are lower bounds for the true elasticities in the production function. With a constant returns to scale production function, we would estimate a sum of elasticities equal to 0.80 when the markup equals 20 percent.

The summary statistics in Table A.2 exclude industries for which at least one of the coefficients estimated with the Wooldridge (2009) extension of the Levinsohn and Petrin (2003)

Table A.2: Summary Statistics of Production Function Estimation

Sum of Elasticities	Spain	Italy	Portugal	Germany	France	Norway
Mean	0.80	0.71	0.79	0.85	0.82	0.76
Median	0.80	0.70	0.77	0.84	0.83	0.76
Max	0.91	0.81	0.87	1.03	0.90	0.90
Min	0.75	0.59	0.72	0.74	0.62	0.58
Standard Deviation	0.04	0.04	0.05	0.07	0.06	0.07

procedure results in a zero, negative, or missing value. Across 6 countries for which we separately estimate elasticities at the two-digit industry level we have few such industries (2 in Spain, 4 in Italy, 6 in Portugal, 3 in Germany, 2 in France, and 5 in Norway). Typically, these industries have a very small number of firms and account for a negligible fraction of total manufacturing activity. Therefore, we do not drop them from our analysis.

C Comparison of Regressions With Finance Literature

Table A.3 compares the investment and debt regressions using our regressors to similar regressions but with regressors more commonly used by the finance literature. All regressions include firm fixed effects and industry-year fixed effects. The regressors that we used in the main text are motivated by our theory in which productivity, net worth, and capital are the state variables that summarize firm capital and debt decisions.

As the table shows, using the sales to capital ratio instead of productivity and the cash flow to capital ratio instead of log net worth leads to highly similar results. With one exception, all coefficient signs are the same across the two types of regressions. All coefficients except for the coefficient on the cash flow to capital ratio in the debt regression in the full sample are statistically significant at the 1 percent level.

Table A.3: Firm-Level Investment and Debt Decisions in the Data (Spain, 1999-2007)

Dependent Variable	Regressors	Permanent Sample	Full Sample
$(k'-k)/k$	$\log Z$	0.10***	0.11***
		(0.011)	(0.007)
	$\log a$	0.09***	0.09***
		(0.006)	(0.003)
	$\log k$	-0.46***	-0.63***
		(0.005)	(0.003)
$(k'-k)/k$	$\log(\text{Sales}/k)$	0.13***	0.14***
		(0.008)	(0.005)
	Cash Flow/k	0.04***	0.05***
		(0.007)	(0.003)
	$\log k$	-0.31***	-0.47***
		(0.008)	(0.005)
$(b'-b)/k$	$\log Z$	-0.38***	-0.48***
		(0.027)	(0.022)
	$\log a$	0.15***	0.14***
		(0.013)	(0.009)
	$\log k$	-0.34***	-0.54***
		(0.011)	(0.010)
$(b'-b)/k$	$\log(\text{Sales}/k)$	-0.45***	-0.49***
		(0.020)	(0.014)
	Cash Flow/k	0.07***	0.00
		(0.020)	(0.013)
	$\log k$	-0.66***	-0.91***
		(0.019)	(0.015)

D Further Results in the Baseline Model

In Table A.4 we present the standard errors in the regressions described in Section 6.1.2. All coefficient estimates are statistically significant at the 1 percent level.

Next, we elaborate on our baseline results in Section 6.1.3 and present aggregate impulses in response to the decline in r_t as a function of the adjustment cost parameter ψ and the borrowing threshold κ. For convenience, we repeat the baseline case with $\psi = 3.1$ and $\kappa = 4.1$ in Figure A.1.

Table A.4: Capital Growth and Initial Net Worth

Dependent Variable	Regressors	Permanent Sample	Full Sample
$(k_{07}-k_{99})/k_{99}$	$\log Z_{99}$	1.14***	1.49***
		(0.08)	(0.06)
	$\log a_{99}$	0.17***	0.15***
		(0.03)	(0.02)
	$\log k_{99}$	-0.96***	-1.11***
		(0.03)	(0.03)
$(b_{07}-b_{99})/k_{99}$	$\log Z_{99}$	1.12***	1.47***
		(0.11)	(0.10)
	$\log a_{99}$	0.20***	0.11***
		(0.04)	(0.03)
	$\log k_{99}$	-0.86***	-0.98***
		(0.05)	(0.04)

Figure A.2 shows impulses in response to the decline in r_t in a model without adjustment costs and financial frictions (corresponding to $\psi = 0.0$ and $\kappa = 0.0$). In this model, capital and debt are growing but there is little change in the dispersion of the $\log(\text{MRPK})$ or in $\log(\text{TFP})$. Figure A.3 shows that the model with only adjustment costs and no financial frictions ($\psi = 3.1$ and $\kappa = 0.0$) also does not yield significant changes in the dispersion of the $\log(\text{MRPK})$ or in $\log(\text{TFP})$. Finally, in Figure A.4 we show the model with financial frictions and no adjustment costs ($\psi = 0.0$ and $\kappa = 4.2$). In this model we obtain a significant increase in the dispersion of the MRPK but a very small decline in $\log(\text{TFP})$. In the absence of adjustment costs, firms with a high permanent productivity component z^P increase significantly their capital stock and overcome instantaneously their borrowing constraints.

E Dispersion of the MRPK in a Simpler Model

In this appendix we use a simpler model to derive in closed-form the response of the dispersion of the $\log(\text{MRPK})$ to various shocks. In this simpler model we show that the dispersion of the MRPK increases when: (i) the cost of capital decreases; (ii) financial frictions increase; (iii) exogenous aggregate productivity or demand increase. All responses have the same sign as the

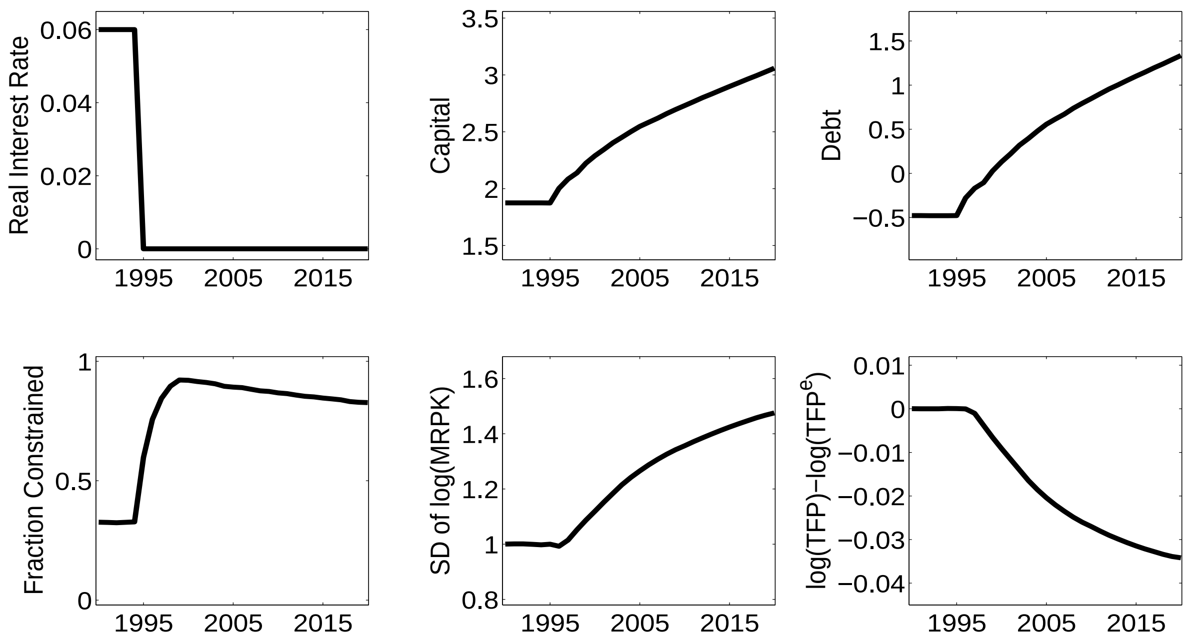


Figure A.1: Decline in the Real Interest Rate ($\psi = 3.1$ and $\kappa = 4.2$)

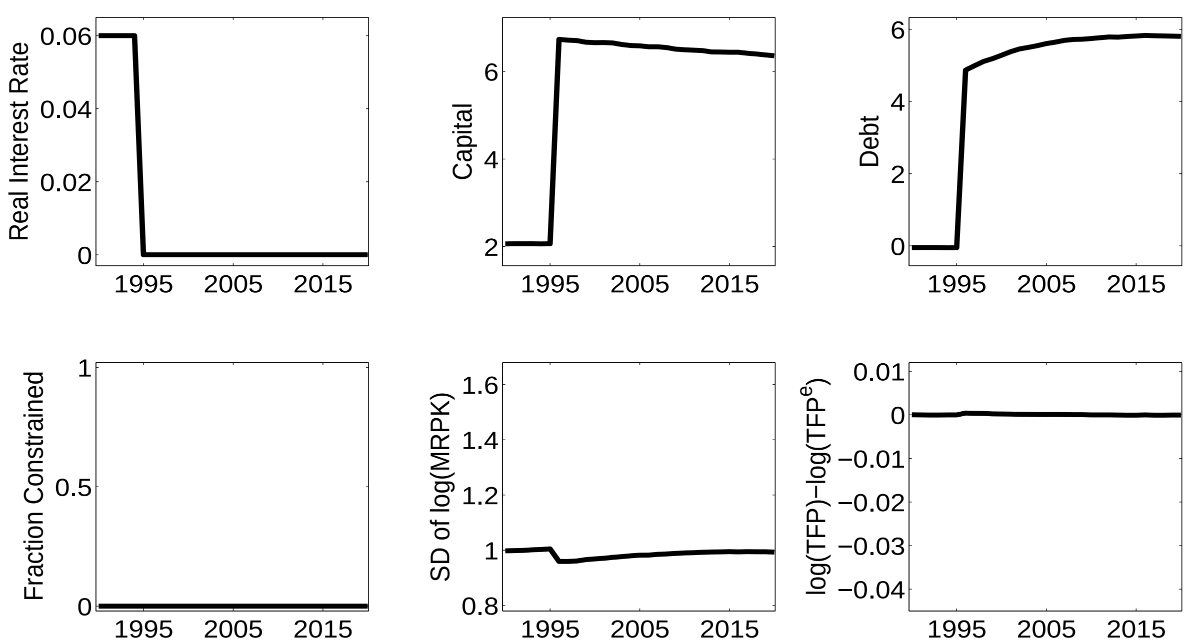


Figure A.2: Decline in the Real Interest Rate ($\psi = 0.0$ and $\kappa = 0.0$)

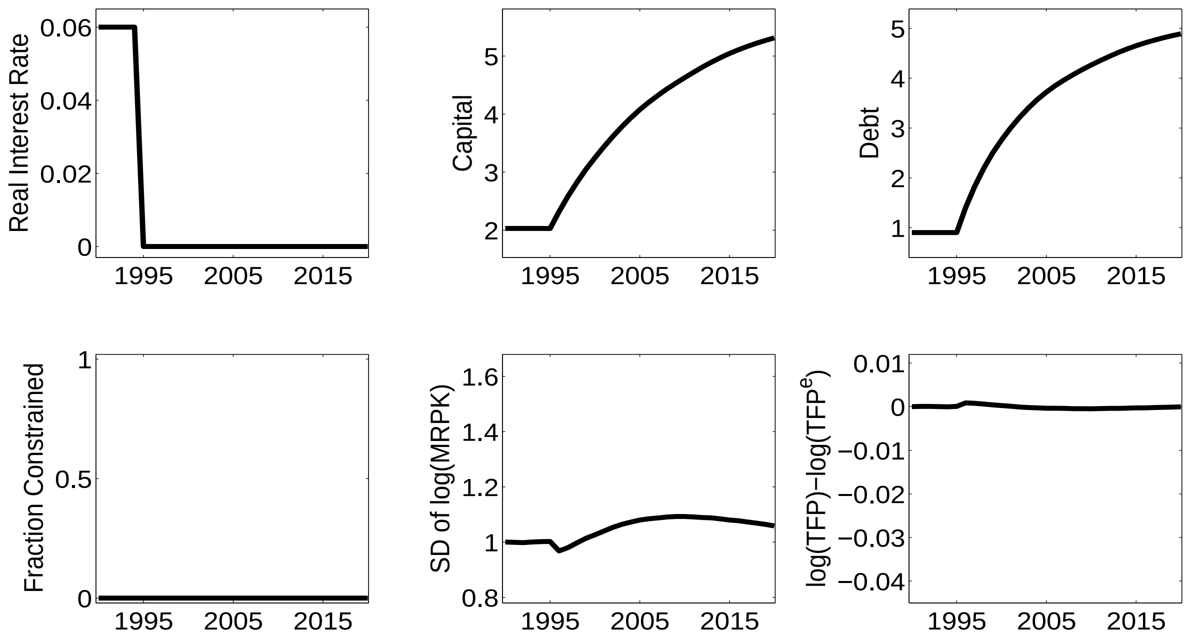


Figure A.3: Decline in the Real Interest Rate ($\psi = 3.1$ and $\kappa = 0.0$)

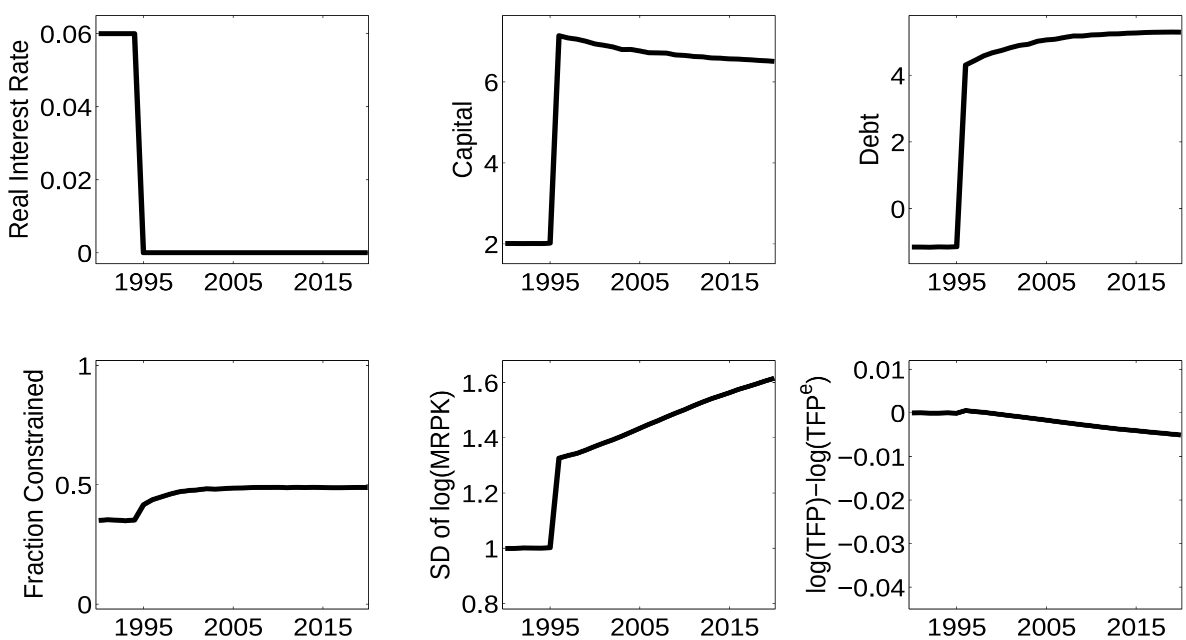


Figure A.4: Decline in the Real Interest Rate ($\psi = 0.0$ and $\kappa = 4.2$)

responses generated by our richer model.

The environment is close to one considered by Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015). Similar to these papers, we assume that firms maximize the discounted present value of utility flows under perfect foresight about next-period's productivity. This assumption implies that debt and capital are perfect substitutes and effectively renders the choice of capital a static decision. A firm's budget constraint is:

$$c + a' = \pi(Z^A z, k) + (1+r)a - Rk. \tag{A.2}$$

where c is consumption, a is assets, π is profits, r is the interest rate, k is capital, and $R = r + \delta$ denotes the cost of capital.

The reduced-form profit function is given by:

$$\pi(Z^A z, k) = \frac{Z^A z}{\eta} k^{\eta}, \tag{A.3}$$

where Z^A is the aggregate component of productivity and z denotes the idiosyncratic component of productivity (which lumps together both the transitory and the permanent component of idiosyncratic productivity). While for simplicity we call it productivity, the product $Z^A z$ represents a reduced-form for productivity, demand, and wages. The concavity of the profit function, $\eta < 1$, can reflect a combination of decreasing returns to scale and a downward sloping demand for a firm's product.[1] The marginal revenue product of capital is:

$$\text{MRPK} = Z^A z k^{\eta - 1}. \tag{A.4}$$

Following Midrigan and Xu (2014), Moll (2014), and Buera and Moll (2015), we specify the borrowing constraint as:

$$k \leq \theta a, \tag{A.5}$$

where the parameter $\theta \geq 1$ captures the degree of financial frictions. A lower θ denotes more severe financial frictions. When $\theta = 1$, firms cannot borrow and have to self-finance capital accumulation. When $\theta \to \infty$, there are no financial frictions in capital accumulation.

[1] The assumption that $\eta < 1$ is an important difference between our model in this section and some of the previous literature. If the profit function was linear, then firm size would be pinned down by the financial constraint. Below we show that when all firms are constrained in the initial equilibrium, small changes in R, θ, or Z^A do not affect MRPK dispersion.

Unconstrained firms equalize the MRPK to the cost of capital $R = r + \delta$. The unconstrained level of capital is:

$$k^* = \left(\frac{Z^A z}{R}\right)^{\frac{1}{1-\eta}}, \tag{A.6}$$

and so capital is given by $k = \min\{k^*, \theta a\}$. Firms with productivity z above some threshold Z^* are constrained and can finance capital only equal to θa. The cutoff productivity level is given by:

$$Z^* = (\theta a)^{1-\eta} \frac{R}{Z^A}. \tag{A.7}$$

We denote the joint distribution of productivity and net worth at any particular point in time by $G(a, z)$. We denote the probability density function of productivity z conditional on assets a by $f(z|a)$, the cumulative density function of z conditional on a by $F(z|a)$, and the marginal probability density function of a by $g(a)$. We denote by z_L and z_H the lowest and highest levels of productivity.

The goal is to solve for changes in the variance of the log (MRPK) in response to changes the cost of capital R, financial frictions θ, and aggregate productivity or demand Z^A. Our solutions should be understood as the first period of an impulse response. We note that assets a are predetermined at the period of the shock, which allows us to treat their distribution as given.

As a preliminary step for our comparative statics we calculate the following quantities:

$$\log(\text{MRPK}) = \log(Z^A z) - (1-\eta)\log(k) = \begin{cases} \log(R), & \text{if } z \leq Z^* \\ \log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right), & \text{if } z > Z^* \end{cases}, \tag{A.8}$$

$$\mathbb{E}\log(\text{MRPK}) = \int_a \left[\int_{z_L}^{Z^*} \log(R) f(z|a)dz + \int_{Z^*}^{z_H} \log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right) f(z|a)dz\right] g(a)da, \tag{A.9}$$

$$(\log(\text{MRPK}))^2 = \left(\log(Z^A z) - (1-\eta)\log(k)\right)^2 = \begin{cases} (\log(R))^2, & \text{if } z \leq Z^* \\ \left(\log\left(Z^A z \theta^{\eta-1} a^{\eta-1}\right)\right)^2, & \text{if } z > Z^* \end{cases}, \tag{A.10}$$

$$\mathbb{E}\left[(\log(\mathrm{MRPK}))^2\right] = \int_a \left[\int_{z_L}^{Z^*} (\log(R))^2 f(z|a)dz + \int_{Z^*}^{z_H} \left(\log\left(Z^A z\theta^{\eta-1}a^{\eta-1}\right)\right)^2 f(z|a)dz\right] g(a)da. \tag{A.11}$$

We use these expectations to calculate the response of the variance $\mathrm{Var}(\log(\mathrm{MRPK}))$ to any shock X:

$$\frac{\partial \mathrm{Var}(\log(\mathrm{MRPK}))}{\partial X} = \frac{\partial \mathbb{E}\left[(\log(\mathrm{MRPK}))^2\right]}{\partial X} - 2\left[\mathbb{E}\log(\mathrm{MRPK})\right]\frac{\partial\left[\mathbb{E}\log(\mathrm{MRPK})\right]}{\partial X}. \tag{A.12}$$

E.1 Changes in the Cost of Capital

We consider how small changes in R impact the dispersion of the MRPK. Using Leibniz's rule we obtain:

$$\frac{\partial\left[\mathbb{E}\log(\mathrm{MRPK})\right]}{\partial R} = \int_a \left[\frac{F(Z^*|a)}{R} + \log(R)f(Z^*|a)\frac{\partial Z^*}{\partial R} - \log\left(Z^A Z^*\theta^{\eta-1}a^{\eta-1}\right) f(Z^*|a)\frac{\partial Z^*}{\partial R}\right] g(a)da.$$

Note that the two last terms in the integral cancel out because at the cutoff Z^* we have $R = Z^A Z^*\theta^{\eta-1}a^{\eta-1}$. Therefore:

$$\frac{\partial\left[\mathbb{E}\log(\mathrm{MRPK})\right]}{\partial R} = \frac{1}{R}\int_a F(Z^*|a)\, g(a)da. \tag{A.13}$$

Similarly:

$$\frac{\partial \mathbb{E}\left[(\log(\mathrm{MRPK}))^2\right]}{\partial R} = \frac{2\log(R)}{R}\int_a F(Z^*|a)\, g(a)da. \tag{A.14}$$

Plugging (A.13) and (A.14) into (A.12) we obtain:

$$\frac{\partial \mathrm{Var}(\log(\mathrm{MRPK}))}{\partial R} = \left(\frac{2}{R}\right)(\log R - \mathbb{E}\log(\mathrm{MRPK}))\int_a F(Z^*|a)\, g(a)da \leq 0. \tag{A.15}$$

The variance is weakly decreasing in R because $\log R \leq \mathbb{E}\log(\mathrm{MRPK})$ and $F(Z^*|a) \geq 0$ at the initial point of differentiation. Note that the variance does not change in the limiting cases of no firm being initially constrained (i.e. $\log R = \mathbb{E}\log(\mathrm{MRPK})$) or all firms being initially constrained (i.e. $F(Z^*|a) = 0$). Finally, we note that locally R does not affect dispersion through the cutoff Z^*. This assumes that there is a smooth distribution of z conditional on a and that there are no mass points.

E.2 Changes in Financial Frictions

We consider how small changes in θ impact the dispersion of the MRPK. We obtain:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial\theta}=\int_a\left[\log(R)f(Z^*|a)\frac{\partial Z^*}{\partial\theta}-\log\left(Z^AZ^*\theta^{\eta-1}a^{\eta-1}\right)f(Z^*|a)\frac{\partial Z^*}{\partial\theta}+\left(\frac{\eta-1}{\theta}\right)\int_{Z^*}^{z_H}f(z)dz\right]g(a)da.$$

Note that the two first terms in the integral cancel out because at the cutoff Z^* we have $R=Z^AZ^*\theta^{\eta-1}a^{\eta-1}$. Therefore:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial\theta}=\left(\frac{\eta-1}{\theta}\right)\int_a\int_{Z^*}^{z_H}f(z)dzg(a)da=\left(\frac{\eta-1}{\theta}\right)\int_a\left(1-F\left(Z^*|a\right)\right)g(a)da. \tag{A.16}$$

We also have:

$$\begin{aligned}\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta}=&\int_a\left[\left(\log(R)\right)^2f(Z^*|a)\frac{\partial Z^*}{\partial\theta}-\left(\log\left(Z^AZ^*\theta^{\eta-1}a^{\eta-1}\right)\right)^2f(Z^*|a)\frac{\partial Z^*}{\partial\theta}\right]g(a)da\\&+\int_a\left[\int_{Z^*}^{z_H}\frac{2(\eta-1)\log\left(Z^Az\theta^{\eta-1}a^{\eta-1}\right)}{\theta}f(z)dz\right]g(a)da.\end{aligned}$$

The first two terms cancel out and therefore this derivative can be simplified to:

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta}=\left(\frac{2(\eta-1)}{\theta}\right)\int_a\int_{Z^*}^{z_H}\log\left(Z^Az\theta^{\eta-1}a^{\eta-1}\right)f(z)dzg(a)da.$$

or

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\mathrm{MRPK}\right)\right)^2\right]}{\partial\theta}=\left(\frac{2(\eta-1)}{\theta}\right)\int_a\mathbb{E}\left(\log\left(\mathrm{MRPK}\right)|z>Z^*,a\right)\left(1-F\left(Z^*|a\right)\right)g(a)da. \tag{A.17}$$

Plugging (A.16) and (A.17) into (A.12) we finally obtain:

$$\frac{\partial\mathrm{Var}\left(\log\left(\mathrm{MRPK}\right)\right)}{\partial\theta}=\left(\frac{2(\eta-1)}{\theta}\right)\int_a\left[\mathbb{E}\left(\log\left(\mathrm{MRPK}\right)|z>Z^*,a\right)-\mathbb{E}\log\left(\mathrm{MRPK}|a\right)\right]\left(1-F\left(Z^*|a\right)\right)g(a)da\leq 0. \tag{A.18}$$

The bracket is weakly positive because the expected marginal revenue product of capital is higher conditional on productivity being above Z^*. Given that $\eta<1$, the derivative of the variance is weakly negative.

E.3 Changes in Aggregate Productivity or Demand

We consider how small changes in Z^A impact the dispersion of the MRPK. We obtain:

$$\frac{\partial\left[\mathbb{E}\log\left(\mathrm{MRPK}\right)\right]}{\partial Z^A}=\int_a\left[\log(R)f(Z^*|a)\frac{\partial Z^*}{\partial Z^A}-\log\left(Z^AZ^*\theta^{\eta-1}a^{\eta-1}\right)f(Z^*|a)\frac{\partial Z^*}{\partial Z^A}+\left(\frac{1}{Z^A}\right)\int_{Z^*}^{z_H}f(z)dz\right]g(a)da.$$

Note that the two first terms in the integral cancel out because at the cutoff Z^* we have $R = Z^A Z^* \theta^{\eta-1} a^{\eta-1}$. Therefore:

$$\frac{\partial\left[\mathbb{E}\log\left(\text{MRPK}\right)\right]}{\partial Z^A} = \left(\frac{1}{Z^A}\right)\int_a\int_{Z^*}^{z_H} f(z)dzg(a)da = \left(\frac{1}{Z^A}\right)\int_a \left(1 - F\left(Z^*|a\right)\right)g(a)da. \quad \text{(A.19)}$$

We also have:

$$\begin{aligned}\frac{\partial\mathbb{E}\left[\left(\log\left(\text{MRPK}\right)\right)^2\right]}{\partial Z^A} &= \int_a \left[\left(\log(R)\right)^2 f(Z^*|a)\frac{\partial Z^*}{\partial Z^A} - \left(\log\left(Z^A Z^*\theta^{\eta-1}a^{\eta-1}\right)\right)^2 f(Z^*|a)\frac{\partial Z^*}{\partial Z^A}\right]g(a)da \\ &\quad + \int_a\left[\int_{Z^*}^{z_H}\frac{2\log\left(Z^A z\theta^{\eta-1}a^{\eta-1}\right)}{Z^A}f(z)dz\right]g(a)da.\end{aligned}$$

The first two terms cancel out and therefore this derivative can be simplified to:

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\text{MRPK}\right)\right)^2\right]}{\partial Z^A} = \left(\frac{2}{Z^A}\right)\int_a\int_{Z^*}^{z_H}\log\left(Z^A z\theta^{\eta-1}a^{\eta-1}\right)f(z)dzg(a)da.$$

or

$$\frac{\partial\mathbb{E}\left[\left(\log\left(\text{MRPK}\right)\right)^2\right]}{\partial Z^A} = \left(\frac{2}{Z^A}\right)\int_a \mathbb{E}\left(\log\left(\text{MRPK}\right)|z > Z^*, a\right)\left(1 - F\left(Z^*|a\right)\right)g(a)da. \quad \text{(A.20)}$$

Plugging (A.19) and (A.20) into (A.12) we finally obtain:

$$\frac{\partial\text{Var}\left(\log\left(\text{MRPK}\right)\right)}{\partial Z^A} = \left(\frac{2}{Z^A}\right)\int_a\left[\mathbb{E}\left(\log\left(\text{MRPK}\right)|z > Z^*, a\right) - \mathbb{E}\log\left(\text{MRPK}|a\right)\right]\left(1 - F\left(Z^*|a\right)\right)g(a)da \geq 0. \quad \text{(A.21)}$$

The bracket is weakly positive because the expected marginal revenue product of capital is higher conditional on productivity being above Z^*. Therefore, the derivative of the variance is weakly positive.

F A Financial Constraint of the Form $k' \leq \theta a'$

In Appendix E we provided analytical solutions for the immediate impact of various shocks on MRPK dispersion within a simplified version of our model with perfect foresight about next period's productivity, no adjustment costs, and a financial constraint of the form $k \leq \theta a$. We now simulate our full model with a risky time-to-build technology of capital accumulation and adjustment costs under the financial constraint $k' \leq \theta a'$. So, the only difference relative to the baseline model is that we replace the size-dependent borrowing constraint in equation (21) with the constraint $k' \leq \theta a'$.

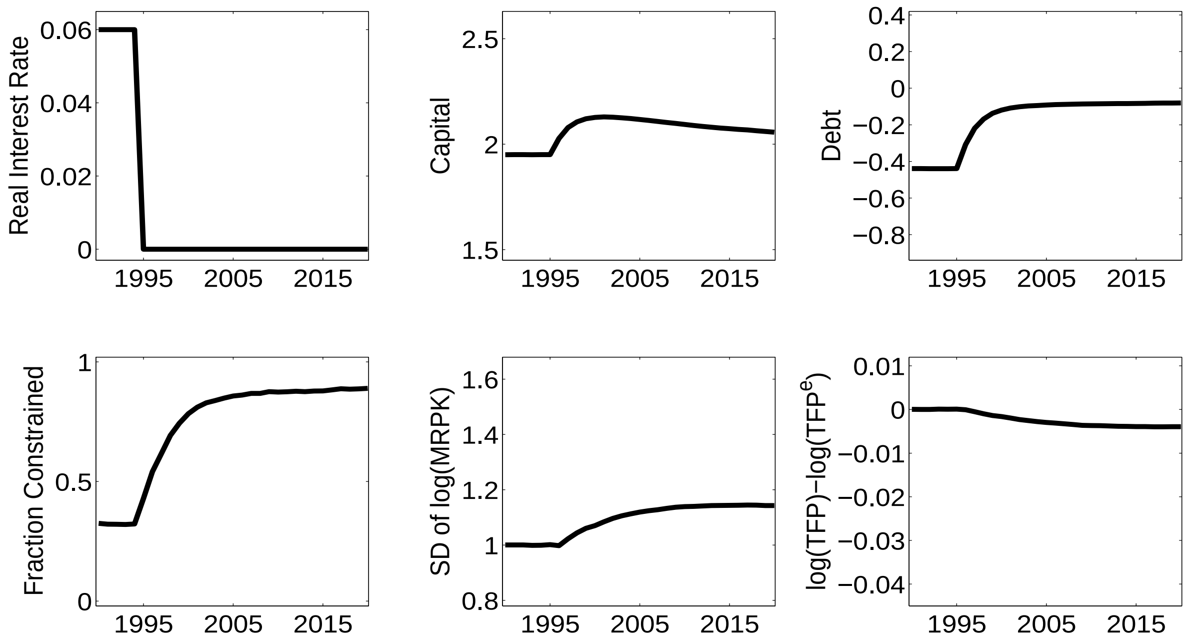


Figure A.5: Decline in the Real Interest Rate ($\psi = 3.1$ and $\theta = 1.0$)

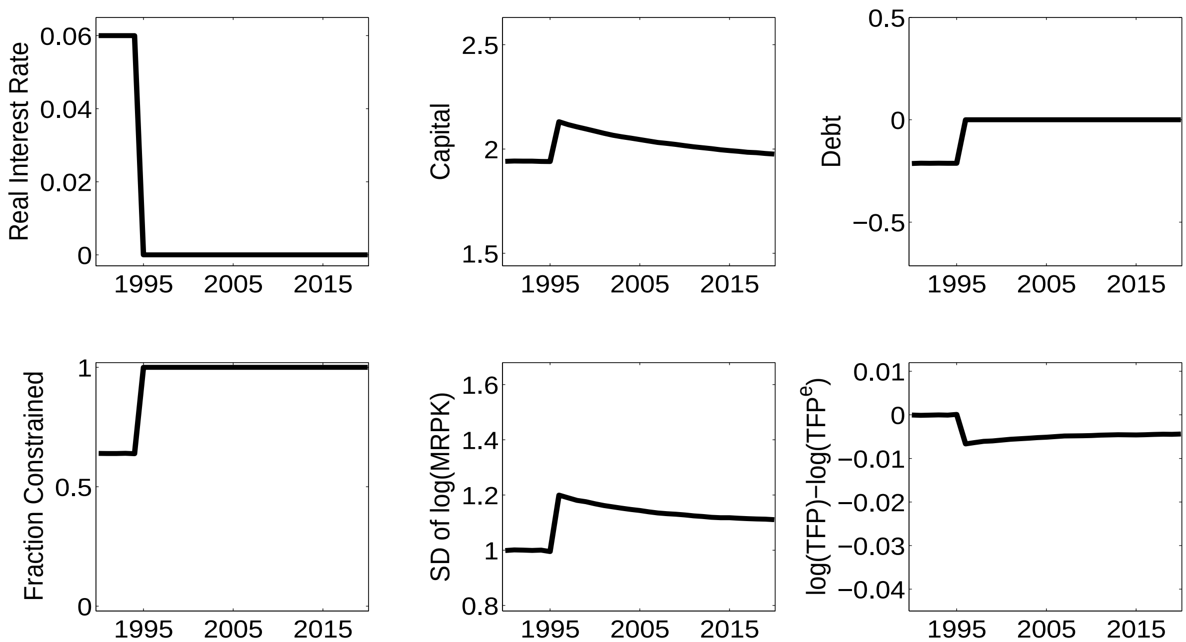


Figure A.6: Decline in the Real Interest Rate ($\psi = 0.0$ and $\theta = 1.0$)

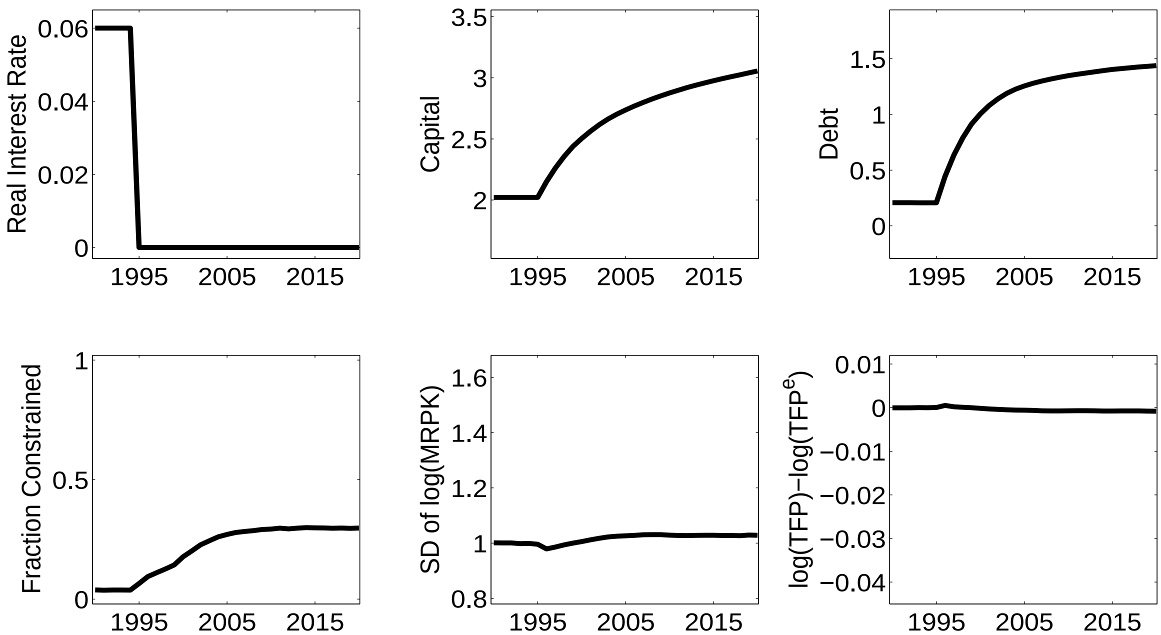


Figure A.7: Decline in the Real Interest Rate ($\psi = 6.5$ and $\theta = 2.2$)

In Figures A.5 and A.6 we present aggregate impulses in response to a decline in the real interest rate from 6 to 0 percent. Figure A.5 uses the adjustment cost parameter $\psi = 3.1$ calibrated from our baseline model and sets $\theta = 1$ which implies that no firm in the economy can borrow. This parameterization is useful because it guarantees that the model with the financial constraint of the form $k' \leq \theta a'$ shares exactly the same initial equilibrium with the model with a size-dependent borrowing constraint. Figure A.6 shuts down adjustment costs ($\psi = 0.0$) and still uses $\theta = 1$. All other parameters are fixed to the values shown in Table 3 for the baseline model. The point of these figures is to show that, in response to the decline in the real interest rate, the model with the alternative financial constraint also generates an increase in MRPK dispersion and a decline in TFP.

Next, we calibrate the model with the financial constraint $k' \leq \theta a'$ in a similar manner to our baseline model with a size-dependent borrowing constraint. Specifically, we set $\psi = 6.5$ and $\theta = 2.2$ to match the responsiveness of firm capital growth to within-firm changes in productivity and net worth. These responses are captured by the coefficients $\beta_z = 0.10$ and $\beta_a = 0.09$ in regression (24) for the permanent sample of firms. In Figure A.7 we present impulses in response to the decline in the real interest rate for this calibrated model. We find

Table A.5: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	$k' \leq \theta a'$	Permanent	Full
Dispersion	Std $(\log \ell)$	0.78	0.70	1.13	1.21
	Std $(\log k)$	0.87	0.64	1.52	1.70
	Std $(\log \text{MRPK})$	0.30	0.22	0.88	1.12
Productivity	Corr $(\log Z, \log \text{MRPK})$	0.13	0.57	0.03	0.05
	Corr $(\log Z, \log \ell)$	0.96	0.98	0.65	0.58
	Corr $(\log Z, \ell/L)$	0.91	0.96	0.54	0.48
	Corr $(\log Z, \log k)$	0.82	0.87	0.62	0.52
	Corr $(\log Z, k/K)$	0.66	0.87	0.53	0.44
	Corr $(\log Z, \log(k/\ell))$	-0.13	-0.57	0.22	0.16
MRPK	Corr $(\log \text{MRPK}, \log \ell)$	-0.13	0.40	-0.03	0.01
	Corr $(\log \text{MRPK}, \ell/L)$	-0.19	0.43	-0.05	-0.03
	Corr $(\log \text{MRPK}, \log k)$	-0.46	0.09	-0.62	-0.68
	Corr $(\log \text{MRPK}, k/K)$	-0.57	0.10	-0.31	-0.28
	Corr $(\log \text{MRPK}, \log(k/\ell))$	-1.00	-1.00	-0.95	-0.96
Financial	Corr $(\log Z, \log a)$	0.81	0.85	0.75	0.65
	Corr $(\log \text{MRPK}, \log a)$	-0.20	0.09	-0.14	-0.14
	Coefficient of b/k on $\log k$	0.14	0.01	0.15	0.23

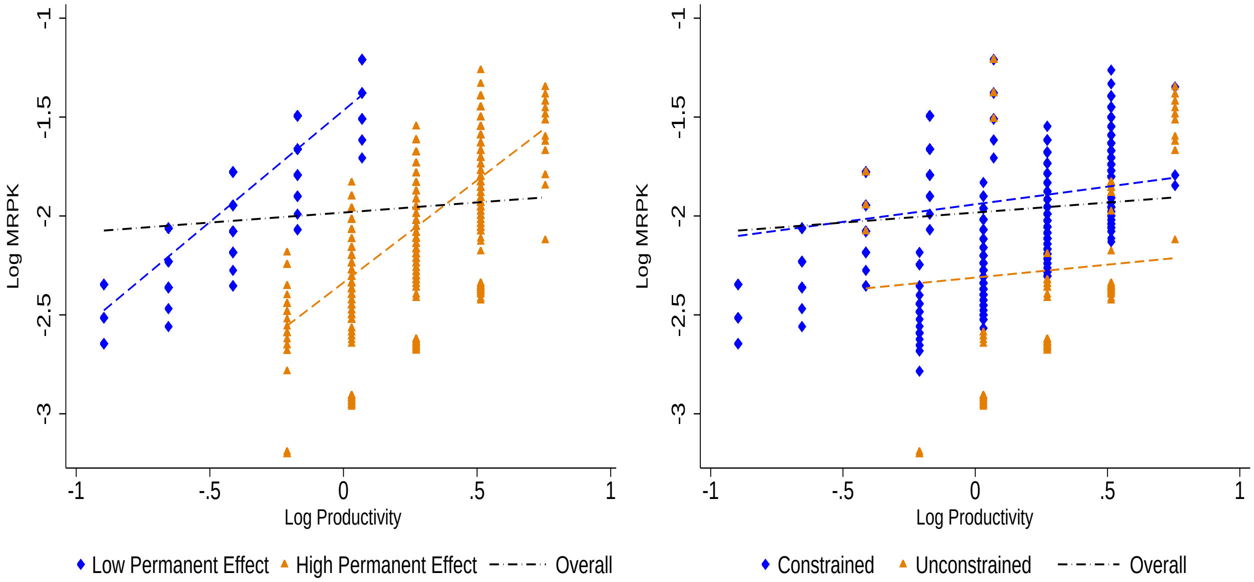
that the model generates a small increase in MRPK dispersion and a negligible decline in TFP. Intuitively, our calibration implies that very few firms are initially constrained before the shock hits. Similar to the analysis in Appendix E, we expect the response of dispersion and TFP to be the smallest when initially all firms are either unconstrained or constrained.

Table A.5 repeats the analysis underlying Table 5 in the main text and compares the model with a financial constraint of the form $k' \leq \theta a'$ to our baseline model with a size-dependent borrowing constraint with respect to various second moments. A key difference between the two models is that the model with the financial constraint $k' \leq \theta a'$ does not generate a negative correlation between measures of revenue-based productivity, such as log (MRPK), and measures of size, such as labor and capital. Additionally, the model with the financial constraint $k' \leq \theta a'$ produces a much stronger correlation between log productivity and log (MRPK) than the model with a size-dependent borrowing constraint.

We now provide intuition for these differences. Figures A.8 and A.9 plot the cross-sectional relationship between log productivity, $\log Z$, and $\log(\text{MRPK})$ in the two models. As we discussed in the main text, within the set of firms with the same permanent productivity z^P, there is a strong correlation between $\log(\text{MRPK})$ and $\log Z$, reflecting transitory productivity shocks in an environment with time-to-build technology and a borrowing constraint. This holds both in the model with a size-dependent borrowing constraint and in the model with a financial constraint of the form $k' \leq \theta a'$.

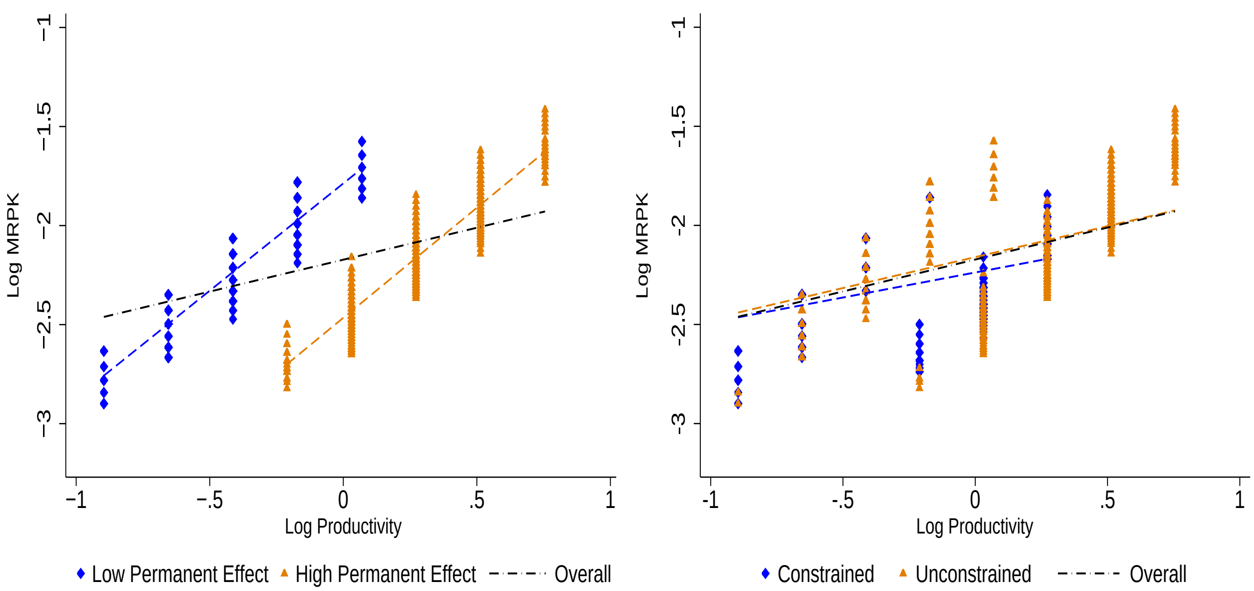
In response to the decline in the real interest rate, the model with a size-dependent borrowing constraint generates a high dispersion of capital across firms with different permanent productivity because high z^P firms increase significantly their capital to overcome permanently the borrowing constraint. Permanent differences in capital increase significantly the variation of MRPK across firms with different z^P components, leading to a low overall correlation between $\log Z$ and $\log(\text{MRPK})$. By contrast, in the model with a financial constraint of the form $k' \leq \theta a'$ there is no additional incentive to increase capital because the borrowing constraint does not depend on size. This greatly reduces capital and MRPK differences across firms with different z^P components. Therefore, the overall correlation between $\log Z$ and $\log(\text{MRPK})$ is high in the model with a financial constraint of the form $k' \leq \theta a'$.

Figures A.10 and A.11 plot the cross-sectional relationship between size (measured with log labor) and $\log(\text{MRPK})$ in the two models. The model with a financial constraint of the form $k' \leq \theta a'$ generates a positive and high correlation between size and $\log(\text{MRPK})$. The intuition is similar to the intuition described above for the relationship between productivity and $\log(\text{MRPK})$, with the time-to-build technology and financial frictions leading to a positive and high correlation between size and $\log(\text{MRPK})$. By contrast, the model with a size-dependent borrowing constraint generates a negative correlation between size and $\log(\text{MRPK})$. This key difference between the two models emerges because, with a size-dependent borrowing constraint, the decline in the real interest rate incentivizes some firms to grow and permanently overcome their borrowing constraint. As a result, in the model with a size-dependent borrowing constraint,



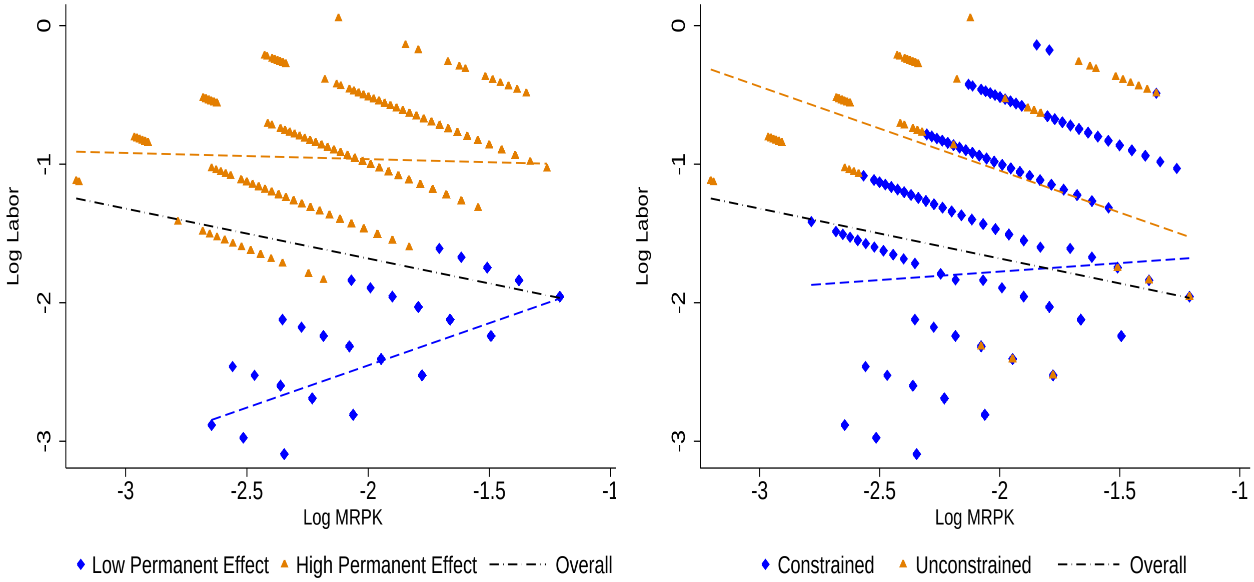
(a) By Permanent Productivity (b) By Constraint Status

Figure A.8: MRPK and Productivity in Model With Size-Dependent Borrowing Constraint



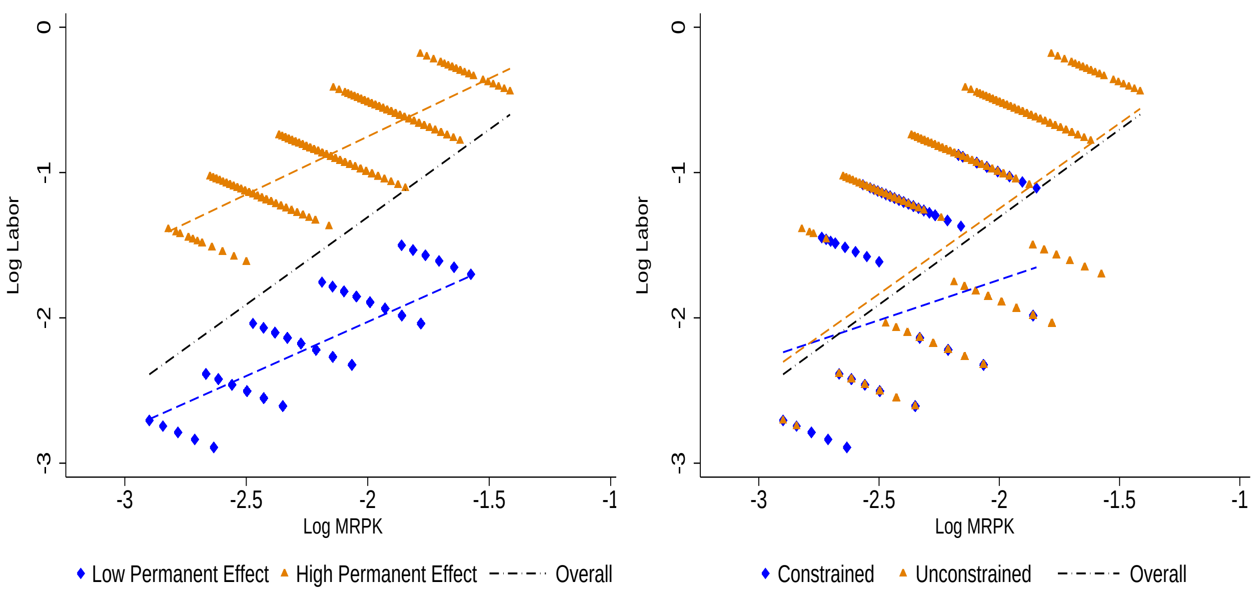
(a) By Permanent Productivity (b) By Constraint Status

Figure A.9: MRPK and Productivity in Model With Financial Constraint $k' \leq \theta a'$



(a) By Permanent Productivity (b) By Constraint Status

Figure A.10: MRPK and Size in Model With Size-Dependent Borrowing Constraint



(a) By Permanent Productivity (b) By Constraint Status

Figure A.11: MRPK and Size in Model With Financial Constraint $k' \leq \theta a'$

larger firms tend to be unconstrained and tend to have a lower return to capital.

G Endogenous Entry and Exit

In this appendix we describe the model with endogenous entry and exit. Let $m_{it} = 0$ denote a firm that operates in the outside sector and let $m_{it} = 1$ denote a firm that produces in manufacturing. The period t status of a firm is a state variable and the period $t+1$ status of a firm is a choice variable. We write the budget constraint of a firm as a function of its state in period t and its entry decision in period $t+1$.

1. When $m_{it} = 1$ and $m_{it+1} = 1$, the budget constraint is:

$$c_{it} + k_{it+1} + (1 + r_t)b_{it} + \frac{\psi\left(k_{it+1} - k_{it}\right)^2}{2k_{it}} = \pi_{it} + (1 - \delta)k_{it} + b_{it+1}, \tag{A.22}$$

   where $\pi_{it} = p_{it}y_{it} - w_t\ell_{it}$ denotes revenues less compensation to labor.

2. When $m_{it} = 1$ and $m_{it+1} = 0$, the budget constraint is:

$$c_{it} + (1 + r_t)b_{it} = \pi_{it} + (1 - \delta)k_{it} + b_{it+1}. \tag{A.23}$$

   Firms that operate in manufacturing and decide to exit are assumed to sell their capital ($k_{it+1} = 0$) without incurring an exit cost.

3. When $m_{it} = 0$ and $m_{it+1} = 0$, the budget constraint is:

$$c_{it} + (1 + r_t)b_{it} = h_t + b_{it+1}, \tag{A.24}$$

   where h_t denotes the income of firms operating in the outside sector.

4. When $m_{it} = 0$ and $m_{it+1} = 1$, the budget constraint is:

$$c_{it} + k_{it+1} + (1 + r_t)b_{it} = h_t - \zeta(k_{it+1}) + b_{it+1}, \tag{A.25}$$

   where $\zeta(k_{it+1})$ denotes an entry cost. We assume that entry costs are an increasing function of the capital stock upon entry.

We now write the problem of a firm in recursive form in the model with endogenous entry and exit. The Bellman equation is:

$$V\left(a, k, m, z^P, z^T, \mathbf{X}\right) = \max_{a', k', m', \ell, p} \left\{U(c) + \beta \mathbb{E} V\left(a', k', m', z^P, (z^T)', \mathbf{X}'\right)\right\}, \tag{A.26}$$

subject to the budget constraint:

$$c + a' = m\left(\pi - (r+\delta)k - m'\frac{\psi\left(k'-k\right)^2}{2k}\right) + (1-m)\left(h - m'\zeta(k')\right) + (1+r)a, \tag{A.27}$$

where $\pi = p(y)y - wl$ and $y = Zk^\alpha \ell^{1-\alpha} = p^{-\varepsilon}$.

In our numerical simulations we work with the quadratic cost $\zeta_{it} = \bar{\zeta} k_{it+1}^2$. We set $\bar{\zeta} = 0.30$ and $h_t = 0.08$. We choose these parameters such that the model replicates the responsiveness of capital growth to within-firm variations in productivity and net worth as observed in the full sample in Table 4.

H Exogenous MRPL Dispersion

Table A.6 repeats the analysis underlying Table 5 in the main text. With this table, we compare the model with exogenous MRPL dispersion to both our baseline model without MRPL dispersion and to the data with respect to various second moments.

We stress three main differences between the model with exogenous MRPL dispersion and the model without MRPL dispersion. First, the model with exogenous MRPL dispersion generates a higher dispersion of firm size (as captured by log labor) and a higher dispersion of log (TFPR) than the baseline model. Second, in the model with exogenous MRPL dispersion there is a weaker correlation between firm log productivity, log Z, and either log labor or the share of firm labor in sectoral labor. This happens because variations of labor across firms in the model with exogenous MRPL dispersion partly reflect variations of the labor wedge. Given that the labor wedge is uncorrelated with firm productivity, the unconditional correlation between firm labor or share in sectoral labor and firm productivity becomes weaker.

The third important difference between the two models is that the model with exogenous MRPL dispersion does not generate the negative correlation between log (MRPK) and either log labor or the share of firm labor in sectoral labor observed in the data. The baseline model

Table A.6: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		Sample	
	Statistic	Baseline	MRPL Dispersion	Permanent	Full
Dispersion	Std ($\log \ell$)	0.78	0.91	1.13	1.21
	Std ($\log$ MRPL)	0.00	0.30	0.30	0.33
	Std ($\log k$)	0.87	0.79	1.52	1.70
	Std ($\log$ MRPK)	0.30	0.39	0.88	1.12
	Std ($\log$ TFPR)	0.10	0.16	0.35	0.42
Productivity	Corr ($\log Z, \log$ TFPR)	0.13	0.22	0.46	0.43
	Corr ($\log Z, \log$ MRPK)	0.13	0.25	0.03	0.05
	Corr ($\log Z, \log \ell$)	0.96	0.78	0.65	0.58
	Corr ($\log Z, \ell/L$)	0.91	0.70	0.54	0.48
	Corr ($\log Z, \log k$)	0.82	0.78	0.62	0.52
	Corr ($\log Z, k/K$)	0.66	0.65	0.53	0.44
	Corr ($\log Z, \log (k/\ell)$)	-0.13	-0.16	0.22	0.16
TFPR	Corr ($\log$ TFPR, $\log \ell$)	-0.13	-0.39	0.02	-0.01
	Corr ($\log$ TFPR, ℓ/L)	-0.19	-0.38	0.01	0.01
	Corr ($\log$ TFPR, $\log k$)	-0.46	-0.24	-0.38	-0.50
	Corr ($\log$ TFPR, k/K)	-0.57	-0.28	-0.14	-0.16
	Corr ($\log$ TFPR, $\log (k/\ell)$)	-1.00	0.28	-0.60	-0.69
MRPK	Corr ($\log$ MRPK, $\log \ell$)	-0.13	0.38	-0.03	0.01
	Corr ($\log$ MRPK, ℓ/L)	-0.19	0.32	-0.05	-0.03
	Corr ($\log$ MRPK, $\log k$)	-0.46	-0.28	-0.62	-0.68
	Corr ($\log$ MRPK, k/K)	-0.57	-0.33	-0.31	-0.28
	Corr ($\log$ MRPK, $\log (k/\ell)$)	-1.00	-0.92	-0.95	-0.96
MRPL	Corr ($\log$ MRPL, $\log \ell$)	—	-0.58	0.31	0.34
	Corr ($\log$ MRPL, ℓ/L)	—	-0.53	0.20	0.22
	Corr ($\log$ MRPL, $\log k$)	—	0.01	0.33	0.34
	Corr ($\log$ MRPL, k/K)	—	0.00	0.22	0.22
	Corr ($\log$ MRPL, $\log (k/\ell)$)	—	0.86	0.18	0.15
Financial	Corr ($\log Z, \log a$)	0.81	0.70	0.75	0.65
	Corr ($\log$ TFPR, $\log a$)	-0.20	-0.05	0.07	0.00
	Corr ($\log$ MRPK, $\log a$)	-0.20	-0.07	-0.14	-0.14
	Corr ($\log$ MRPL, $\log a$)	—	0.01	0.45	0.44
	Coefficient of b/k on $\log k$	0.14	0.24	0.15	0.23

without MRPL dispersion generates a negative correlation because smaller firms are more likely to be constrained. In the model with exogenous MRPL dispersion, an increase in the labor wedge τ causes both firm labor and MRPK to decrease (the latter decreases because k is predetermined and revenues decrease). This tends to increase the overall correlation between the two variables in the cross section of firms. We also note that, conditional on a labor wedge shock, labor and log (MRPL) are negatively correlated in the model. However, in the data this correlation is positive.

I Overhead Labor

A model with overhead labor, such as the one developed by Bartelsman, Haltiwanger, and Scarpetta (2013), can match the observed positive correlation between firm size and measured log (MRPL). Consider the production function:

$$y_{it} = Z_{it} k_{it}^{\alpha} \left(\ell_{it} - \phi_{\ell}\right)^{1-\alpha}, \tag{A.28}$$

where ϕ_ℓ denotes overhead labor. With this production function, all firms equalize the true marginal revenue product of labor to the common wage. However, the *measured* marginal revenue product of labor varies across firms. To see this, we write:

$$\text{MRPL}_{it} := \left(\frac{1-\alpha}{\mu}\right)\left(\frac{p_{it} y_{it}}{\ell_{it}}\right) = \left(1 - \frac{\phi_\ell}{\ell_{it}}\right) w_t. \tag{A.29}$$

Firms with higher labor also have higher measured MRPL.

Next, we calibrate and simulate the model with overhead labor.[2] The economic environment is similar to our baseline model with the only exception that we use the production function with overhead labor in equation (A.28) instead of the Cobb-Douglas production function. We calibrate jointly the adjustment cost parameter ψ, the borrowing threshold κ, and overhead labor ϕ_ℓ to match three moments. As before, the two moments are the responsiveness of capital growth to within-firm variations in productivity and net worth as observed in the permanent

[2] In parallel to the model with exogenous labor taxes, we rebate $\phi_\ell w$ back to each firm. This allows us to make consistent comparisons between the model with overhead labor and our baseline model without MRPL dispersion. In the absence of this rebate some small firms would eventually be forced to shut down which, in turn, would change the distribution of productivity across firms in the model.

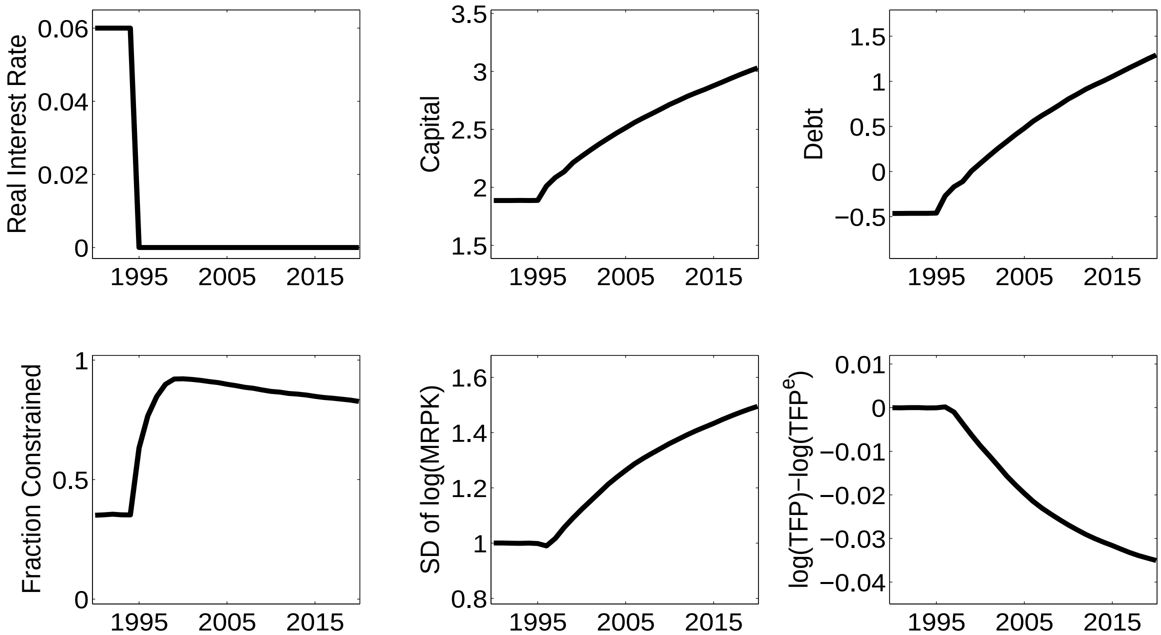


Figure A.12: Decline in the Real Interest Rate: Model With Overhead Labor

sample of firms in Table 4. The third moment is the standard deviation of log (MRPL) which in the data equals 0.30. We find that $\psi = 3.0$, $\kappa = 4.3$, and $\phi_\ell = 0.11$. All other parameters are fixed to the values shown in Table 3 for the baseline model.

In Figure A.12 we present impulses in response to a decline in the real interest rate in the model with overhead labor. We note that the impulses are almost identical to those in the baseline model presented in Figure 11.[3] This is not surprising because our calibrated values of $\psi = 3.0$ and $\kappa = 4.3$ are very close to the calibrated values $\psi = 3.1$ and $\kappa = 4.2$ in the baseline model. Overhead labor does not interact in an important quantitative way with firm investment and debt decisions as captured by the regressions that use within-firm variation in Table 4.

In Table A.7 we compare the model with overhead labor to our baseline model without MRPL dispersion and to the data with respect to various second moments. Consistent with the logic that overhead labor does not interact quantitatively with investment and debt decisions, various moments related to leverage, net worth, capital, and MRPK are similar between the model with overhead labor and the baseline model without MRPL dispersion. While the model with

[3]We define aggregate total factor productivity as $\text{TFP}_t := Y_t / \left(K_t^\alpha \left(L_t - \phi_\ell N_t \right)^{1-\alpha} \right)$, where $\phi_\ell N_t$ denotes total overhead labor in the economy. That is, we do not allow overhead labor to artificially bias measured TFP in the model.

Table A.7: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	Overhead Labor	Permanent	Full
Dispersion	Std $(\log \ell)$	0.78	0.49	1.13	1.21
	Std $(\log \text{MRPL})$	0.00	0.30	0.30	0.33
	Std $(\log k)$	0.87	0.87	1.52	1.70
	Std $(\log \text{MRPK})$	0.30	0.29	0.88	1.12
	Std $(\log \text{TFPR})$	0.10	0.21	0.35	0.42
Productivity	Corr $(\log Z, \log \text{TFPR})$	0.13	0.95	0.46	0.43
	Corr $(\log Z, \log \text{MRPK})$	0.13	0.12	0.03	0.05
	Corr $(\log Z, \log \ell)$	0.96	0.95	0.65	0.58
	Corr $(\log Z, \ell/L)$	0.91	0.91	0.54	0.48
	Corr $(\log Z, \log k)$	0.82	0.82	0.62	0.52
	Corr $(\log Z, k/K)$	0.66	0.66	0.53	0.44
	Corr $(\log Z, \log(k/\ell))$	-0.13	0.57	0.22	0.16
TFPR	Corr $(\log \text{TFPR}, \log \ell)$	-0.13	0.81	0.02	-0.01
	Corr $(\log \text{TFPR}, \ell/L)$	-0.19	0.74	0.01	0.01
	Corr $(\log \text{TFPR}, \log k)$	-0.46	0.61	-0.38	-0.50
	Corr $(\log \text{TFPR}, k/K)$	-0.57	0.42	-0.14	-0.16
	Corr $(\log \text{TFPR}, \log(k/\ell))$	-1.00	0.32	-0.60	-0.69
MRPK	Corr $(\log \text{MRPK}, \log \ell)$	-0.13	-0.17	-0.03	0.01
	Corr $(\log \text{MRPK}, \ell/L)$	-0.19	-0.19	-0.05	-0.03
	Corr $(\log \text{MRPK}, \log k)$	-0.46	-0.46	-0.62	-0.68
	Corr $(\log \text{MRPK}, k/K)$	-0.57	-0.56	-0.31	-0.28
	Corr $(\log \text{MRPK}, \log(k/\ell))$	-1.00	-0.74	-0.95	-0.96
MRPL	Corr $(\log \text{MRPL}, \log \ell)$	—	0.97	0.31	0.34
	Corr $(\log \text{MRPL}, \ell/L)$	—	0.91	0.20	0.22
	Corr $(\log \text{MRPL}, \log k)$	—	0.92	0.33	0.34
	Corr $(\log \text{MRPL}, k/K)$	—	0.76	0.22	0.22
	Corr $(\log \text{MRPL}, \log(k/\ell))$	—	0.74	0.18	0.15
Financial	Corr $(\log Z, \log a)$	0.81	0.81	0.75	0.65
	Corr $(\log \text{TFPR}, \log a)$	-0.20	0.69	0.07	0.00
	Corr $(\log \text{MRPK}, \log a)$	-0.20	-0.23	-0.14	-0.14
	Corr $(\log \text{MRPL}, \log a)$	—	0.86	0.45	0.44
	Coefficient of b/k on $\log k$	0.14	0.13	0.15	0.23

overhead labor generates a positive correlation between log (MRPL) and size in the cross section of firms, there are two important discrepancies relative to the data. First, overhead labor reduces the log labor dispersion across firms. This happens because less productive firms that would otherwise optimally choose to be small are forced to hire more labor than the overhead. Second, the model with overhead labor generates a strong positive correlation between log (TFPR) and firm size as measured either by labor or capital. In our data for Spain, however, this correlation is close to zero or negative.

J Unmeasured Inputs and Higher MRPK Dispersion

In our baseline model we obtained a lower level of cross-sectional dispersion of capital and especially of MRPK relative to the data. In this appendix we describe a model with an unmeasured input that allows us to rationalize a higher level of capital and MRPK dispersion. We argue that such a modification does not change significantly our main results.

Consider the production function:

$$y_{it} = Z_{it} \left(k_{it} + \phi_k\right)^{\alpha} \ell_{it}^{1-\alpha}, \tag{A.30}$$

where ϕ_k denotes some unmeasured input that enters additively with capital in production. This input could represent some form of intangible capital that is not well measured in the data. Note that the production function (A.30) is similar to the production function in equation (A.28) in the model with overhead labor, with the difference being that in the former we add ϕ_k to capital whereas in the latter we subtract ϕ_ℓ from labor.[4] Similarly to our baseline model, we calibrate values of $\psi = 3.2$, $\kappa = 3.7$, and $\phi_k = 0.30$ to match the responsiveness of capital growth to productivity and net worth using within-firm variation. All other parameters are set at their baseline values shown in Table 3.

Table A.8 repeats the analysis underlying Table 5 in the main text and compares the model with the unmeasured input to our baseline model and to the data along various second moments

[4] Similarly to the model with overhead labor, we tax lump-sum each firm an amount equal to $(r_t + \delta)\phi_k$. Also, we define aggregate total factor productivity as $\text{TFP}_t := Y_t / \left(\left(K_t + \phi_k N_t\right)^{\alpha} L_t^{1-\alpha}\right)$, where $\phi_k N_t$ denotes the total unmeasured input in the economy. That is, we do not allow this input to artificially bias measured TFP in the model.

Table A.8: Summary Statistics in the Cross Section of Firms (1999-2007)

		Model		**Sample**	
	Statistic	Baseline	Unmeasured Input	Permanent	Full
Dispersion	Std ($\log \ell$)	0.78	0.74	1.13	1.21
	Std ($\log k$)	0.87	1.37	1.52	1.70
	Std ($\log$ MRPK)	0.30	0.71	0.88	1.12
Productivity	Corr ($\log Z, \log$ MRPK)	0.13	-0.63	0.03	0.05
	Corr ($\log Z, \log \ell$)	0.96	0.97	0.65	0.58
	Corr ($\log Z, \ell/L$)	0.91	0.92	0.54	0.48
	Corr ($\log Z, \log k$)	0.82	0.85	0.62	0.52
	Corr ($\log Z, k/K$)	0.66	0.68	0.53	0.44
	Corr ($\log Z, \log (k/\ell)$)	-0.13	0.63	0.22	0.16
MRPK	Corr ($\log$ MRPK, $\log \ell$)	-0.13	-0.79	-0.03	0.01
	Corr ($\log$ MRPK, ℓ/L)	-0.19	-0.77	-0.05	-0.03
	Corr ($\log$ MRPK, $\log k$)	-0.46	-0.94	-0.62	-0.68
	Corr ($\log$ MRPK, k/K)	-0.57	-0.85	-0.31	-0.28
	Corr ($\log$ MRPK, $\log (k/\ell)$)	-1.00	-1.00	-0.95	-0.96
Financial	Corr ($\log Z, \log a$)	0.81	0.82	0.75	0.65
	Corr ($\log$ MRPK, $\log a$)	-0.20	-0.79	-0.14	-0.14
	Coefficient of b/k on $\log k$	0.14	0.43	0.15	0.23

in the cross section of firms. There are two key differences between the two models. First, as shown in the first panel of the table, the model with the unmeasured input comes much closer than the baseline model in matching the level of capital and MRPK dispersion observed in the data.

The second important difference between the two models is that the model with the unmeasured input generates a more negative correlation between MRPK and measures of size or productivity across firms. To understand this point, we write the *true* MRPK as:

$$\overline{\text{MRPK}}_{it} := \left(\frac{\alpha}{\mu}\right)\left(\frac{p_{it}y_{it}}{k_{it}+\phi_k}\right) = \left(1+\tau_{it}^k\right)(r_t+\delta). \tag{A.31}$$

where τ_{it}^k denotes the percent deviation of the true MRPK from the frictionless cost of capital $r_t+\delta$. As in our baseline analysis, the wedge τ_{it}^k arises because of a binding borrowing constraint,

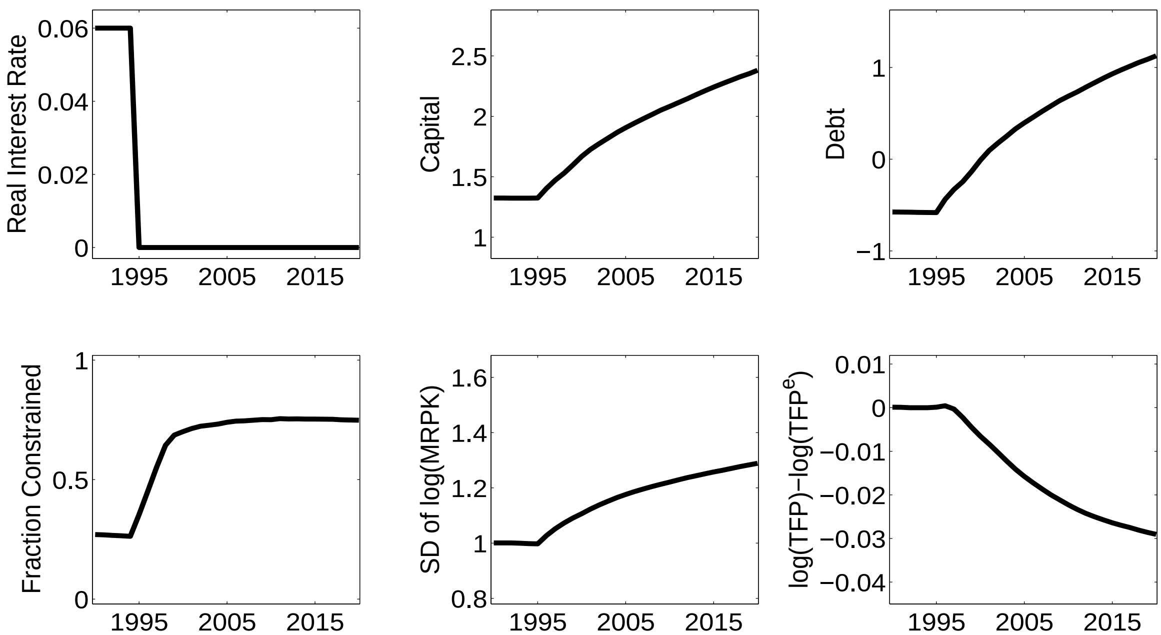


Figure A.13: Decline in the Real Interest Rate: Model With Unmeasured Input

a risky time-to-build technology of capital accumulation, and investment adjustment costs.

Next, consider the *measured* MRPK:

$$\text{MRPK}_{it} := \left(\frac{\alpha}{\mu}\right)\left(\frac{p_{it}y_{it}}{k_{it}}\right) = \left(1+\frac{\phi_k}{k_{it}}\right)\overline{\text{MRPK}}_{it} = \left(1+\frac{\phi_k}{k_{it}}\right)\left(1+\tau_{it}^k\right)(r_t+\delta). \tag{A.32}$$

Equation (A.32) shows that ϕ_k introduces an additional wedge between the frictionless cost of capital and the measured MRPK. Firms with higher capital k_{it} will tend to have lower measured MRPK. The existence of this additional wedge explains why the model with the unmeasured input generates more negative cross-sectional correlations between MRPK and either productivity or size.

In Figure A.13 we present impulses in response to the decline in the real interest rate in the model with the unmeasured input. The impulses look quite similar to the impulses generated by our baseline model. We, therefore, argue that the relatively low level of capital and MRPK dispersion generated by the baseline model is not crucial for the main results that emerge from our analysis.