APPENDIX

Resurrecting the Role of the Product Market Wedge in Recessions

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This Appendix provides a detailed description of our data, elaboration on the problem of the self-employed, a more thorough explanation of some calculations, and some robustness results.

A1 The Aggregate Wedge

A1.1 Representative Agent Wedge

Variables used to construct the RAW include:

- $\frac{y_t}{n_t}$: (Real) Output per hour; BLS, Labor Productivity & Costs, Business Sector.

- c_t: (Real) Nondurables and services consumption per adult equivalent; NIPA consumption data, adjusted for indirect taxes following Prescott (2004) and McDaniel (2007). Adult-equivalent population = (Population ≥ 16) + 0.5 * (Population ≤ 15).

- n_t: Hours worked per capita; Hours worked from BLS (LPC, Business Sector), and population is Civilian Pop 16+.

- $\tau_t = ((\tau_t^c + \tau_t^n)/(1 + \tau_t^c))$, where τ_t^c is the average tax rate on consumption, following McDaniel (2007), and τ_t^n is the average marginal labor tax rate, using NBER TaxSim to extend Barro and Redlick (2011) through 2012.

In Section 2 of the main paper, we refer to a robustness exercise in which we depart from the assumption of perfect consumption sharing. Specifically, we allow the consumption of the employed and unemployed to differ and use the cyclical elasticity of employed consumption as the relevant input into the RAW. We note that per-capita consumption, c_t, can be decomposed as

$$c_t = e_t c_t^e + u_t c_t^u + (1 - e_t - u_t) c_t^{nlf},$$

where e_t is the employment-population ratio, u_t is the unemployment rate, and c_t^e, c_t^u, and c_t^{nlf} are average consumption of the employed, unemployed, and those not in the labor force, respectively.

We assume that unemployed consumption is a fraction φ of employed consumption and consider two cases for c^{nlf}: $c^{nlf} = c^e$ and $c^{nlf} = c^u$.

Case 1: $c^{nlf} = c^e$, $c^u = \varphi c^e$

$$\begin{aligned} c_t &= e_t c_t^e + u_t \varphi c_t^e + (1 - e_t - u_t) c_t^e \\ &= [1 - (1 - \varphi) u_t] c_t^e \\ \Rightarrow \quad & ln(c_t^e) \approx ln(c_t) + (1 - \varphi) u_t \end{aligned} \tag{1}$$

Case 2: $c^{nlf} = c^u = \varphi c^e$

$$\begin{aligned} c_t &= e_t c_t^e + u_t \varphi c_t^e + (1 - e_t - u_t) \varphi c_t^e \\ &= [\varphi + (1 - \varphi) e_t] c_t^e \\ \Rightarrow \quad & ln(c_t^e) \approx ln(c_t) - \frac{(1 - \varphi)}{\varphi} e_t + const \end{aligned} \tag{2}$$

We set $\varphi = 0.83$ based on Saporta-Eksten (2014). Using quarterly data from 1987 to 2012, the cyclical elasticity (with respect to real GDP) of per-capita consumption, the unemployment rate, and the employment-to-population ratio are 0.61 (s.e. 0.03), -0.55 (0.03) and 0.42 (0.03), respectively. Using either equation (1) or (2), the cyclical elasticity of employed consumption is 0.52. Thus, the RAW's cyclical elasticity (wrt GDP) is slightly less countercyclical than in the perfect consumption sharing case: -2.51 (s.e. 0.20) instead of -2.69.

A1.1.1 Hamilton Filter

We replicate each table in the main paper but using Hamilton's (Forthcoming.) detrending approach rather than the HP filter. Specifically, Hamilton's trend is calculated by estimating an OLS regression of y_{t+h} on a constant and the p most recent values of y as of date t. For quarterly data, we use $h = 8$ and $p = 4$.

Table A1 shows that the Hamilton-detrended RAW continues to be strongly countercyclical. We also see a difference between the Hamilton and HP filter; namely, the Hamilton filter is is not a linear operator. Thus, after applying the Hamilton filter, the cyclical elastcities of the RAW's components (scaled by the appropriate coefficients) do not add up to the cyclical elasticity of the RAW.

Table A1: **Representative Agent Wedge: Hamilton Filter**

	Elasticity wrt	
	GDP	Total Hours
Representative agent wedge	-2.36 (0.19)	-1.97 (0.07)
Labor productivity	0.03 (0.08)	-0.26 (0.05)
Hours per capita	1.25 (0.07)	0.94 (0.04)
Consumption per capita	0.65 (0.03)	0.37 (0.03)
Tax rates	0.00 (0.07)	-0.05 (0.05)

Note: Each entry is from a separate regression. Sample covers 1987Q1–2012Q4. All variables are in logs and detrended following Hamilton (Forthcoming.). The wedge calculation assumes $\sigma = 0.5$ and $\eta = 1.0$.

A1.2 Extensive and Intensive Margin Wedges

Some variables (for example, y_t/n_t and c_t) used to construct the IMW and EMW are the same as used for the RAW. Additional (seasonally-adjusted) variables include:

- h_t: Average weekly hours worked (per worker); BLS, LPC, Business Sector.
- v_t: Vacancies (per capita); Pre-1995 is help-wanted index, and post-1995 is Barnichon's (2010) spliced series of help-wanted and JOLTS. Population is 16+.

- m_t: Matches (per capita); Post-1994 from Fallick and Fleischman (2004), and pre-1994 is backcast using data on unemployment and vacancies, following Blanchard and Diamond (1989).

The calibration is described in the text with the exception of ψ, the fixed (utility) cost of employment. One can derive an expression for ψ by combining the steady-state optimality conditions for the extensive and intensive margins and assuming the EMW and IMW are the same in steady state. The result is $\psi \equiv \frac{h^{1+1/\eta}}{\eta+1}\left[1-(\eta+1)\left[1-\beta(1-\delta)\right]\left[\frac{\kappa v}{\phi m}+\gamma\right]\right]$.

The EMW includes expectational terms in S_t, for example, $\mathbb{E}_t\left\{\frac{u'(c_{t+1})}{u'(c_t)}\frac{y_{t+1}/n_{t+1}}{y_t/n_t}\right\}$. We construct these using three-variable, four-lag VARs consisting of real GDP growth, aggregate (log) hours worked, and the respective expectational term. We estimate the VAR using data over the entire sample period, and then use the estimated coefficients to construct time series of the expectational terms.

Finally, as a robustness exercise, we constructed the EMW using alternative data. We assumed $\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{1+r_{t+1}}$ and measured the (ex post) real interest rate, r_{t+1}, as the three-month T-bill rate less (realized) core PCE inflation at $t+1$. Our results change little. The cyclical elasticity of the EMW with respect to GDP was -1.89 (s.e. 0.28) and with respect to aggregate hours was -1.54 (0.15).

A1.2.1 Hamilton Filter

Table A2 reports cyclical elasticities for the Hamilton-detrended EMW and IMW. As was the case for the HP-filtered data in the main text, the two wedges have very similar cyclical behavior and smaller elasticity than the RAW.

A1.3 Aggregate Wedge Decomposition

The decomposition requires wage measures. For our baseline (labeled AHE), we assume $\frac{w_t n_t}{p_t y_t}$ is the labor share of income as measured in the BLS's LPC Business Sector. Because we also have a series for labor productivity $\frac{y_t}{n_t}$, we can back out the average real wage in the economy.

Table A2: **Extensive and Intensive Margin Wedges: Hamilton Filter**

	Elasticity wrt	
	GDP	Total Hours
Extensive margin wedge	-1.31 (0.18)	-1.36 (0.13)
Intensive margin wedge	-1.29 (0.12)	-1.34 (0.06)

Note: Each entry is from a separate regression. Sample covers 1987Q1–2012Q4. All variables are in logs and detrended following Hamilton (Forthcoming.).

Kudlyak (2014) estimated the semi-elasticities of average hourly earnings, new hire wages, and the user cost of labor, respectively, to the unemployment rate. We use these estimated elasticities, along with the time series of unemployment and our (baseline) average wage measure, to construct time series for new hire wages and the user cost of labor.

A1.3.1 Hamilton Filter

Table A3 reports results of decomposing the Hamilton-filtered EMW and IMW using different wage measures. As in the main text, using average hourly earnings (AHE) attributes little (less than 20 percent) of the wedge to the price markup, while decomposing using the user cost of labor (UC) attributes almost all of the wedge to the price markup.

A2 Self-Employed Problem

We present a static decision problem of a person deciding between (1) working for someone else as an employee; (2) sole proprietorship (self-employment with no employees); or (3) self-employment with employees. It features no

Table A3: **Wedge Decomposition, Alternative Wage Measures: Hamilton Filter**

	Elasticity wrt	
	GDP	Total Hours
Extensive margin wedge	-1.31 (0.18)	-1.36 (0.13)
Price markup (AHE)	-0.23 (0.10)	-0.24 (0.09)
Price markup (NH)	-0.66 (0.13)	-0.61 (0.11)
Price markup (UC)	-1.37 (0.19)	-1.21 (0.15)
Intensive margin wedge	-1.29 (0.12)	-1.34 (0.06)
Price markup (AHE)	-0.08 (0.07)	-0.10 (0.06)

Note: Each entry is from a separate regression. Sample covers 1987Q1–2012Q4. All variables are in logs and detrended following Hamilton (Forthcoming.).

extensive margin frictions; the person works in one of the three ways. All earnings are consumed. The person draws a talent triple $\{z_e, z_p, z_s\}$ governing their efficiency as an employee, a sole proprietor, or as a self-employed person with employees. These talent draws determine whether the person works as an employee (sufficiently high z_e/z_p and z_e/z_s), a sole proprietor (sufficiently high z_p/z_e and z_p/z_s), or as a self-employed person with employees (sufficiently high z_s/z_e and z_s/z_p). The person takes as given their talents, the market wage per efficiency unit (w) of hours worked as an employee, aggregate consumption C, and the aggregate price level P. Aggregate consumption is a CES composite of each firm's output with elasticity of substitution ϵ. The aggregate price index is normalized to 1.

A2.1 Working for someone else

This is the same as the representative agent decision problem in Section 2 of the paper, except that each worker has efficiency units z_e and faces a wage per efficiency unit instead of a wage per hour.

The person maximizes

$$u(c,h) = \frac{c^{1-1/\sigma}}{1-1/\sigma} - \nu\frac{h^{1+1/\eta}}{1+1/\eta}$$

subject to

$$c \leq wz_e h.$$

Here c denotes composite consumption of the individual, h is their hours worked, z_e is their efficiency units as a worker, and w is the market real wage per efficiency unit.

This leads to optimal hours worked as a worker of

$$h^* = \left(\frac{z_e w}{\nu^{\sigma/(\sigma-1)}}\right)^{\frac{1-1/\sigma}{1/\eta+1/\sigma}}$$

One can then calculate $c^* = z_e w h^*$ and the $u(c^*, h^*)$ associated with being a worker.

A2.2 Self-employed with employees

Assume production for the self-employed *with workers* is

$$y = z_s h^{1-\alpha} n^{\alpha}$$

where z_s is the person's talent as a self-employed person and n is the total efficiency units of labor hired (the sum across hired workers of the product of their talent and hours).

A person with workers will hire them to maximize profits:

$$\pi = py - wn.$$

Assuming the firm faces CES demand with elasticity ϵ ($y/C = p^{-\epsilon}$), profits can be re-expressed as

$$\pi = C^{1/\epsilon} y^{1-1/\epsilon} - wn = C^{1/\epsilon} \left(z_s h^{1-\alpha} n^{\alpha}\right)^{1-1/\epsilon} - wn.$$

This leads to the usual first order condition setting the marginal revenue product of efficiency units of labor equal to the wage per efficiency unit:

$$\alpha(1-1/\epsilon) C^{1/\epsilon} \left(z_s h^{1-\alpha}\right)^{1-1/\epsilon} n^{\alpha(1-1/\epsilon)-1} = w.$$

It then follows that the labor share (wage bill relative to revenue) is

$$\frac{wn}{py} = \alpha(1-1/\epsilon).$$

Thus self-employed profits (or earnings) can be expressed as

$$\pi = \left[1-\alpha(1-1/\epsilon)\right] C^{1/\epsilon} \left(z_s h^{1-\alpha} n^{\alpha}\right)^{1-1/\epsilon}$$

when n is chosen optimally. The self-employed person with employees therefore chooses their hours to maximize

$$u(c,h) = \frac{c^{1-1/\sigma}}{1-1/\sigma} - \nu \frac{h^{1+1/\eta}}{1+1/\eta}$$

subject to

$$c \leq \left[1-\alpha(1-1/\epsilon)\right] C^{1/\epsilon} \left(z_s h^{1-\alpha} n^{\alpha}\right)^{1-1/\epsilon}.$$

The first order condition for self-employed hours is

$$(1-1/\epsilon)\frac{\pi}{h} c^{-1/\sigma} = \nu h^{1/\eta}. \tag{3}$$

Note that marginal earnings per hour worked is proportional to average earnings per hour worked. The factor of proportionality is the inverse of the price-cost markup. Fluctuations in average earnings thus capture movements in marginal earnings if the price markup is constant. Procyclical ϵ (countercyclical markups) might account for the cyclical labor wedge in the data for the self-employed with employees.

One can combine the two first order conditions (for efficiency units hired and own self-employed hours) to determine h^* and n^*. They will be functions of exogenous (to the individual) variables w, C, z_s, and ϵ, and of the fixed parameters η, σ, ν, and α.

One can then calculate $c^* = \pi(n^*, h^*)$ and the $u(c^*, h^*)$ associated with being self-employed with employees.

A2.3 Sole proprietors

Assume production for the sole proprietors (self-employed *with no workers*) is

$$y = z_p h.$$

With this production function, the self-employed person with no employees chooses their hours to maximize

$$u(c,h) = \frac{c^{1-1/\sigma}}{1-1/\sigma} - \nu \frac{h^{1+1/\eta}}{1+1/\eta}$$

subject to

$$c \leq C^{1/\epsilon} \left(z_p h\right)^{1-1/\epsilon}.$$

The first order condition is as before:

$$(1-1/\epsilon)\frac{\pi}{h} c^{-1/\sigma} = \nu h^{1/\eta}. \tag{4}$$

Note that marginal earnings per hour worked continue to be proportional to

average earnings per hour worked, conditional on the price-cost markup.

One can similarly solve for the hours worked and consumption that satisfy the budget constraint and first order condition to arrive at h^*, c^*, and in turn $u(c^*, h^*)$.

Again, procyclical ϵ might account for the cyclical labor wedge in the data for sole proprietors. Conditioning on self-employment in consecutive periods, as opposed to on self-employed with employees versus sole proprietors separately, does not pose a selection problem for inferring *changes* in the labor wedge — as long as ϵ is the same whether a person has employees or is a sole proprietor. This follows from the identical form of the first order condition in (3) and (4).

A3 Self-Employed Empirics

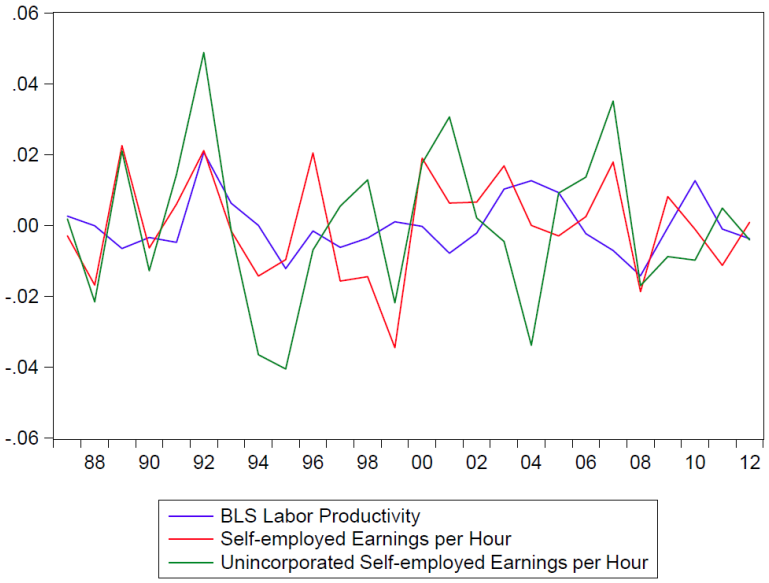
Figures A1–A4 display the time series of the variables that underlie our estimates of the cyclicality of the all-worker and self-employed wedges, respectively. All series are HP-filtered. Figure A1 displays (log) indices for hours per week for both the self-employed and wage earners, while Figure A2 presents the same comparison for annual hours. Figure A3 presents aggregate labor productivity, self-employed income per hour, and income per hour for the unincorporated self-employed. We use March CPS data to construct the time series in these three figures, as described in Section 3 of the main paper.

Figure A4 presents our consumption series for the self-employed together with aggregate consumption. To construct consumption for the self-employed, we use the Consumer Expenditure Surveys (CE) from 1987 through 2012 to get a quarterly series for the growth rate of self-employed consumption relative to a representative sample of CE households. The relative growth rate, in turn, is integrated to obtain a series for relative self-employed consumption, indexed to the beginning of 1987. We add this relative estimate to NIPA aggregate consumption to arrive at an estimate of the cyclicality of consumption for the self-employed. The following paragraphs describe the construction of the

Figure A1: Weekly Hours: Self-Employed vs. Wage Earners

Figure A2: Annual Hours: Self-Employed vs. Wage Earners

Figure A3: Alternative Productivity Measures

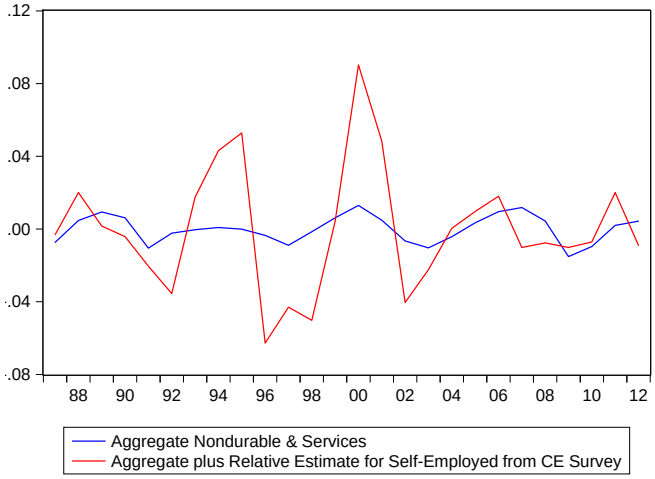


quarterly growth rates of consumption for the self-employed.

The CE has been an ongoing quarterly survey since 1980, with about 5,000 households interviewed each quarter. Households are asked about their detailed expenditures for the previous three months. Each household is surveyed up to four consecutive quarters, allowing construction of up to three observations on quarterly growth. We focus on expenditures on nondurables and services, which we construct by aggregating individual categories that are clearly not durables by NIPA standards. We include expenditures on housing: for renters this is captured by household rent; for home owners it reflects the owner's estimate of its rental value (rental equivalence). The categories we can classify as nondurables and services constitute about two-thirds of household expenditures. We deflate these expenditures by the GDP deflator for nondurables and services. Individual growth rates across any two quarters are calculated by the midpoint formula to reduce the impact of extreme values.

During each of the first and fourth quarterly interviews on expenditures,

Figure A4: Alternative Consumption Measures



households are surveyed about the work experience of their household members during the past 12 months. We focus on the work history in the latter survey, as the work history over the prior 12 months conforms to the time frame for reported expenditures. (For a small number of households, we fill in for missing employment information from responses collected in earlier quarters.) We create a sample of workers from the CE households, including all members that meet our sample requirements. These requirements are chosen to mimic our treatment of the CPS data: (i) individuals must be between ages 20-70; (ii) they must report working at least 10 weeks during the year, at a workweek of 10 hours or more when working; (iii) we exclude workers in the top or bottom 9.6 percent of the income distribution and the top 1.2 percent of hours per week. These last exclusions are chosen to match those we made on the CPS data, dictated by its top-coding of income and hours. We make two other sample restrictions in order to measure quarterly growth rates of household consumption. We exclude households in the top and bottom 1 percent of expenditures in any quarter in order to eliminate top-coded expenditures and outliers. We exclude households that exhibited a change in

household size across the quarters that are the basis for the growth rate. In all calculations we employ the CE sampling weight that is designed to make the sample representative of the U.S. civilian noninstitutionalized population.

We classify workers as self-employed, as opposed to wage earners, if they report that the job for which they received most income was self-employment and, in fact, at least 95 percent of their reported income over the past 12 months is from (nonfarming) self-employment. This conforms well to our definition in Section 3 of the main paper based on CPS data. We do not observe consumption at the individual level (e.g., for a self-employed member versus a wage-earning member). Thus, we have to make the simplifying assumption that households equate consumption across members. For example, if a household has one self-employed worker and one wage earner, then that household contributes two members to our overall sample and one member to our self-employed sample. But the growth rate in consumption in any quarter will be the same for both members of that household. We have 11,849 quarterly observations on consumption growth that apply for self-employed workers, which equals 115 per quarter on average.

A3.1 Hamilton Filter

We have annual data for the self-employed wedge and thus construct the Hamilton trend using $h = 2$ (i.e., forecast two-year-ahead wedge) and $p = 0$ (i.e., no lags) as suggested by Hamilton (forthcoming). To calculate the trend for the first few years of our sample, we first need to backcast values of the wedge for 1985 and 1986, which we do with an AR(1) specification.

Table A4 replicates Table 5 from the main text. Our main result continues to hold; namely, the self-employed labor wedge (shown in Column 3) has similar cyclicality as the all-worker wedge (in Column 1).

Table A5 replicates Table 6 from the main text, in which we explored alternative measurements for the self-employed wedge. In all cases but one

Table A4: **Cyclicality of the Labor Wedge, All Workers vs. Self-Employed: Hamilton Filter**

Elasticity wrt	(1)	(2)	(3)	(4)
Real GDP	-1.66 (0.23)	-1.93 (0.23)	-1.44 (0.25)	-2.59 (0.84)
Total hours	-1.20 (0.19)	-1.56 (0.13)	-1.20 (0.16)	-1.62 (0.63)

Notes: Replicates Table 5 of the main text, except that all data have been detrended following Hamilton (forthcoming).

(i.e., column 3 with respect to GDP), the cyclical elasticity of the self-employed wedge is at least half of the cyclical elasticity of the all-worker wedge (i.e., Column 1 of Table A4). And, in all cases, the cyclical elasticity of the self-employed wedge is significantly different than zero.

A4 Intermediates

We first derive an industry-level, intensive margin labor wedge using more general technology and preferences than we used for our baseline results. We then derive the industry-level, *extensive margin* labor wedge. Finally, we describe the data used in our calculations.

The *gross output* production function implies a marginal product of labor on the intensive margin of

$$mpn_{it}^{int} = (1-\alpha)(1-\theta)\left(\frac{y_{it}}{v_{it}}\right)^{\frac{1}{\varepsilon}}\left(\frac{v_{it}}{n_{it}}\right)^{\frac{1}{\omega}}\left(z_{v,it}z_{n,it}\right)^{\frac{\omega-1}{\omega}}e_{it}.$$

For our baseline, $\varepsilon = \omega = 1$, this simplifies to $mpn_{it}^{int} = (1-\alpha)(1-\theta)\frac{y_{it}}{n_{it}}e_{it}$.

Table A5: **Labor Wedge, Self-Employed Alternatives: Hamilton Filter**

Elasticity wrt	(1)	(2)	(3)	(4)
Real GDP	-1.44 (0.25)	-0.83 (0.25)	-0.73 (0.25)	-0.94 (0.38)
Total hours	-1.20 (0.16)	-0.69 (0.20)	-0.80 (0.20)	-1.01 (0.28)

Notes: Replicates Table 6 of the main text, except that all data have been detrended following Hamilton (forthcoming).

Our baseline used the following preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-1/\sigma}}{1-1/\sigma} - \nu \sum_i \left[\left(\frac{h_{it}^{1+1/\eta}}{1+1/\eta} + \psi \right) e_{it} \right] \right\},$$

so the marginal rate of substitution of consumption for an extra hour per worker in industry i is $mrs_{it}^{int} = \nu h_{it}^{1/\eta} c_t^{1/\sigma} e_{it}$.

Thus, our baseline industry-i (intensive margin) labor wedge is (up to an additive constant)

$$\begin{aligned} ln(\mu_i^{int}) &= ln\left(\frac{p_i mpn_i^{int}}{p\, mrs_i^{int}}\right) = ln\left(\frac{p_i}{p}\frac{y_i}{n_i}\right) - \left[\frac{1}{\sigma} ln(c) + \frac{1}{\eta} ln(h_i)\right] \\ &= ln\left(\frac{p_i \frac{v_i}{n_i}}{p\frac{v}{n}}\right) + ln\left(\frac{y_i}{v_i}\right) - \frac{1}{\eta} ln\left(\frac{h_i}{h}\right) + ln\left(\frac{mpn^{int}}{mrs^{int}}\right), \end{aligned} \tag{5}$$

where $mpn_t^{int} \equiv (1-\alpha)(1-\theta)\frac{v_t}{n_t} e_t$ and $mrs_t^{int} \equiv \nu h_t^{1/\eta} c_t^{1/\sigma} e_t$ are based on aggregate data. For $\varepsilon, \omega \neq 1$, it is straightforward to see how the labor wedge would be altered. Specifically, for $\varepsilon < 1$, the labor wedge becomes less countercyclical if gross output is more procyclical than value added.

Note that our preferences assume separability across labor supply in

different industries. This seems reasonable because workweeks are person-specific. But, we could consider alternative preferences, say,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-1/\sigma}}{1-1/\sigma} - \nu \left(\frac{h_t^{1+1/\eta}}{1+1/\eta} + \psi \right) e_t \right\},$$

where $h_t \equiv \frac{\sum_i h_{it} e_{it}}{\sum_i e_{it}}$ and $e_t \equiv \sum_i e_{it}$. In this case, $mrs_{it}^{int} = \nu h_t^{1/\eta} c_t^{1/\sigma} e_{it}$. The industry-$i$ labor wedge is thus (for baseline technology, $\varepsilon = \omega = 1$)

$$ln(\mu_i^{int}) = ln\left(\frac{p_i \frac{v_i}{n_i}}{p \frac{v}{n}} \right) + ln\left(\frac{y_i}{v_i} \right) + ln\left(\frac{mpn^{int}}{mrs^{int}} \right).$$

Under these preferences, labor supply is perfectly substitutable across industries and only aggregate labor supply, e and h, matters. The labor wedge no longer needs an adjustment for industry-specific workweeks. Table A6 shows how replacing industry-specific workweeks with aggregate average weekly hours worked affects our results (compare with Table 8 in the main paper). Manufacturing industries exhibit more procyclical workweeks, and the labor wedge is thus less countercyclical for manufacturing. On the other hand, it is more countercyclical for nonmanufacturing and all industries. (For the latter, recall that the 60 industries covered by KLEMS are not necessarily representative of the entire economy.)

Moving to the extensive margin, $mpn_{it}^{ext} = mpn_{it}^{int} \frac{h_{it}}{e_{it}}$ and $mrs_{it}^{ext} = mrs_{it}^{int} \frac{\Omega_{it}}{h_{it}^{1/\eta}} \frac{h_{it}}{e_{it}}$. The industry-$i$ extensive margin labor wedge is

$$ln(\mu_i^{ext}) = ln\left(\frac{p_i mpn_i^{ext}}{p\, mrs_i^{ext}} \right) - S_i = ln(\mu_i^{int}) - ln\left(\frac{\Omega_i}{h_i^{1/\eta}} \right) - S_i,$$

or, for our baseline case, it is

$$ln(\mu_i^{ext}) = ln\left(\frac{p_i \frac{v_i}{n_i}}{p \frac{v}{n}} \right) + ln\left(\frac{y_i}{v_i} \right) - ln\left(\frac{\Omega_i}{\Omega} \right) + ln\left(\mu^{ext} \right) - (S_i - S). \quad (6)$$

Because of data limitations (i.e., vacancies and matches are not available for

Table A6: **Cyclicality of (Common MRS) Intensive Margin Labor Wedge**

	Elasticity wrt	
	GDP	Total Hours
All Industries	-1.10 (0.26)	-0.72 (0.13)
Manufacturing	-0.55 (0.39)	-0.35 (0.20)
Non-Manufacturing	-1.25 (0.24)	-0.82 (0.12)

Note: Each entry is from a separate regression. Annual data are from 1987 to 2012 for 60 industries (1,560 industry-year observations): 18 manufacturing and 42 nonmanufacturing. All variables are in logs and HP-filtered. Regressions include industry fixed effects and use industry average value-added shares as weights. Standard errors are clustered by year.

industries), we assume S_{it} differs across industry only because of industry-specific workweek movements. That is, $S_{it} = \left[\frac{h_i/h_{it}}{h/h_t}\right] S_t$. Table A7 displays the cyclicality of the extensive margin labor wedge. The results are similar to Table A6.

To construct the industry-level labor wedge and the intermediates-based price markup, some variables (e.g., c_t) are the same as used earlier in the paper. Additional variables include:

- p_t: Price deflator for nondurables and services consumption; Tornqvist index of NIPA implicit price deflators for nondurables and services.

- $p_{it}, y_{it}, n_{it}, p_{mit}, m_{it}$: Respectively, the gross output deflator, real gross output, hours worked, intermediates deflator, and quantity of intermediates (Tornqvist index of materials, energy and services) by industry from BLS KLEMS.

- h_{it}: Average weekly hours worked (per worker); ratio of hours worked (from BLS KLEMS) to industry-specific employment (calculated with data underlying BLS LPC dataset).

Table A7: **Cyclicality of (Common MRS) Extensive Margin Labor Wedge**

	Elasticity wrt	
	GDP	Total Hours
All Industries	-1.14 (0.52)	-0.86 (0.28)
Manufacturing	-0.59 (0.66)	-0.49 (0.35)
Non-Manufacturing	-1.28 (0.49)	-0.96 (0.26)

Note: Each entry is from a separate regression. Annual data are from 1987 to 2012 for 60 industries (1,560 industry-year observations): 18 manufacturing and 42 nonmanufacturing. All variables are in logs and HP-filtered. Regressions include industry fixed effects and use industry average valu- added shares as weights. Standard errors are clustered by year.

The KLEMS data incorporate survey information from a number of BLS and BEA (Census) programs. The methodolgy is described in the BLS Handbook of Methods, Chapter 11 (http://www.bls.gov/opub/hom/pdf/homch11.pdf). We note in particular that its industry measures for gross output, value added, and intermediate inputs employ Census, BLS, and IRS data sources, with data sources prioritized to reflect the underying qualilty of data by industry. (See Moyer et al., 2004 for a detailed discussion of methods for selecting data sources and for how measures are harmonized across industries and with NIPA accounts.)

A4.1 Hamilton Filter

Because the intermediates-based price markup is constructed at an annual frequency, we apply the Hamilton filter in the same way we did for the self-employed wedge (i.e., $h = 2$, $p = 0$, and pad beginning of time series by backcasting).

Table A8 replicates Table 7 from the main text. The price markup remains highly countercyclical for the case of all industries together and for

Table A8: **Cyclicality of the Price Markup Using Intermediates: Hamilton Filter**

	$\varepsilon = 1$		$\varepsilon = 0.75$	
Elasticity wrt →	GDP	Hours	GDP	Hours
All Industries	-0.74 (0.14)	-0.55 (0.10)	-0.98 (0.20)	-0.74 (0.14)
NonMfg.	-0.77 (0.11)	-0.56 (0.09)	-1.06 (0.17)	-0.77 (0.13)
Manufacturing	-0.61 (0.30)	-0.54 (0.19)	-0.70 (0.49)	-0.64 (0.34)
Materials	-0.82 (0.44)	-0.75 (0.29)	-1.17 (0.67)	-1.01 (0.48)
Services	0.61 (0.39)	0.43 (0.24)	1.09(0.49)	0.71 (0.28)
Energy	-1.38 (0.95)	-1.00 (0.67)	-2.54 (1.43)	-2.03 (0.95)

Note: Replicates Table 7 of the main text, except that all data have been detrended following Hamilton (forthcoming).

nonmanufacturing industries. For manufacturing industries, the (absolute value of the) elasticities are somewhat smaller than in the HP-filtered case and not as statistically significant. This is also the case when we break intermediates into materials, energy and services for the manufacturing industries.

Table A9 replicates Table 8 from the main text. The cyclicality of the industry-specific labor wedge is quite similar to the HP-filtered version in the main text. Comparing Tables A8 and A9, we conclude that the intermediates-based price markup continues to account for the bulk (i.e., typically around 80 to 85 percent) of the cyclical labor wedge.

A5 Other Nonwage Decompositions

A5.1 Advertising

Hall (2014) considers a simple theory of advertising (further simplified here), in

Table A9: **Cyclicality of the Intensive Margin Labor Wedge: Hamilton Filter**

	Elasticity wrt	
	GDP	Total Hours
All industries	-0.87 (0.20)	-0.70 (0.13)
Manufacturing	-0.73 (0.31)	-0.51 (0.18)
Nonmanufacturing	-0.90 (0.18)	-0.75 (0.12)

Note: Replicates Table 8 of the main text, except that all data have been detrended following Hamilton (forthcoming).

which a firm's objective is

$$max_{p,A}\,(p - mc)\,\frac{A^{\alpha}}{p^{\epsilon}} - \kappa A,$$

where p is the firm's price, A its advertising volume, mc the marginal cost of production, κ the cost of a unit of advertising, and $-\epsilon$ and α are the elasticities of demand with respect to price and advertising. The first-order condition for advertising yields an expression for the ratio of advertising expenditure to revenue:

$$\frac{\kappa A}{pQ} = \alpha\left[1 - \frac{1}{p/mc}\right]. \tag{7}$$

Hall's finding that the advertising expenditure share of revenue is acyclical, combined with equation (7), suggests that markups are also acyclical.

But, as stated in the main text, if advertising spending displays a constant elasticity impact on consumers' reservation prices, rather than on quantity demanded, that implication no longer holds.[1] The firm's objective then

[1]Assume a fixed population and that individual $i's$ willingness to pay for a good is given by $x_i = ZA^{\alpha}\Omega_i$, where Z is an aggregate shifter, A is advertising for the good, and Ω_i is the individual preference. If Ω_i is distributed basic Pareto, $f(\Omega_i) = \epsilon\Omega_i^{-(1+\epsilon)}$ for $\Omega_i \geq 1$, then demand for the good is $Z^{\epsilon}A^{\alpha\epsilon}p^{-\epsilon}$, where p is the price of the good.

becomes

$$max_{p,A}\,(p-mc)\left(\frac{p}{A^{\alpha}}\right)^{-\epsilon}-\kappa A,$$

and optimal advertising requires

$$\frac{\kappa A}{pQ}=\alpha\epsilon\left[1-\frac{1}{p/mc}\right]. \tag{8}$$

In this case, an increase in the price elasticity of demand lowers the price markup — that is, $p/mc=\epsilon/(\epsilon-1)$ – but has no effect on the advertising share. This is because the reduced benefit of advertising, from the decline in p/mc, is exactly canceled by the more elastic response of sales to that advertising.

A5.2 Inventories

Here, we show how data on work-in-process (WIP) inventories can be used to infer a price markup. Following Christiano (1988), we assume a production function that uses WIP inventories as one of its inputs. For a firm in industry i,

$$y_{it}=g(z_{it},n_{it},k_{it}){q_{it}}^{\varphi_{it}},$$

where y_{it} denotes output, q_{it} is beginning-of-period inventories, and z_{it}, n_{it}, and k_{it} are TFP, hours worked, and capital, respectively. The elasticity of output with respect to inventories, φ_{it}, is allowed to vary across both industry and time. The law of motion for inventories is assumed to be

$$q_{i,t+1}=(1-\delta_q)q_{it}+y_{it}-y_{it}^{f},$$

where δ_q is the depreciation rate of inventories, $y_{it}^{f}\geq 0$ is output of finished goods, and $q_{i,t+1}\geq 0$. That is, total output y_{it} is the sum of gross investment in WIP inventories and finished-good output. The latter includes both final sales and (gross) investment in finished-goods inventories, but it is not necessary to separate these two for our purposes.

An optimizing firm minimizes the expected present discounted cost of

producing a given path of finished goods. One perturbation on its cost-minimizing strategy would be to produce an additional unit of output in the form of WIP inventories at time t and then reduce production just enough at $t+1$ — that is, by $\left(1-\delta_q+\varphi_{i,t+1}\frac{y_{i,t+1}}{q_{i,t+1}}\right)$ — to keep inventories unaffected at $t+2$ forward. At an optimum,

$$\frac{mc_{it}}{p_t}=\mathbb{E}_t\left[M_{t,t+1}\frac{mc_{i,t+1}}{p_{t+1}}\left(1-\delta_q+\varphi_{i,t+1}\frac{y_{i,t+1}}{q_{i,t+1}}\right)\right], \tag{9}$$

where $\frac{mc_{it}}{p_t}$ is the (real) marginal cost of production and $M_{t,t+1}$ is the firm's discount factor. In words, the firm equates the marginal cost of output to its marginal benefit, which is reduced future production costs.[2] Because the industry price markup is $\mu^p_{it}\equiv\frac{p_{it}}{mc_{it}}$, we can write (9) as

$$\frac{p_{it}/p_t}{\mu^p_{it}}=\mathbb{E}_t\left[M_{t,t+1}\frac{p_{i,t+1}/p_{t+1}}{\mu^p_{i,t+1}}\left(1-\delta_q+\varphi_{i,t+1}\frac{y_{i,t+1}}{q_{i,t+1}}\right)\right]. \tag{10}$$

We assume the stochastic discount factor is given by $M_{t,t+1}\equiv\beta\frac{u'(c_{t+1})}{u'(c_t)}$ and that the joint conditional distribution of $\frac{u'(c_{t+1})}{u'(c_t)}$, $\frac{1}{\mu^p_{i,t+1}}$, $\frac{p_{i,t+1}}{p_{t+1}}$, and $1-\delta_q+\varphi_{i,t+1}\frac{y_{i,t+1}}{q_{i,t+1}}$ is log-normal and homoskedastic.[3] We then take logs of equation (10) and get (up to a constant)

$$ln(\mu^p_{it})\approx ln\left(\frac{p_{it}}{p_t}\right)+\mathbb{E}_t\left\{-ln\left(\frac{u'(c_{t+1})}{u'(c_t)}\right)+ln\left(\mu^p_{i,t+1}\right)-ln\left(\frac{p_{i,t+1}}{p_{t+1}}\right)-\frac{\varphi_{i,t+1}}{1-\delta_q}\frac{y_{i,t+1}}{q_{i,t+1}}\right\},$$

where $\frac{\varphi_{i,t+1}}{1-\delta_q}\frac{y_{i,t+1}}{q_{i,t+1}}\approx ln\left(1+\frac{\varphi_{i,t+1}}{1-\delta_q}\frac{y_{i,t+1}}{q_{i,t+1}}\right)$. Iterating forward for $ln(\mu^p_{i,t+s})$ and using

[2]Note that an optimizing firm will *always* produce to the point that the marginal value of an extra unit of output equals its marginal cost. In a model in which the firm can adjust sales at the margin, the marginal value of output is simply marginal revenue. If the firm cannot adjust sales, the additional unit of output is held as an inventory and valued accordingly. The value of a finished-good inventory is the expected discounted revenue it generates when it is eventually sold. The value of a WIP inventory, on the other hand, is that the firm enters the next period with a larger stock of WIP inventories.

[3]As explained by Campbell (2003), log-normality implies the log of an expectation can be expressed as an expectation of the log plus a variance term. The conditional homoskedasticity means the variance term is not time-varying.

$u'(c_t) = c_t^{-1/\sigma}$ yields the inventory-based price markup:

$$ln(\mu_{it}^p) \approx -\frac{1}{\sigma} ln(c_t) + ln\left(\frac{p_{it}}{p_t}\right) - \mathbb{E}_t \sum_{s=1}^{\infty} \frac{\varphi_{i,t+s}}{1-\delta_q} \frac{y_{i,t+s}}{q_{i,t+s}} + \text{constant terms.} \tag{11}$$

The intuition for equation (11) is as follows. Suppose the economy is in a recession in period t, so the log marginal utility of consumption, $-ln(c_t)/\sigma$, is high. If the firm's price markup and relative price are not cyclical, then (11) says the path of future output-to-inventory ratios must be high. That is, the firm should be depleting future WIP inventories in order to push output out the door today and boost consumption.

Alternatively, if the expected path of output-to-inventory ratios is not cyclical, then for equation (11) to hold, the firm's real marginal cost (mc_{it}/p_t) must be low in recessions. In turn, either the price markup (μ_{it}^p) is high or the firm's relative price (p_{it}/p_t) is low in recessions. That is, if firms do not deplete inventory investment in recessions, one explanation is that product market distortions keep the firm's price high relative to its marginal cost.

To measure the price markup according to equation (11), we turn to NIPA, which provides quarterly and monthly measures of inventories, sales, and sales price deflators by industry. We define industry output as sales plus the change in (total) inventories, and we use quarterly data from 1987 to 2012 for comparison with previous sections.[4] WIP inventories are available for 22 (roughly two-digit) manufacturing industries, but the industry classification changed from the SIC to NAICS in 1997. To create consistent industry definitions, we aggregate some industries, leaving 14 sectors.[5]

[4]Specifically, the output-to-WIP-inventory ratio, $\frac{y_{it}}{q_{it}}$, and price deflator for (industry) sales, p_{it}, are taken from the NIPA Underlying Detail Tables, Real Inventories and Sales.

[5]We use a Tornqvist index to construct chain-weighted growth rates of real sales, real inventories, and price deflators for the combined industries. For bridging across the 1996-97 break, we made two assumptions. For inventories, we assume the industry shares of nominal inventories do not change between December 1996 and January 1997. (This is feasible, since the inventory data are reported for both classifications in 1997, but there is no such overlap for the sales.) For sales, we assume the growth rate in the nominal inventory-to-shipments ratio is the same as that of the real inventory-to-shipments ratio (in January 1997). The former

To calibrate the parameters in equation (11), we first note that inventory-to-output ratios exhibited significant low-frequency movement over our sample period. We thus let φ_{it} vary over time and set $\varphi_{it} = \left[\frac{1}{\beta} - (1-\delta_q)\right] \frac{\bar{q}_{it}}{\bar{y}_{it}}$, where $\frac{\bar{q}_{it}}{\bar{y}_{it}}$ is a quadratic trend fitted to the inventory-output ratio.[6] Our quarterly calibration sets $\beta = 0.996$ and $\delta_q = 0.01$. As a result φ_{it}, which measures the share of output attributable to inventories, is quite low, about 0.2 percent, on average.

Constructing the inventory-based price markup requires computing, at each point in time, the sum of expected future output-to-inventory ratios. We estimate industry-specific, three-variable, 12-(monthly)-lag VARs consisting of real GDP growth, aggregate (log) hours worked, and the industry-specific output-to-inventory ratio. The latter two variables are quadratically detrended. We estimate the VARs using data over the entire sample period, and then use the estimated coefficients to produce a time series for the expected sum of future output-to-inventory ratios.[7]

Figure A5 plots the weighted-average industry μ^p against GDP. As shown, the price markup is quite countercyclical. This is also true if we define the cycle in terms of hours worked. Figure A6 plots μ^p again, but now aggregated to an annual frequency and plotted against the weighted-average manufacturing-industry labor wedge constructed in Section 4 of the main paper rather than against GDP. The price markup accounts for most of the cyclical variation in the labor wedge.

We next run regressions of the industry-level price markup on the cycle

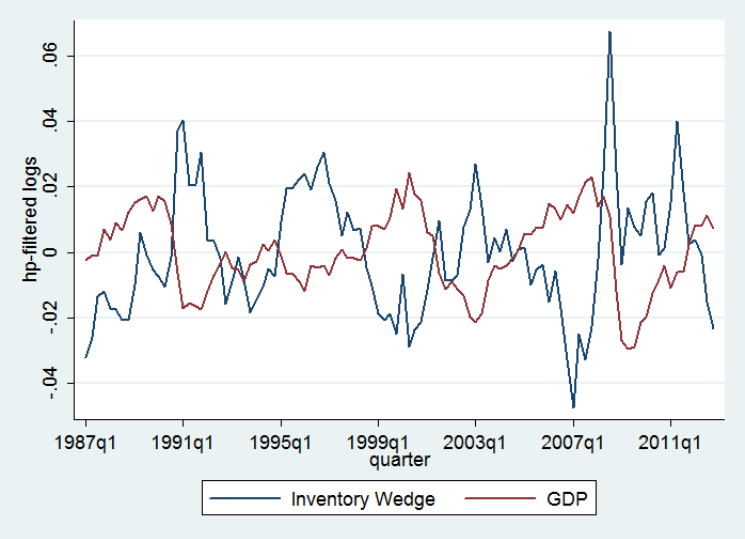
$$log\left(\mu^p_{it}\right) = \alpha_i + \beta^p log\left(cyc_t\right) + \varepsilon_{it},$$

is constructed using data from the Census M3 survey, which has a consistent NAICS industry classification across 1996-97.

[6] This specification for φ_{it} ensures that equation (10) holds in (detrended) steady state.

[7] We considered a second approach to calculating the expected sum of future output-to-input ratios, which involved truncating the sum at either four or eight quarters and calculating the (ex post) realized sum. Because we project the constructed price markup on the time-t business cycle, using the (ex post) realized values is valid for our purposes. It does require using a one-sided HP-filter for the business cycle, so the difference between expected and realized values of the output-to-inventory ratios is orthogonal to the time-t cycle. This second approach produced results for the price markup that were very similar to the VAR approach.

Figure A5: Cyclicality of Inventory-Based μ^p



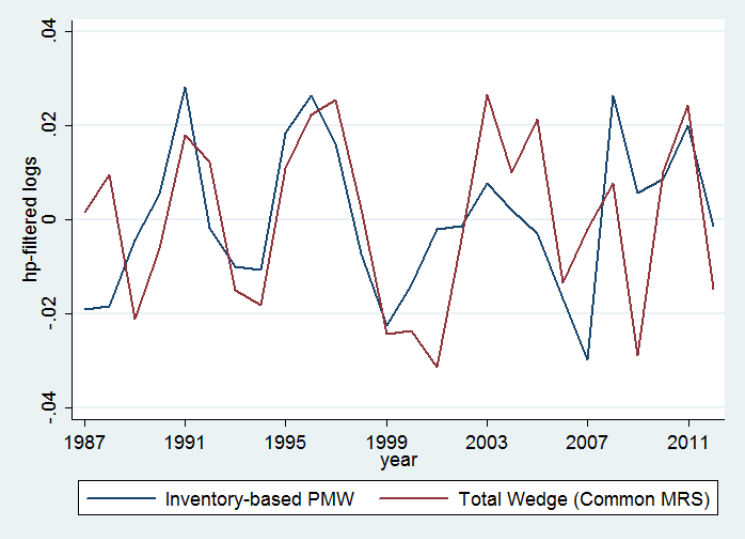
where the weights are the industry's average share of output and standard errors are clustered by period. Table A10 displays the results at an *annual* frequency for comparison with the labor wedge.[8] The strongly countercyclical μ^p (-0.70 elasticity with respect to GDP) accounts for nearly all of the cyclicality in the labor wedge (-0.72).

Finally, we have used WIP inventories for our calculations because these align most closely with the theory, which posits a role for inventories in *production*. Christiano (1988) argues for total inventories (i.e., including materials, WIP, and final goods inventories), noting that labor inputs can be conserved by transporting materials in bulk and holding finished inventories. For robustness, we redo our calculations using total inventories instead of WIP inventories. The results are fairly similar to those reported in Table A10: the cyclical elasticity of the price markup is -0.56 with respect to GDP and -0.22 with respect to hours.[9]

[8]The quarterly elasticities are more precisely estimated: -0.80 (s.e. 0.12) with respect to GDP and -0.33 (0.08) with respect to hours.

[9]Using total inventories enables one to consider industries outside of manufacturing (e.g.,

Figure A6: Inventory-Based μ^p vs. Labor Wedge μ



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wholesale and retail trade).

Table A10: **Cyclicality of Inventory-Based** μ^p

	Elasticity wrt	
	GDP	Total Hours
Price markup	-0.70 (0.26)	-0.26 (0.15)
Marginal utility of consumption	-1.33 (0.06)	-0.76 (0.08)
Relative price	0.83 (0.20)	0.60 (0.11)
Expected output/inventory path	0.21 (0.05)	0.10 (0.04)

Note: Each entry is from a separate regression. Annual data are from 1987 to 2012 for 14 manufacturing industries (364 industry-years). Variables are in logs and HP-filtered. Regressions include industry fixed effects and use industry average value-added shares as weights. Standard errors are clustered by year. See equation (11) for the components.

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