Aggregate Issuance and Savings Waves: Online Appendix

Andrea L. Eisfeldt — UCLA Anderson

Tyler Muir — Yale SOM

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Abstract

This Appendix provides supplementary material to the main text. In particular, we provide and analyze a simple two-date version of our dynamic model, provide a detailed description of data sources, compute additional robustness checks and comparative statics, show firm policy functions, and include mathematical proofs.

I. Two Date Model

We develop and analytically characterize the relationship between the cost of external finance, the amount of external finance raised, and investment in capital and liquid assets in a simple two date model, in order to provide some basic intuition for our main results. In particular, we illustrate the reason why firms' use of the external finance they raise provides information about the cost of those funds. The intuition in both the simple and the full models is as follows: The marginal benefit of investing in physical capital is high at low levels, but decreases with the level of investment. Liquidity accumulation displays less decreasing returns to scale, but has a marginal benefit that is on average lower than the marginal benefit of investment in physical capital. As a result, firms will only invest in liquid assets once they have invested enough in physical capital to push the marginal return below that on liquid assets, but will typically invest a positive amount in physical capital. Importantly, the lower the cost of external finance is, the more likely it is that the marginal return to physical capital investment will be pushed down to the marginal return to cash because a lower cost of funds increases investment.

In the simple two date model, the decreasing marginal return to physical capital is due to decreasing returns to scale, and we fix the marginal return on liquid assets. Clearly, for positive liquidity accumulation to occur, this fixed return must exceed the discount rate, and for positive capital accumulation it must be lower than the marginal return on investment below some level of investment. In the two date model, we simply assume a fixed return that exceeds the discount rate, but is lower than the marginal return on capital for low levels of investment. In our dynamic model, physical capital investment has decreasing marginal returns due to both decreasing returns to scale, as well as to convex adjustment costs. The physical return to liquid assets is fixed at a rate *lower* than the discount rate, and thus positive liquidity accumulation only arises when the total return is endogenously pushed above the discount rate due to the benefit of liquid assets' use as future internal funds for investment.

In this section, we study a firm which maximizes the present value of cash flows over two dates, zero and one. For simplicity, we set the interest rate to zero. At date zero, the firm receives an endowment of liquid assets, and internal funds from operating cash flows, both of which we normalize to zero without (qualitative) loss of generality. The firm then chooses how much to invest in both physical capital (i_k) and liquid assets (i_l). At date one, the firm receives cash flows from its productive physical capital and from its liquid assets. Liquid assets produce $r_l > 1$ at date one. Physical capital produces output according to zi_k^θ and does not depreciate. We define payouts gross of financing costs, e, as internal funds minus investment in physical capital and liquid assets. If $e < 0$, the firm is raising external finance, and pays a cost $\frac{\xi e^2}{2}$, where

ξ is interpreted as the current "level" of the cost of external finance.

The firm's objective over date zero and date one cash flows, respectively, is then:

$$\max_{i_k, i_l} \left\{ \left[e - \mathbb{1}_{\{e<0\}} \frac{\xi e^2}{2} \right] + \left[(i_k + z i_k^\theta) + (l + i_l) r_l \right] \right\} \tag{1}$$

$$s.t. \quad e = -i_l - i_k$$

$$i_l \geq 0.$$

where $\mathbb{1}_{\{e<0\}}$ is an indicator equal to one if the firm is raising external finance. Since we normalize operating cash flows to zero, this indicator will always equal one due to the inada condition on the production function. We use ψ_l to denote the multiplier on the constraint $i_l \geq 0$. The first order condition with respect to investment in liquid assets, i_l, is:

$$r_l - 1 + \psi_l = -\xi \left(-i_l - i_k \right).$$

The first order condition with respect to capital investment, i_k, is:

$$\theta z \left(i_k \right)^{\theta - 1} = -\xi \left(y - i_l - i_k \right).$$

There are two cases depending on whether the firm's constraint on negative liquidity accumulation is binding or not. In the case that the firm both invests and accumulates liquidity, $\psi_l = 0$ and the first order conditions equate the marginal product of capital, the return on liquid assets, and the marginal cost of raising external finance. In the case that $i_l = 0$, the marginal return on capital is set equal to the marginal cost of external finance, but exceeds the marginal return on liquidity accumulation with ψ_l capturing the wedge between the two.

These first order conditions imply the following optimal financing and investment policies:

$$i_l = \frac{r_l - 1}{\xi} - \left(\frac{r_l - 1 + \psi_l}{\theta z} \right)^{\frac{1}{\theta - 1}}$$

$$i_k = \left(\frac{r_l - 1 + \psi_l}{\theta z} \right)^{\frac{1}{\theta - 1}}$$

$$-e = \frac{r_l - 1 + \psi_l}{\xi}.$$

Figure 1 illustrates the firm's investment and financing decisions decisions graphically by plotting the net marginal benefit of capital investment and investment in liquid assets, along with the marginal cost of external finance for three levels of ξ, high, medium, and low. Consider the middle panel, which depicts the case for a medium level of ξ, ξ_M, and start from zero external finance raised, zero investment, and zero liquidity accumulation. Because the production function satisfies an inada condition, and the marginal cost of external finance at zero is zero, the firm will raise a positive amount of external finance, and spend the first funds it raises on investment. The firm will then raise external finance and invest the funds, moving down the marginal benefit of investment curve and up the marginal cost of external finance curve, until the marginal return on an additional dollar of investment is either driven below the rising marginal cost of an additional dollar of external finance, or below the constant marginal return of a dollar invested in liquid assets, whichever happens first. As long as ξ is low enough, so that the marginal cost of external finance does not increase too quickly, there will be positive liquidity accumulation as depicted in the middle panel. The left panel of Figure 1 graphs the case in which $i_l = 0$ because the marginal cost curve is steep enough so that the firm chooses not to drive the marginal return to investment down to the marginal return on liquidity accumulation. In this case, ψ_l is positive and drives a wedge between the marginal returns on the two assets. The right hand panel depicts the case for a low level of ξ, ξ_L. One can see that as ξ decreases from ξ_M to ξ_L, the firm raises additional external finance $-e$, and accumulates each unit of additional funds in the liquid asset.

With these firm policies conditional on z in hand, we can consider how the level of ξ affects the correlation between liquidity accumulation and external finance in a panel of firms described by the problem in equation (1) by considering a cross section of firms with heterogeneous z_i. Figure 2 graphs a panel of three firms with heterogeneous levels of productivity z_i, along with the three levels of ξ from Figure 1. For the highest level of ξ, the with the steepest marginal cost, only the firm with the lowest level of productivity accumulates liquidity. As ξ decreases, first the middle productivity firm, and then the high productivity firm, accumulate liquidity. Moreover, each dollar of liquidity accumulated comes from funds raised externally. Although the specifics of this example rely on the assumption that internal funds are zero, because funds are spent first on investment, the qualitative intuition generalizes to the case in which firms have positive internal funds. For this panel of firms, the cross sectional correlation between liquidity accumulation and external finance is:

$$\rho_{e,i_l}^{xs} = corr\left(\frac{r_l - 1 + \psi_{li}}{\xi}, \frac{r_l - 1 + \psi_{li}}{\xi} - \left(\frac{r_l}{\theta z_i}\right)^{\frac{1}{\theta-1}}\right). \tag{2}$$

Clearly, this correlation is zero when ξ is high enough such that $i_l = 0$ for all firms. As ξ decreases, more firms will save some of the proceeds from the external finance they raise. Moreover, equation (2) illustrates that the lower the level of ξ, the more the firms' choices for liquidity accumulation and external finance will both be dominated by the shared term with ξ in the denominator. Thus, as a result of both an extensive and an intensive margin for liquidity accumulation, as ξ decreases, these two flows will be more correlated.

Finally, we construct the investment returns for physical capital and liquid assets. We present these returns for this simple model in order to connect the intuition developed in this section to that for the dynamic model, for which we provide analogous returns. Each return is the physical return times an external finance discount factor. The external finance discount factor is the ratio of a firm's marginal value of funds tomorrow relative to today. In this model, all else equal, the external finance discount factor is high when ξ is large, and this high cost reduces the return to investment and liquidity accumulation. Specifically:

$$R_k = \frac{1}{1-\xi e}\frac{\theta z i_k^{\theta-1}+1}{1}.$$

We can interpret the first term as the external finance discount factor and the second term as the physical return. A high current cost of external finance reduces the marginal return to investment. For the return on liquidity accumulation, we have:

$$R_l = \frac{1}{1-\xi e}\frac{r_l+\psi_l}{1}.$$

At the optimum, all returns are equated, and equal to one since there is no discounting. If liquidity accumulation is positive at the optimum, then $\psi_l = 0$ and $\frac{r_l}{1-\xi e}$ is both the return to liquidity accumulation and the return to an additional dollar of external finance, since that dollar will be invested in liquid assets.

Figure 1: Two period model intuition for ξ_H, ξ_M, and ξ_L. $\theta z {i_k}^{\theta-1}$ represents the marginal product of capital, $\xi(-e)$ the marginal cost of external finance, and $r_l - 1$ the net return on liquid assets. i_l and i_k represent investment in liquid assets and physical capital, $-e$ represents external finance raised.

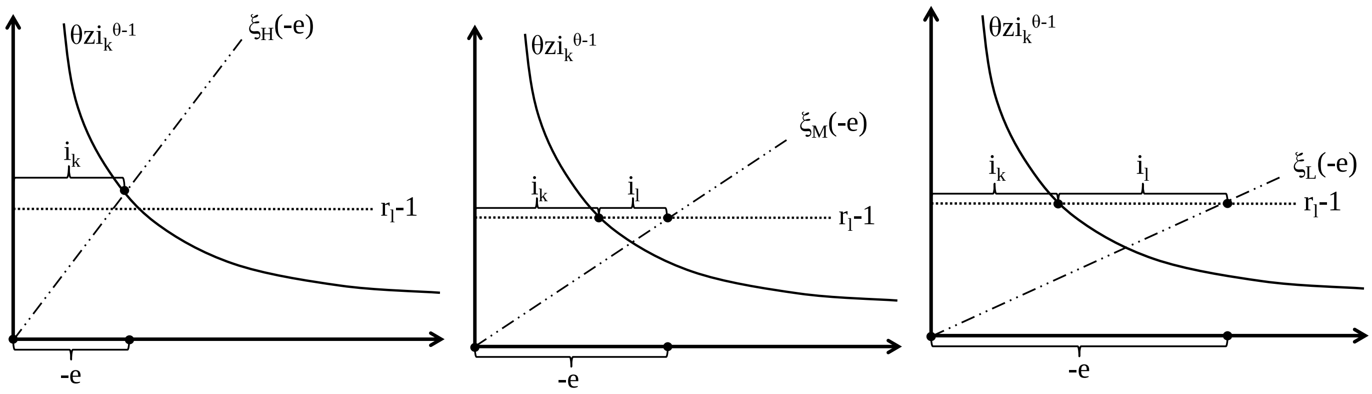
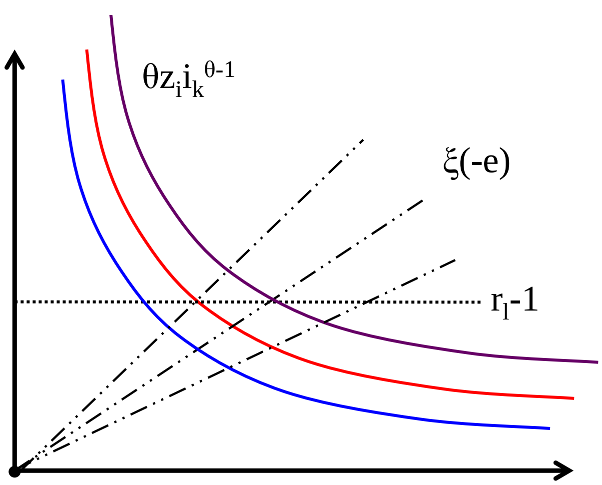


Figure 2: Two period model intuition for a panel of firms with heterogeneous levels of productivity z_i, and three levels of ξ, ξ_H, ξ_M, and ξ_L. $\theta z {i_k}^{\theta-1}$ represents the marginal product of capital, $\xi(-e)$ the marginal cost of external finance, and $r_l - 1$ the net return on liquid assets. i_l and i_k represent investment in liquid assets and physical capital, $-e$ represents external finance raised.



II. Data Description and Sources

Our data construction closely follows Covas and Den Haan (2011). Our primary source of data is the Compustat fundamentals annual file. Our main results use data from 1980-2010. We exclude financials, utilities and firms with SIC codes starting with 9. We also exclude firms with missing assets, equity, debt, and those with missing or negative PPE and cash balances. As in Covas and Den Haan (2011), we also remove GM, GE, Chrysler, and Ford, since these firms were the most affected by the accounting change in 1988 requiring firms to consolidate the balance sheets of their wholly owned subsidiaries.

Computstat Data

We first define liquidity accumulation, investment, and external finance as:

$Investment = i_k =$CAPEX (Capital Expenditures)
$Liquidity\ Accumulation = i_l =$ CHECH (Cash and cash equivalents, change)

$External\ Finance = -e = -(CF_D + CF_E)$

For flows to debt and equity and operating cash flows we use the statement of cash flows:

For statments of cash flows:

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For statments by source and use of funds:

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For working capital statementes:

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For cash statements by activity:

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

Flow of Funds Data

We use annual data from the electronic ASCII flow of funds seasonally adjusted annual rates table F.102 available at
http://www.federalreserve.gov/Releases/z1/Current/data.htm.
Refer to the coded tables for definitions and relationships between entries. Codes appear in parentheses after variable names. Interest payments, not reported in table F.102, are from NIPA table 1.14 line 25 "Net interest and miscellaneous payments" for nonfinancial corporate business.

CF_D =Commercial paper (FA103169700) + Mortgages (FA103065003) - Credit market instruments (FA104104005) + NIPA interest

CF_E =Net dividends (FA106120005) - Net new equity issues (FA103164003)

Liquidity Accumulation = i_l = Net acquisition of financial assets - Commercial paper - Mortgages - Trade receivables - Other Assets

Investment = i_k =Capital expenditures

Other Data

The following series used can be found in the FRED database at the St Louis Fed website.

Gdp= Real gross domestic product

Default Spread= Difference between Moody's Seasoned Baa and Aaa yield. We use end of year values.

Lending Standards= Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Large and Medium Firms (DRTSCILM). We use end of year values.

Sentiment= The University of Michigan Consumer Sentiment Index based on household surveys about the state of the economy. We use end of year values.

Finally, we obtain TFP data from John Fernald's website at: `http://www.frbsf.org/economics/economists/staff.php?jfernald`. We construct the log level series from the series of annual changes provided, and detrend the series with two breaks as in Fernald (2007), which advocates breaks after 1974 and 1995. Shocks are then residuals from an AR(1) regression on the log level series.

III. Comparative Statics and Full Policy Functions

Full policy functions as a function of capital and liquid assets are provided across four productivity and cost of external finance states in Figure 3. We use the most extreme four exogenous states (highest and lowest productivity and highest and lowest cost of external finance) for an illustration of how firm policies change across productivity and cost of external finance states. The states, from left to right are: (high productivity, low cost of external finance), (high productivity, high cost of external finance), (low productivity, low cost of external finance), and (low productivity, high cost of external finance). The first row plots investment, and shows that investment is most constrained in the high productivity, high cost of external finance state. There is not much difference between investment in columns three and four, the two states in which productivity is low, but investment is constrained in column two relative to column one, the two states in which productivity is high. The second row plots external finance, and shows that firms raise more external finance (e is more negative) in the low cost of external finance state, especially when productivity is relatively high. External finance raised is also naturally decreasing in cash and capital. Finally, the third row plots liquidity accumulation, and shows a similar pattern to the investment plots. It is useful to keep in mind that the average firm size as measured by the capital stock is around 100. We plot the stationary distribution of firm size in our model in Figure 5. This informs us that the constrained region in investment, for example, is very relevant for the average size firm.

We plot comparative statics for the five key parameters plus the risk free rate and four key moments in Figure 4. These plots illustrate how the key moments are affected by changes in key parameter values, holding all other parameters constant at their estimated values. Thus, one can think of these plots as showing, starting from the moments implied by the model at estimated parameters, how far and in what direction can changes in a given key parameter move each of these moments. There are small differences between these plots and the estimates in the paper, because the estimates are constructed using a finer grid, and a longer simulation length. To construct these illustrative plots, we use a slightly coarser grid, and shorter sample length, in order to solve the model for a wide range of parameter values. For illustrative moments, we plot the fraction of liquid assets to total assets as well as the pairwise correlations between liquidity accumulation, external finance, and investment.

The upper left panels show the effects of changing the fixed cost of external finance λ_1. Liquid assets are increasing in this fixed cost as the fixed cost induces a hedging motive for holding cash. The more costly it is to raise external funds, the more valuable it is to hold cash. This is especially true with the fixed cost because it induces external finance to be "lumpy" while the firm wants investment to be smooth. The correlation between liquidity accumulation and external finance is also largely increasing in this parameter as issuing and saving is more likely when there is a high fixed cost. This is to avoid paying the fixed cost in the future. The correlation between external finance and investment is U-shaped in this parameter which highlights why we can not simply increase this parameter to achieve larger cash holdings as we would end up making external finance more correlated with investment. Investment also generally becomes more correlated with liquidity as we increase this parameter as more investment is funded by liquid assets rather than external finance.

The upper right panels show the effect of changing the quadratic cost of external finance λ_2. This parameter has a fairly minor effect on cash holdings but does affect each of the correlations, although the effects are not quantitatively large, holding other parameters constant. This parameter is most useful in matching the volatility of external finance as the size of issuances will be much larger when it is lower. Therefore, one can think of λ_2 mostly as being chose to match the volatility of external finance.

The middle left panels show the effect of changing the quadratic adjustment cost of investment a. In addition to the effects displayed, this also has a sizeable effect on the volatility of aggregate investment with larger values leading to smoother investment. This parameter also helps in matching the high observed autocorrelation of investment in the data. We can also see the correlation between liquidity accumulation and external finance is increasing in a as firms will want to save more of the proceeds from raising external finance when investment smoothing motives are higher.

The middle right panels show the effect of changing the volatility of the stochastic cost shocks σ_η. Increasing this volatility leads to larger cash holdings, and a larger correlation betwen liquidity and external finance. This is because the additional uncertainty leads to a larger demand for cash holdings, and the additional variation in the cost leads firms to time the market more. Firms are much more likely to issue external finance when it is cheap because the difference in high and low cost states is much larger. External finance becomes more correlated with liquidity and less correlated with investment as we increase this parameter. The comparative statics make clear why this parameter is crucial in matching our key moments of interest and thus give the intuition for why a model with stochastic costs is necessary to fit the data. This is shown more formally in the text with the $J - test$ that rejects a restricted model that restricts this parameter to zero.

The lower left panels show the effect of changing the persistence (or auto-correlation) of the stochastic cost series γ. Decreasing the persistence makes taking advantage of low cost states more important because it is more likely the economy could transition to a higher cost state next period. Accordingly, lowering the persistence increases liquid asset holdings (for precautionary motives), increases issuance and savings waves, and decreases the correlation between external finance and investment.

The lower right panels show the effect of changing the interest rate r. This parameter is estimated separately in our model to be consistent with the empirical risk free rate. However, it is interesting to see how changes in this parameter could affect our results and this gives us a way to think about how a stochastic interest rate might change our results. It is important to note, however, that variation in the real risk free rate is quantitatively negligible, and in our model purely nominal variation should not matter. For cash holdings, there are two effects when changing the interest rate. First, it would seem higher liquid assets will make the return on liquid assets larger (and in physical terms it does) which may lead to higher share of liquid assets. However, changes in the interest rate also have a large effect on how the firm discounts the future. In particular, the firm cares less about future states of the world when r is higher and cares a lot about such states when r is low. This greatly affects the demand for liquid assets becuase liquidity is primarily used as a hedge against future states (particularly states when the firm would have to raise costly external finance). Thus a decrease in the risk free rate leads to much larger cash to assets (these rise to 6% when $r = 2\%$ and up to 10% when $r = 1\%$). Decreasing the risk free rate also decreases most of the correlations, but its primary impact is on cash balances. Thus the low cash balances in our model can potentially be solved by having a lower risk free rate. Our main correlation of interest, that between liquidity accumulation and external finance, varies between 0.5 and 0.7 with reasonable changes in the interest rate. Empirically this number is 0.6 so we find that the model can roughly match this value with alternative assumptions about the interest rate. Lastly, recall that these numbers do not take into account the fact that the estimation procedure would choose different parameter values if we change the interest rate parameter. Thus, this exercise should mostly be thought of as understanding qualtiatively how the parameter changes will affect certain moments, but the quantities may in fact not change as substantially given that the estimation is designed to minimize the distance between the model and the data.

IV. SMM

The SMM procedure chooses parameters b to minimize $g_T(b)'Wg_T(b)$, where $g_T(b) = M_T(x) - M_N(y(b))$ is the difference between data and model generated moments. For details on the procedure, constructing standard errors, etc, see Lee and Ingram (1991). In our simulation, we use a matrix of shocks of size $(K+2, N)$ where N is the simulated length of the time-series and K is the number of firms. There are $K+2$ shock series because each firm will have an idiosyncratic shock and there will be 2 aggregate shocks (the shocks to productivity and the stochastic cost). We set $K = 1000$ and $N = 700$, but we throw away the first 200 observations to remove initial dependence. We aggregate our panel of firms and compute aggregate statistics when comparing to the data. We use as the weighting matrix (W) the inverse variance covariance matrix of the moments. To construct standard errors, we also need the derivative of the moments with respect to changes in the parameters. Specifically, we need $\frac{\partial g(b)}{\partial b}$ which we compute numerically. Then $\left(1 + \frac{1}{N}\right)\left(\frac{\partial g(b)}{\partial b} W \frac{\partial g(b)}{\partial b}\right)^{-1}$ gives the variance of the estimator.

Given this, we can compute standard errors for parameters as well as the $J - test$ for over-identifying restrictions $(TJ_T(restricted) - TJ_T(unrestricted) \sim \chi^2(\#of restrictions))$. An alternative approach to constructing standard errors is via bootstrap where we re-estimate the model many times using T observations. We can then directly plot a histogram of estimates. This gives confidence intervals very similar to the standard approach outlined above and these are what we report in the main text. Essentially both methods tell us the same thing: since the moments are fairly sensitive to small changes in parameter values, the standard errors are relatively small because the model could not match observed moments with substantially different parameter values.

V. Proof of Proposition 1

Lemma 1. *For each level of capital, liquid assets, and level of the cost of external finance, there is a cutoff for productivity $\hat{z}$ above which the firm finds it optimal to pay the fixed cost and raise external finance.*

Proof. By standard arguments, the fixed cost of external finance leads to inaction regions, and a threshold rule for e. We use a variational argument to show that there is a threshold $\hat{z}$ above which the firm will raise external finance. Consider a firm with productivity z which is deciding whether to raise external finance and invest. Suppose the firm increases k to k^* by raising external finance today, but leaves capital unchanged at all other dates. The firm will get $\frac{1}{1+r}E[z'k^{*\theta}] - \frac{1}{1+r}E[z'k^{\theta}] = \rho\frac{1}{1+r}z(k^{*\theta} - k^{\theta})$ in terms of present discounted productivity gains and would have $(k^* - k)(1 - \delta)$ extra depreciated capital. However, the firm would pay today investment adjustment costs and external finance costs of $\frac{a}{2}\left(\frac{i^{*2}-i^2}{k}\right) + \lambda_1 + \lambda_2\frac{1}{2}\xi e^{*2}$. The firm would also have lower investment adjustment costs next period because we assume it leaves the future path of capital unchanged $\frac{a}{2}\left(\frac{i^{*'2}-i^{'2}}{k}\right)$. It is also possible that the policy for raising external finance next period will change. However, since the firm has extra capital, the pseudo constraint on internal funds associated with ψ_e will be more slack and/or the costs of external finance will be lower. Importantly, only the gain in output next period depends on z. Moreover, because the output gain increases monotonically in z, for given parameters and initial levels of capital and liquid assets, there is always a z for which the above strategy is positive NPV. Thus, there exists a $\hat{z}$, conditional on the firm's other state variables, such that the firm will raise external finance and invest whenever $z > \hat{z}$.

References

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VI. Figures, Tables. View in color.

Figure 3: We plot firm policies as functions of capital and liquid assets for 4 states in our model from left to right: high TFP, low cost of external finance, high TFP, high cost of external finance, low TFP, low cost of external finance, and low TFP, high cost of external finance.

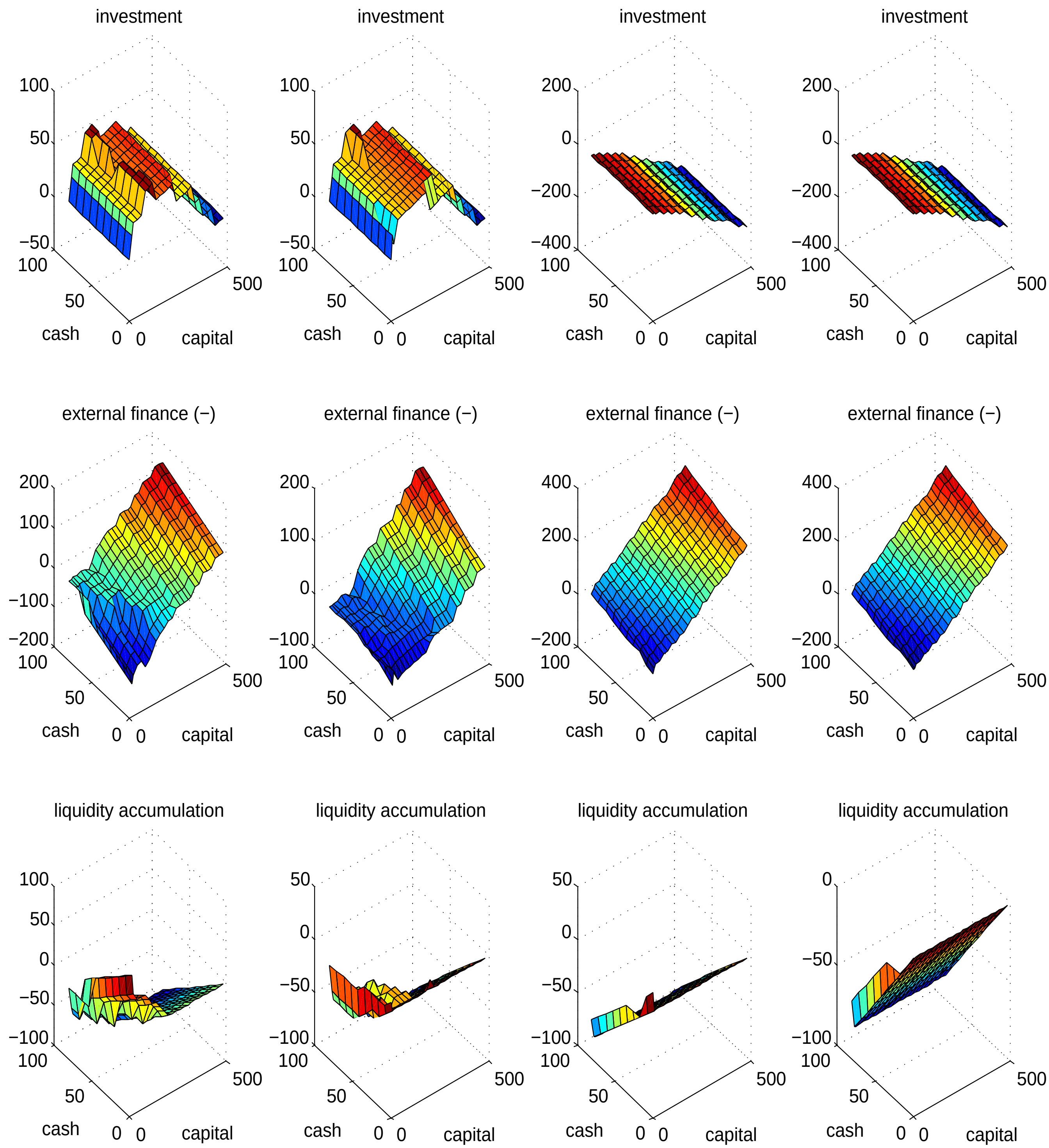

Figure 4: We plot comparative statics of each parameter to four key moments: the pairwise correlations between liquidity, investment, and external finance, as well as the level of liquid assets. We do comparative statics for our two cost parameters λ_1 and λ_2, the investment adjustment cost a, the volatility of external finance shocks σ, the persistent of external finance shocks γ, and the risk free rate r.

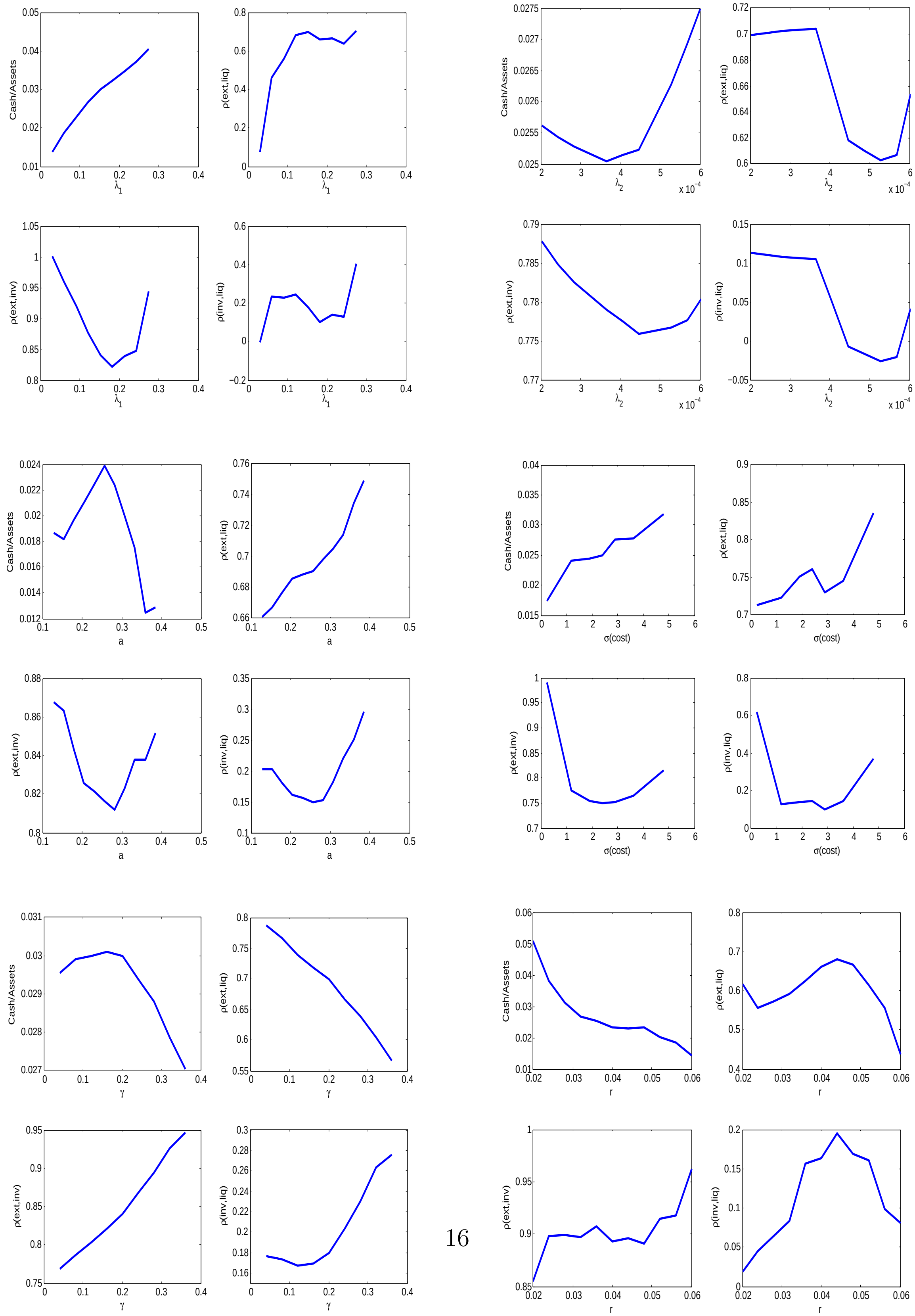

Figure 5: This figure plots the distribution of firm size (capital stock) that comes out of our model. Firm size is on average around 100. This is useful for interpreting where the distribution of firms will be in Figure 3.