Numerical Simulations for "Matching and Sorting in a Global Economy" by G. Grossman, E. Helpman and P. Kircher

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This document serves two purposes. First, in section 1, it describes the computational algorithms employed to solve various cases of the Grossman, Helpman and Kircher (henceforth, GHK) model numerically. Second, in section 2, it discusses comparative static results obtained by numerical simulation, for the case of the model with heterogeneous managers and workers, and strictly log supermodular productivity functions that exhibit a constant elasticity of substitution between manager and worker ability.

1 Solution Approach & Numerical Algorithms

In this section, we summarize the equations defining an equilibrium for various cases of the GHK model, and then discuss how to solve these equations numerically. We begin in subsection 1.1 by examining the case in which managers are homogeneous and workers are heterogeneous, so that the sorting of workers across sectors is a meaningful equilibrium outcome, but the matching of workers to managers is not determined by the model. Subsection 1.2 then considers the case in which both managers and workers are heterogeneous, which introduces the added complexity of solving for the matching between workers and managers in equilibrium.

1.1 Homogeneous Managers

1.1.1 Two Regions of Sorting

Suppose that managers are homogeneous and that productivity in sector i as a function of worker type is denoted by $\tilde{\psi}_i(q_L)$. As shown in section 3 of GHK, the first-order condition

for the firm's optimal choice of worker type implies that the wage function must take the following form:

$$w(q_L) = w_i \tilde{\psi}_i(q_L)^{\frac{1}{\gamma_i}}, \ \forall q_L \in Q_{L_i}, \tag{1.1}$$

where w_i is a constant wage anchor and Q_{L_i} is the set of worker types employed in sector i. Now, we seek an equilibrium in which $Q_{L_1} = [q_L^*, q_{Lmax}]$ and $Q_{L_2} = [q_{Lmin}, q_L^*]$ for some cutoff value $q_L^* \in S_L$, so that there are two regions of sorting. We know from Proposition 1 in GHK that a sufficient condition for the equilibrium to take this form is:

$$\frac{\varepsilon_{\tilde{\psi}_1}(q_L)}{\gamma_1} > \frac{\varepsilon_{\tilde{\psi}_2}(q_L)}{\gamma_2}, \ \forall q_L \in S_L, \tag{1.2}$$

where $\varepsilon_{\tilde{\psi}_i}$ is the elasticity of the function $\tilde{\psi}_i$. We therefore assume in this section that condition (1.2) is satisfied by the choice of parameter values.

An equilibrium of this sort is characterized by the following conditions. First, using (1.1), the zero-profit condition for firms requires:

$$\bar{\gamma}_1 p_1^{\frac{1}{1-\gamma_1}} w_1^{-\frac{\gamma_1}{1-\gamma_1}} = \bar{\gamma}_2 p_2^{\frac{1}{1-\gamma_2}} w_2^{-\frac{\gamma_2}{1-\gamma_2}} \tag{1.3}$$

where $\bar{\gamma}_i \equiv \gamma_i^{\frac{\gamma_i}{1-\gamma_i}}(1-\gamma_i)$. Second, the wage function must be continuous at q_L^*. Again using (1.1), we can express this condition as:

$$w_1 \tilde{\psi}_1(q_L^*)^{\frac{1}{\gamma_1}} = w_2 \tilde{\psi}_2(q_L^*)^{\frac{1}{\gamma_2}} \tag{1.4}$$

Third, using (1.1) and the first-order condition for the firm's optimal choice of the number of workers, we can write the labor market clearing condition as:

$$\mathcal{L}_S \equiv \frac{\bar{H}}{\bar{L}} - \left(\frac{w_1}{\gamma_1 p_1}\right)^{\frac{1}{1-\gamma_1}} \int_{q_L^*}^{q_{Lmax}} \tilde{\psi}_1(q_L)^{\frac{1}{\gamma_1}} \phi_L(q_L) dq_L - \left(\frac{w_2}{\gamma_2 p_2}\right)^{\frac{1}{1-\gamma_2}} \int_{q_{Lmin}}^{q_L^*} \tilde{\psi}_2(q_L)^{\frac{1}{\gamma_2}} \phi_L(q_L) dq_L = 0, \tag{1.5}$$

where $\mathcal{L}_S$ denotes the (scaled) excess supply of effective labor.[1] Equations (1.3)-(1.5) jointly determine the three variables w_1, w_2, and q_L^*.

An outline of the solution algorithm is as follows:

1. Guess a value for the wage anchor w_1.

2. Solve equation (1.3) for the implied value of w_2, given w_1.

3. Solve equation (1.4) for the implied value of q_L^*, given w_1 and w_2.

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[1]As in GHK, the effective labor hired per manager by a firm that employs workers with ability q_L in sector i is defined as $\tilde{\psi}_i(q_L)^{\frac{1}{\gamma_i}} l(q_L)$, where $l(q_L)$ is the actual quantity of labor demanded.

4. Substitute the values for w_1, w_2, and q_L^* into equation (1.5) and check if the labor market clearing condition is satisfied within some tolerance $\varepsilon > 0$.

   (a) If $\mathcal{L}_S > \varepsilon$, increase w_1 and repeat from step 1.

   (b) If $\mathcal{L}_S < -\varepsilon$, reduce w_1 and repeat from step 1.

Note that the direction of adjustment for the guess w_1 in step 4 is based on the observation that if excess effective labor supply is positive (negative), it implies that wages are too low (high) for the labor market to clear, and therefore need to be adjusted upwards (downwards). More precisely, we can show that the left-hand side of (1.5) is strictly decreasing in w_1, once w_2 and q_L^* are also treated as functions of w_1 defined by equations (1.3) and (1.4). To see this, first take logs of (1.3), differentiate with respect to w_1, and rearrange terms to obtain:

$$\frac{dw_2}{dw_1} = \left(\frac{1-\gamma_2}{1-\gamma_1}\right)\left(\frac{\gamma_1}{\gamma_2}\right)\left(\frac{w_2}{w_1}\right) \tag{1.6}$$

Next, take logs of (1.4), differentiate with respect to w_1, substitute equation (1.6), and rearrange terms to obtain:

$$\frac{dq_L^*}{dw_1} = \frac{q_L^*}{w_1}\left[\left(\frac{1-\gamma_2}{1-\gamma_1}\right)\left(\frac{\gamma_1}{\gamma_2}\right) - 1\right]\left[\frac{\varepsilon_{\tilde{\psi}_1}(q_L)}{\gamma_1} - \frac{\varepsilon_{\tilde{\psi}_2}(q_L)}{\gamma_2}\right]^{-1} \tag{1.7}$$

Finally, differentiate equation (1.5) with respect to w_1 and substitute equations (1.3), (1.4), (1.6) and (1.7) to obtain:

$$-\frac{d\mathcal{L}_S}{dw_1} = \left[\frac{w_1^{\frac{\gamma_1}{1-\gamma_1}}}{(1-\gamma_1)(\gamma_1 p_1)^{\frac{1}{1-\gamma_1}}}\right]\int_{q_L^*}^{q_{Lmax}} \tilde{\psi}_1(q_L)^{\frac{1}{\gamma_1}}\phi_L(q_L)dq_L \tag{1.8}$$
$$+ \left[\frac{w_2^{\frac{\gamma_2}{1-\gamma_2}}}{(1-\gamma_2)(\gamma_2 p_2)^{\frac{1}{1-\gamma_2}}}\right]\left(\frac{1-\gamma_2}{1-\gamma_1}\right)\left(\frac{\gamma_1}{\gamma_2}\right)\left(\frac{w_2}{w_1}\right)\int_{q_{Lmin}}^{q_L^*} \tilde{\psi}_2(q_L)^{\frac{1}{\gamma_2}}\phi_L(q_L)dq_L$$
$$+ \left(\frac{w_1}{p_1\gamma_1}\right)^{\frac{1}{1-\gamma_1}}\tilde{\psi}_1(q_L^*)^{\frac{1}{\gamma_1}}\phi_L(q_L^*)\left(\frac{q_L^*}{w_1}\right)\left[\left(\frac{1-\gamma_2}{1-\gamma_1}\right)\left(\frac{\gamma_1}{\gamma_2}\right) - 1\right]^2\left[\frac{\varepsilon_{\tilde{\psi}_1}(q_L)}{\gamma_1} - \frac{\varepsilon_{\tilde{\psi}_2}(q_L)}{\gamma_2}\right]^{-1}$$

The first two terms on the right-hand side of (1.8) are strictly positive, and under condition (1.2), so too is the third term. Therefore, $\frac{d\mathcal{L}_S}{dw_1} < 0$.

   The Matlab file **masterSolver_homMan_2reg.m** implements the algorithm described above for the case in which $\tilde{\psi}_i(q_L) = q_L^{\alpha_i}$. Note that, with this specification, the elasticity $\varepsilon_{\tilde{\psi}_i}$ is a constant and the assumption that condition (1.2) holds can be made without loss of generality, since this amounts only to a specific labeling of the sectors. The search routine on the guess w_1 then proceeds as follows. First, choose some lower and upper bounds $[w_1^{min}, w_1^{max}]$ for the wage anchor w_1. Then, set the initial guess to be $w_1 = \frac{w_1^{min}+w_1^{max}}{2}$. To

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raise the guess in step 4 of the algorithm, set w_1^{min} equal to the current value of the guess for w_1; to reduce the guess, set w_1^{max} equal to the current value of the guess for w_1. This routine thereby halves the search region for w_1 with every iteration, and is guaranteed to converge as long as the equilibrium value of w_1 is less than w_1^{max}.

An example of parameter values that generate such an equilibrium is listed in Table 1. The cutoff worker ability is

$$q_L^* = 0.9502$$

and the resulting wage function is shown in Figure 1. In this figure, shadow wages depict plots of $w_i \tilde{\psi}_i(q_L)^{\frac{1}{\gamma_i}}$ for $q_L \in Q_{Lj}$, $j \neq i$. Note that the shadow wages are always lower than actual equilibrium wages, indicating that firms optimally hire the workers that sort to their sector. The values of the wage anchors are

$$w_1 = 0.6180,$$
$$w_2 = 0.5556,$$

and the implied salary for managers is

$$r = 0.4490.$$

The measure of managers assigned to each sector is:

$$H_1 = 0.6771,$$
$$H_2 = 0.3229.$$

[Table 1 about here.]

[Figure 1 about here.]

1.1.2 More than Two Regions of Sorting

Now, suppose that condition (1.2) does not hold, so that the equilibrium of the model is not necessarily characterized by two regions of sorting as in section 1.1.1. In this case, the equilibrium sorting pattern of the model could potentially take a multitude of forms. Without prior knowledge about the qualitative characteristics of the sorting pattern, however, it is difficult to solve the model numerically for a given set of parameter values. Therefore, a more practical approach is to first fix the sorting pattern of interest and then try to determine whether a given set of parameter values produces an equilibrium of that form. In this

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section, we illustrate this solution approach for a three-region sorting equilibrium in which $Q_{L_1} = [q_{Lmin}, q_L^*] \cup [q_L^{**}, q_{Lmax}]$ and $Q_{L_2} = (q_L^*, q_L^{**})$ for some pair of cutoff values q_L^* and q_L^{**}.

An equilibrium of this sort is characterized by similar conditions to those in Section 1.1.1, except for the following modifications. First, the wage function must be continuous at not only q_L^*, but also at q_L^{**}. Therefore, in addition to (1.4), we also require

$$w_1 \tilde{\psi}_1(q_L^{**})^{\frac{1}{\gamma_1}} = w_2 \tilde{\psi}_2(q_L^{**})^{\frac{1}{\gamma_2}}. \tag{1.9}$$

Second, the labor market clearing condition is now written as

$$\mathcal{L}_S \equiv \frac{\bar{H}}{\bar{L}} - (\frac{w_1}{\gamma_1 p_1})^{\frac{1}{1-\gamma_1}} [\int_{q_{Lmin}}^{q_L^*} \tilde{\psi}_1(q_L)^{\frac{1}{\gamma_1}} \phi_L(q_L) dq_L + \int_{q_L^{**}}^{q_{Lmax}} \tilde{\psi}_1(q_L)^{\frac{1}{\gamma_1}} \phi_L(q_L) dq_L] \tag{1.10}$$
$$- (\frac{w_2}{\gamma_2 p_2})^{\frac{1}{1-\gamma_2}} [\int_{q_L^*}^{q_L^{**}} \tilde{\psi}_2(q_L)^{\frac{1}{\gamma_2}} \phi_L(q_L) dq_L].$$

Finally, we must check that firms (or equivalently, workers) cannot earn positive profits by switching the sector in which they operate, given the conjectured equilibrium sorting regions and wage function. To do so, we simply compute what profits each firm would earn if it hired workers in the other sector, and check that these profits are non-positive.[2]

Equations (1.3), (1.4), (1.9), and (1.10) jointly determine the four variables w_1, w_2, q_L^*, and q_L^{**}, and if the solution of this system also does not allow for profitable deviations by firms, they constitute an equilibrium in which the sorting pattern is as initially postulated.

The outline of the solution algorithm in this case is as follows:

1. Guess a value for the wage anchor w_1.

2. Solve equation (1.3) for the implied value of w_2, given w_1.

3. Solve equations (1.4) and (1.9) for the implied values of q_L^* and q_L^{**}, given w_1 and w_2.

4. Substitute the values for w_1, w_2, q_L^*, and q_L^{**} into equation (1.10) and check if the labor market clearing condition is satisfied within some tolerance $\varepsilon > 0$.

   (a) If $\mathcal{L}_S > \varepsilon$, raise w_1 and repeat from step 1.

   (b) If $\mathcal{L}_S < -\varepsilon$, reduce w_1 and repeat from step 1.

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[2]Note that equation (1.2) automatically rules out such profitable deviations for the case with two regions of sorting. A sufficient (but not necessary) condition for the case with three regions of sorting is $\frac{\varepsilon_{\tilde{\psi}_1}(q_L')}{\gamma_1} \leq \frac{\varepsilon_{\tilde{\psi}_2}(q_L)}{\gamma_2} \leq \frac{\varepsilon_{\tilde{\psi}_1}(q_L'')}{\gamma_1}$, $\forall q_L' \leq q_L^*$, $q_L^* \leq q_L \leq q_L^{**}$, $q_L^{**} \leq q_L''$.

5. Once $|\mathcal{L}_S| < \varepsilon$, check that no profitable deviations by firms are possible.

Note that the direction of adjustment for w_1 in step 4 is based on the same reasoning as in section 1.1.1. Furthermore, it is possible that the solution obtained from steps 1 to 4 still allows for profitable deviations by firms. In this case, the postulated sorting pattern is not consistent with the true equilibrium of the model for the given set of parameter values.

The Matlab file **masterSolver_homMan_3reg.m** implements the algorithm described above for the case in which $\tilde{\psi}_i(q_L) = (\alpha_i q_L^{\rho_i} + 1)^{\frac{1}{\rho_i}}$, using the same search routine on the initial wage guess w_1 as described in section 1.1.1. Note that with this specification of the productivity function, the elasticity $\varepsilon_{\tilde{\psi}_i}$ is no longer a constant, and therefore it is possible for equation (1.2) not to hold.

An example of parameter values that generate an equilibrium of the type considered here is listed in Table 2. The cutoff values of worker ability are

$$(q_L^*, q_L^{**}) = (0.9080, 1.8000)$$

and the resulting wage function is shown in Figure 2. The wage anchors are

$$w_1 = 0.6432,$$
$$w_2 = 0.5210,$$

and the implied salary for managers is

$$r = 0.3887.$$

The measure of managers assigned to each sector is:

$$H_1 = 0.8111,$$
$$H_2 = 0.1889.$$

[Table 2 about here.]

[Figure 2 about here.]

1.2 Heterogeneous Managers with Strictly Log Supermodular Productivity Functions

1.2.1 Two Regions of Sorting

Suppose now that both workers and managers are heterogeneous and that productivity in sector i as a function of manager and worker types is denoted by $\psi_i(q_H, q_L)$. As in section 4 of GHK, we assume here that ψ_i is strictly increasing, continuously differentiable, and strictly log supermodular for $i = 1, 2$, so that the solutions for the matching, wage, and salary functions must satisfy the following conditions:

$$r\left[\mu\left(q_L\right)\right] = \bar{\gamma}_i p_i^{\frac{1}{1-\gamma_i}} \psi_i\left[\mu\left(q_L\right), q_L\right]^{\frac{1}{1-\gamma_i}} w\left(q_L\right)^{-\frac{\gamma_i}{1-\gamma_i}}, \quad \forall q_L \in Q_{Li}, \, i = 1, 2; \tag{1.11}$$

$$\frac{w'\left(q_L\right)}{w\left(q_L\right)} = \frac{\psi_{iL}\left[\mu\left(q_L\right), q_L\right]}{\gamma_i \psi_i\left[\mu\left(q_L\right), q_L\right]}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int}, \, n \in N_i, \, i = 1, 2; \tag{1.12}$$

$$\mu'\left(q_L\right) = \frac{\left(1 - \gamma_i\right) \bar{L} \phi_L\left(q_L\right) w\left(q_L\right)}{\gamma_i \bar{H} \phi_H\left[\mu\left(q_L\right)\right] r\left[\mu\left(q_L\right)\right]}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int}, \, n \in N_i, \, i = 1, 2. \tag{1.13}$$

Here, $\mu\left(\cdot\right)$ denotes the inverse matching function, so that $\mu\left(q_L\right)$ is the ability of managers that are matched to workers of ability q_L. $M_i^{n,int}$ denotes the interior of a connected subset of the equilibrium allocation graph M_i in sector i, and N_i denotes the set of all such connected subsets.

To solve the system of equations (1.11)-(1.13), we first substitute (1.11) into (1.13) to eliminate the salary function $r\left(\cdot\right)$, obtaining:

$$\mu'\left(q_L\right) = \left[\frac{\bar{L}\phi_L\left(q_L\right)}{\bar{H}\phi_H\left[\mu\left(q_L\right)\right]}\right]\left[\frac{w\left(q_L\right)}{\gamma_i p_i \psi_i\left[\mu\left(q_L\right), q_L\right]}\right]^{\frac{1}{1-\gamma_i}}, \quad \forall \left\{\mu\left(q_L\right), q_L\right\} \in M_i^{n,int}, \, n \in N_i, \, i = 1, 2 \tag{1.14}$$

Equations (1.12) and (1.14) give a system of two differential equations in the unknown functions $w\left(\cdot\right)$ and $\mu\left(\cdot\right)$. With the appropriate boundary conditions, we can solve these equations numerically, and then use equation (1.11) to recover the salary function. As with the case of homogeneous managers, however, the equilibrium sorting of workers and managers potentially takes a multitude of forms. Therefore, it is again more practical to first fix the sorting pattern of interest and then try to determine whether a given set of parameter values is consistent with an equilibrium of that form.

In this section, we discuss the solution approach for the case in which each of the sets Q_{Li} and Q_{Hi} is an interval, such that each graph M_i consists of a single connected set (section 1.2.2 discusses the solution approach for more complicated sorting patterns). In this case, there exist cutoff ability levels $q_L^* \in S_L$ and $q_H^* \in S_H$, with workers of ability $q_L \geq q_L^*$

sorting into one sector and workers of ability $q_L < q_L^*$ sorting into the other sector, and similarly for managers. Furthermore, within this class of equilibria there are two qualitatively distinguishable patterns of sorting. First, the equilibrium could have the best workers and best managers sorting to the same sector, which we will refer to as a high-high/low-low (HH/LL) equilibrium. Second, the best workers and the worst managers could sort to the same sector, which we will refer to as a high-low/low-high (HL/LH) equilibrium. For the sake of consistency and without loss of generality, we label the sectors such that the best workers always sort to sector 1. Then, as stated in Proposition 7 of GHK, sufficient conditions guaranteeing that the equilibrium sorting pattern is of the HH/LL form are

$$\frac{\psi_{1H}(q_H, q_L)}{(1 - \gamma_1)\psi_1(q_H, q_L)} > \frac{\psi_{2H}(q_H, q_L)}{(1 - \gamma_2)\psi_2(q_H, q_L)}, \ \ \forall q_H \in S_H, q_L \in S_L \tag{1.15}$$

$$\frac{\psi_{1L}(q_H, q_L)}{\gamma_1\psi_1(q_H, q_L)} > \frac{\psi_{2L}(q_H, q_L)}{\gamma_2\psi_2(q_H, q_L)}, \ \ \forall q_H \in S_H, q_L \in S_L \tag{1.16}$$

while Propositions 5 and 6 imply that sufficient conditions for an HL/LH equilibrium are

$$\frac{\psi_{2H}(q_H, q_{Lmin})}{(1 - \gamma_2)\psi_2(q_H, q_{Lmin})} > \frac{\psi_{1H}(q_H, q_{Lmax})}{(1 - \gamma_1)\psi_1(q_H, q_{Lmax})}, \ \ \forall q_H \in S_H \tag{1.17}$$

$$\frac{\psi_{1L}(q_{Hmin}, q_L)}{\gamma_1\psi_1(q_{Hmin}, q_L)} > \frac{\psi_{2L}(q_{Hmax}, q_L)}{\gamma_2\psi_2(q_{Hmax}, q_L)}, \ \ \forall q_L \in S_L \tag{1.18}$$

We can use these conditions to guarantee that the choice of parameter values for the model do in fact generate sorting patterns of either the HH/LL or HL/LH form, and then proceed to solve for equilibria with these qualitative characteristics.

Now, when the equilibrium is of the HH/LL form, the boundary conditions that accompany the system of differential equations (1.12) and (1.14) are the following: (i) continuity of $w(\cdot)$ at q_L^*, (ii) continuity of $\mu(\cdot)$ at q_L^*, (iii) $\mu(q_{Lmin}) = q_{Hmin}$, and (iv) $\mu(q_{Lmax}) = q_{Hmax}$. Alternatively, when the equilibrium is of the HL/LH form, we still require continuity of the wage function at q_L^*, but the matching function is no longer continuous at q_L^*. Instead, boundary conditions (ii)-(iv) are replaced by the following: (ii) $\mu(q_{Lmin}) = q_H^*$, (iii) $\mu(q_{Lmax}) = q_H^*$, and (iv) $\mu(q_L^{*-}) = q_{Hmax}$, $\mu(q_L^{*+}) = q_{Hmin}$, where $q_L^{*-} = \lim_{q \nearrow q_L^*} q$ and $q_L^{*+} = \lim_{q \searrow q_L^*} q$.

Regardless of whether the equilibrium is of the HH/LL or the HL/LH form, the boundary conditions (i)-(iv) allow us to solve equations (1.12) and (1.14) numerically for a given value of q_L^*. In the Matlab file **algorithm_hetMan_2reg.m**, this computation is performed using the **bvp4c** solver, which is capable of solving multipoint boundary value problems such as the one described above. The solver requires separate functions that specify (i) the differential equations, (ii) the boundary conditions, and (iii) initial guesses for the wage

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and matching functions. In the Matlab file, the differential equations are specified in the function **odefun_2sec**, while the boundary conditions and initial guesses are specified in the functions **bcfun_2sec_HHLL** and **yinit_2sec_HHLL** respectively for the HH/LL equilibrium case, and **bcfun_2sec_HLLH** and **yinit_2sec_HLLH** for the HL/LH case.[3]

For any given value of q_L^*, the **bvp4c** solver yields solutions for the matching, wage, and salary functions that are consistent with equations (1.11)-(1.13) and the boundary conditions. However, the zero-profit condition (1.11) only ensures that a manager of a given ability $q_H \in Q_{Hi}$ employed in a sector i cannot earn positive profits by hiring workers of any ability, *if that manager remains in sector i*. That is, $\Pi_1 (q_H) = 0$ for all $q_H \in Q_{H1}$ but not necessarily for all $q_H \in Q_{H2}$, and similarly $\Pi_2 (q_H) = 0$ for all $q_H \in Q_{H2}$ but not necessarily for all $q_H \in Q_{H1}$, where the profit functions are defined by:

$$\Pi_i (q_H) \equiv \max_{q_L \in S_L} \pi_i (q_H, q_L) \tag{1.19}$$

$$\pi_i (q_H, q_L) \equiv \bar{\gamma}_i p_i^{\frac{1}{1-\gamma_i}} \psi_i (q_H, q_L)^{\frac{1}{1-\gamma_i}} w (q_L)^{-\frac{\gamma_i}{1-\gamma_i}} - r (q_H) \tag{1.20}$$

Therefore, in solving for the matching, wage, and salary functions, we must adjust the worker ability cutoff q_L^* until the solutions obtained do not enable managers to make positive profits by hiring workers of any ability, even after allowing managers to switch the sector in which they operate. The outline of this algorithm is summarized below:

1. Guess a value for the worker ability cutoff $q_L^* \in S_L$.

2. Given this value of q_L^*, solve the system of differential equations (1.12) and (1.14) using the appropriate boundary conditions, and compute the implied salary function using equation (1.11).

3. Using the solutions for $\mu (\cdot)$, $w (\cdot)$, and $r (\cdot)$, compute the profit differentials for managers from switching sectors, $\Delta\Pi_i [\mu (q_L)] = \pi_i [\mu (q_L), q_L] - \Pi_j [\mu (q_L)], j \neq i$, and check that these differentials are non-positive within some tolerance $\varepsilon > 0$.[4]

   (a) If $\Delta\Pi_1 [\mu (q_L)] \leq \varepsilon$ for all $q_L \in Q_{L1}$ but $\Delta\Pi_2 [\mu (q_L)] > \varepsilon$ for some $q_L \in Q_{L2}$, adjust q_L^* upwards and repeat from step 1.

   (b) If $\Delta\Pi_2 [\mu (q_L)] \leq \varepsilon$ for all $q_L \in Q_{L2}$ but $\Delta\Pi_1 [\mu (q_L)] > \varepsilon$ for some $q_L \in Q_{L1}$, adjust q_L^* downwards and repeat from step 1.

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[3]See the Matlab help file on the **bvp4c** function for more details about the syntax and implementation of the solver.

[4]Note that when a manager with ability $\mu (q_L)$ switches sectors, he does not necessarily employ workers of ability q_L, but rather the best workers given his type.

4. Once $\Delta\Pi_i\left[\mu\left(q_L\right)\right] \leq \varepsilon$ for all $q_L \in Q_{Li}$ for both $i = 1, 2$, check that $\Pi_i\left(\mu\left(q_L\right)\right) = 0$ for all $q_L \in Q_{Li}$ for both $i = 1, 2$.

Note that, in determining the direction of adjustment for q_L^* in step 3 of the algorithm, it is possible in principle that there exists some $q_L \in Q_{Li}$ such that $\Delta\Pi_i\left[\mu\left(q_L\right)\right] > \varepsilon$, for *both* $i = 1, 2$. In this case, the algorithm breaks down. However, we find that whenever the sufficient conditions (1.15)-(1.16) or (1.17)-(1.18) are satisfied and we search for an equilibrium with the appropriate sorting pattern, this problem never is encountered in practice.

Also, note that the final check on the zero-profit condition in step 4 is needed because equation (1.12) is a first-order condition that is necessary but not sufficient to ensure zero profits for any manager (the typical second order condition depends on $w\left(\cdot\right)$ and $r\left(\cdot\right)$, which are endogenous). Therefore, while equations (1.11) and (1.12) guarantee that $\pi_i\left[q_H, \mu^{-1}\left(q_H\right)\right] = 0$ for all $q_H \in Q_{Hi}$, they do not rule out the possibility that $\mu^{-1}\left(q_H\right)$ is a local but not global maximizer of (1.19), so that $\Pi_i\left(q_H\right) > \pi_i\left(q_H, \mu^{-1}\left(q_H\right)\right)$ for some $q_H \in Q_{Hi}$. Nonetheless, any solution for the matching, wage, and salary functions obtained via the algorithm described above is by design consistent with equations (1.11)-(1.13), the appropriate boundary conditions, as well as zero maximal profits for all firms, and therefore accurately characterizes an equilibrium of the model.

The Matlab file **masterSolver_hetMan_2reg.m** (which calls **algorithm_hetMan _2reg.m**) implements the algorithm described above for the case in which $\psi_i(q_H, q_L) = \left(\alpha_i q_L^{\rho_i} + \beta_i q_H^{\rho_i}\right)^{\frac{\alpha_i + \beta_i}{\rho_i}}$ with $\rho_i < 0$. Note that this specification of the productivity function is strictly log supermodular for any $\rho_i < 0$, and approaches the Cobb-Douglas specification studied in Appendix B of GHK as ρ_i approaches 0. The search routine on q_L^* is similar to the one used for the search on the wage anchor w_1 for the case with homogenous managers. First, set the bounds for the cutoff worker ability to be $[q_{Lmin}^*, q_{Lmax}^*] = [q_{Lmin}, q_{Lmax}]$. Then, set the initial guess to be $q_L^* = \frac{q_{Lmin}^* + q_{Lmax}^*}{2}$. To adjust the guess upwards in step 3 of the algorithm, set q_{Lmin}^* equal to the current value of the guess for q_L^*; to adjust the guess downwards, set q_{Lmax}^* equal to the current value of the guess for q_L^*. This routine halves the search region for q_L^* with every iteration.

An example of parameter values that generate an equilibrium with the HH/LL sorting pattern is listed in Table 3. The cutoff values for the manager and worker qualities are

$$q_H^* = 1.3115,$$
$$q_L^* = 1.3190,$$

and the resulting matching, wage, and salary functions are shown in Figure 3. Similarly, an example of parameter values that generate an equilibrium with the HL/LH sorting pattern

is listed in Table 4. The cutoff values for the manager and worker qualities are

$$q_H^* = 1.2006,$$
$$q_L^* = 1.2006,$$

and the resulting matching, wage, and salary functions are shown in Figure 4.

[Table 3 about here.]

[Figure 3 about here.]

[Table 4 about here.]

[Figure 4 about here.]

1.2.2 More than Two Regions of Sorting

When neither conditions (1.15) and (1.16) nor (1.17) and (1.18) are satisfied, we can no longer be sure *a priori* about the sorting pattern of managers and workers in equilibrium, which makes numerical solution of the model a more challenging problem. Specifically, the difficulty arises from the fact that implementation of the **bvp4c** solver requires identification of the number of distinct regions that characterize the differential equation system, as well as specification of the boundary conditions that automatically fix the sorting pattern being considered. Nonetheless, the approach to solving the model numerically for the case in which there are more than two regions of sorting is qualitatively similar to the case with only two regions of sorting.

First, for a given number of sorting regions, we identify all possible types of sorting patterns that could obtain in equilibrium. For example, with three regions of sorting, there are two pairs of ability cutoffs $\{q_L^*, q_H^*\}$ and $\{q_L^{**}, q_H^{**}\}$, with $q_{Lmin} \leq q_L^* < q_L^{**} \leq q_{Lmax}$ and $q_{Hmin} \leq q_H^* < q_H^{**} \leq q_{Hmax}$. The fact that any equilibrium must exhibit positive assortative matching within each sector then implies that there are three possible patterns of sorting:

1. Workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \leq q_H^*$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$; workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H > q_H^{**}$. (low-low/mid-mid/high-high equilibrium, LL/MM/HH)

2. Workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \leq q_H^*$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H > q_H^{**}$; workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$. (low-low/mid-high/high-mid equilibrium, LL/MH/HM)

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3. Workers of ability $q_L \leq q_L^*$ sort to sector 1 and match with managers of ability $q_H \in (q_H^*, q_H^{**}]$; workers of ability $q_L \in (q_L^*, q_L^{**}]$ sort to sector 2 and match with managers of ability $q_H \leq q_H^*$; workers of ability $q_L > q_L^{**}$ sort to sector 1 and match with managers of ability $q_H > q_H^{**}$. (low-mid/mid-low/high-high equilibrium, LM/ML/HH)

Next, for each possible sorting pattern, we specify the boundary conditions for the numerical solver. For example, for the LL/MM/HH equilibrium, the six boundary conditions would be (i) continuity of $w(\cdot)$ at q_L^*, (ii) continuity of $w(\cdot)$ at q_L^{**}, (iii) continuity of $\mu(\cdot)$ at q_L^*, (iv) continuity of $\mu(\cdot)$ at q_L^{**}, (v) $\mu(q_{Lmin}) = q_{Hmin}$, and (vi) $\mu(q_{Lmax}) = q_{Hmax}$. We then proceed using the same algorithm as in the previous section, guessing values for the cutoff worker ability levels q_L^* and q_L^{**}, and adjusting these guesses until profitable deviations are ruled out for all firms.

An example of parameter values that generate an equilibrium with three regions of sorting (specifically, one of the LM/ML/HH form) is listed in Table 5. The cutoff values for the manager and worker qualities are

$$q_H^* = 1.0584,$$
$$q_H^{**} = 1.0853,$$
$$q_L^* = 1.1577,$$
$$q_L^{**} = 1.5115,$$

and the resulting matching, wage, and salary functions are shown in Figure 5.

[Table 5 about here.]

[Figure 5 about here.]

2 Comparative Static Results with Heterogeneous Workers and Managers and CES Productivity Functions

In this section, we employ the numerical algorithm discussed in section 1.2 for the case with heterogeneous workers and managers and CES productivity functions, and solve the model for different values of parameters to study various comparative static responses. Subsection 2.1 studies comparative statics with respect to the relative factor endowment $\bar{H}/\bar{L}$, subsection 2.2 considers shifts in the supports of the manager and worker ability distributions S_H and S_L, and subsection 2.3 examines changes in the relative goods price p_2/p_1.

For each of these analyses, the approach taken is to first identify all qualitatively distinct cases of parameter values that might be of interest and then to characterize the comparative static properties of the model for each case. We also restrict attention here to equilibria with two regions of sorting, and are particularly interested in determining how a change in parameter values affects the following four key characteristics of the equilibrium:

1. Sorting: do more workers and managers sort to a particular sector following the change in parameters?

2. Matching: does the quality of the match for a given worker or manager improve or worsen?

3. Inter-sector inequality: do real wages and salaries of workers and managers in one sector increase more than real wages and salaries of workers and managers in the other sector?

4. Intra-sector inequality: do real wages and salaries of high ability workers and managers increase more than real wages and salaries of low ability workers and managers within the same sector?

2.1 Comparative Statics with Respect to $\bar{H}//\bar{L}$

Here, we examine the responses of the matching, wage, and salary functions to a change in the relative factor endowment $\frac{\bar{H}}{\bar{L}}$, and interpret the comparative static results as a comparison of two countries A and B that differ only in $\frac{\bar{H}}{\bar{L}}$. Without loss of generality, we label the countries such that country A always has a higher ratio of managers to workers than country B. Subsection 2.1.1 considers equilibria in which the best workers and managers sort to the same sector in both countries (HH/LL sorting), while subsection 2.1.2 considers equilibria in which the best workers and the worst managers sort to the same sector in both countries (HL/LH sorting). Again, we label the sectors such that the best workers always sort to sector 1.

2.1.1 HH/LL Sorting

In this subsection, we use parameter values listed in Table 6. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that the inequalities in Proposition 7 of GHK (guaranteeing sorting of the best workers and managers to sector 1) are satisfied. Since these inequalities require $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$ when $\gamma_1 = \gamma_2$, we fix $\alpha_1 + \beta_1 = 2$ and $\alpha_2 + \beta_2 = 1$.

[Table 6 about here.]

To summarize the results of this subsection, the following always obtain regardless of parameter values:

- wages in country A are always higher than in country B for all worker types, while salaries in country A are always lower than in country B for all manager types

- if the two sectors have the same factor intensities ($\gamma_1 = \gamma_2$), then the matching functions and intra-sector inequality are identical in the two countries

- if factor intensities differ in the two sectors ($\gamma_1 \neq \gamma_2$), then:

  - country A always has more workers and managers sorting to the sector that is relatively manager intensive (i.e. has the smaller γ) than country B
  - the quality of the match for any given worker in sector 2 is better in B than in A
  - intra-sector wage inequality in sector 2 is greater in B than in A, while intra-sector salary inequality in sector 2 is greater in A than in B

However, in terms of the quality of matches and intra-sector inequality in sector 1, there are 2 possible cases:

1. the match quality is better in A than in B for all workers in sector 1, intra-sector wage inequality in sector 1 is greater in A than in B, and intra-sector salary inequality in sector 1 is greater in B than in A

2. the match quality is better in B than in A for all workers in sector 1, intra-sector wage inequality in sector 1 is greater in B than in A, and intra-sector salary inequality in sector 1 is greater in A than in B

Hence, overall there are 5 qualitatively distinguishable cases for this sorting pattern, which are described in Table 7. We now examine under what types of parameter values each case is more likely to obtain.

[Table 7 about here.]

First, case 1 obtains whenever $\gamma_1 = \gamma_2$, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 8, and Figure 6 shows a typical example of differences in the matching, wage, and salary functions.[5] Here, we see that the matching functions and intra-sector inequality

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[5]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$ and $\bar{H}^B = 0.5$.

are identical in the two countries. Furthermore, wages are higher in A than in B for all worker types, while salaries are lower in A than in B for all manager types.

[Table 8 about here.]

[Figure 6 about here.]

Second, case 2 obtains whenever $\gamma_1 < \gamma_2$, and at least one of the following is true: (i) γ_1 and γ_2 are not too small, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and the two ratios are not too large, or (iii) $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\beta_2}$ where $\frac{\alpha_1}{\beta_1}$ is not too large and $\frac{\alpha_2}{\beta_2}$ is not too small. Examples for this case are listed in Table 9, and Figure 7 shows a typical example of differences in the matching, wage, and salary functions.[6] Here, we see that, as in case 1, wages are higher in A than in B for all worker types, whereas salaries are lower in A than in B for all manager types. Now, however, more workers and managers sort to the manager-intensive sector (1) in the manager-abundant country (A) than in the labor-abundant country (B), and the quality of the match is better in B than in A for a given worker in sector 2, but is better in A than in B for a given worker in sector 1. Regarding intra-sector inequality, we see that intra-sector wage inequality is greater in A than in B for workers in sector 1, but is greater in B than in A for workers in sector 2; conversely, intra-sector salary inequality is greater in B than in A for managers in sector 1, but is greater in A than B for managers in sector 2.

[Table 9 about here.]

[Figure 7 about here.]

Third, case 3 obtains whenever (i) $\gamma_1 < \gamma_2$, (ii) γ_1 and γ_2 are both small, and (iii) either $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are large, or $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\beta_2}$ with $\frac{\alpha_1}{\beta_1}$ large and $\frac{\alpha_2}{\beta_2}$ small. Examples for this case are listed in Table 10, and Figure 8 shows a typical example of differences in the matching, wage, and salary functions.[7] Here, we see that the results are qualitatively similar to case 2, except that now the quality of the match is better in B than in A for all worker types, intra-sector wage inequality is greater in B than in A in both sectors, and intra-sector salary inequality is greater in A than in B in both sectors.

[Table 10 about here.]

[Figure 8 about here.]

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[6]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\bar{H}^B = 0.5$.

[7]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1.8, 0.2\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.9, 0.1\}$, and $\bar{H}^B = 0.1$.

Fourth, case 4 obtains whenever $\gamma_1 > \gamma_2$, and at least one of the following is true: (i) γ_1 and γ_2 are not too large, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and the two ratios are not too small, or (iii) $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\beta_2}$ where $\frac{\alpha_1}{\beta_1}$ is not too small and $\frac{\alpha_2}{\beta_2}$ is not too large. Examples for this case are listed in Table 11, and Figure 9 shows a typical example of differences in the matching, wage, and salary functions.[8] Here, we see that the results are qualitatively similar to case 2, except for the fact that country A has more workers and managers sorting to sector 2 than country B, since now sector 2 is the manager-intensive sector.

[Table 11 about here.]

[Figure 9 about here.]

Finally, case 5 obtains whenever (i) $\gamma_1 > \gamma_2$, (ii) γ_1 and γ_2 are both large, and (iii) either $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are small, or $\frac{\alpha_1}{\beta_1} \neq \frac{\alpha_2}{\beta_2}$ with $\frac{\alpha_1}{\beta_1}$ small and $\frac{\alpha_2}{\beta_2}$ large. Examples for this case are listed in Table 12, and Figure 10 shows a typical example of differences in the matching, wage, and salary functions.[9] Here, we see that the results are qualitatively similar to case 4, except that now the quality of the match is better in B than in A for all worker types, intra-sector wage inequality is greater in B than in A in both sectors, and intra-sector salary inequality is greater in A than in B in both sectors.

[Table 12 about here.]

[Figure 10 about here.]

2.1.2   HL/LH Sorting

In this subsection, we use parameter values listed in Table 13. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that the inequalities in Propositions 5 and 6 of GHK (guaranteeing sorting of the best workers and the worst managers to sector 1) are satisfied. Since these inequalities do not require $\alpha_1 + \beta_1 \neq \alpha_2 + \beta_2$ for particular values of γ_1 and γ_2, we fix $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = 1$ to keep things simple.

[Table 13 about here.]

To summarize the results of this subsection, the following always obtain regardless of parameter values:

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[8]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$, and $\bar{H}^B = 0.5$.

[9]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.2, 1.8\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.1, 0.9\}$, and $\bar{H}^B = 0.1$.

- wages in country A are always higher than in country B for all worker types, while salaries in country A are always lower than in country B for all manager types

- if the two sectors have the same factor intensities ($\gamma_1 = \gamma_2$), then the matching functions and intra-sector inequality are identical in the two countries

- if factor intensities differ in the two sectors ($\gamma_1 \neq \gamma_2$), then country A always has more workers and managers sorting to the sector that is relatively manager intensive (i.e. has the smaller γ) than country B

However, in terms of the quality of matches and intra-sector inequality, there are 2 possible cases:

1. the match quality is better in A than in B for all workers except those that are employed in different sectors in A and B, intra-sector wage inequality is greater in A than in B in both sectors, and intra-sector salary inequality is greater in B than in A in both sectors

2. the match quality is better in B than in A for all workers except those that are employed in different sectors in A and B, intra-sector wage inequality is greater in B than in A in both sectors, and intra-sector salary inequality is greater in A than in B in both sectors

Hence, overall there are three qualitatively distinguishable cases for this sorting pattern, which are described in Table 14. We now examine under what kinds of parameter values each case is more likely to obtain.

[Table 14 about here.]

First, case 1 obtains whenever $\gamma_1 = \gamma_2$, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Propositions 5 and 6 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 15, and Figure 11 shows a typical example of differences in the matching, wage, and salary functions.[10] Here, we see that the matching functions and intra-sector inequality are identical in the two countries. Furthermore, wages are higher in A than in B for all worker types, while salaries are lower in A than in B for all manager types.

[Table 15 about here.]

[Figure 11 about here.]

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[10]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$, and $\bar{H}^B = 0.8$.

Second, case 2 obtains whenever $\gamma_1 < \gamma_2$, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 16, and Figure 12 shows a typical example of differences in the matching, wage, and salary functions.[11] Here, we see that, as in case 1, wages are higher in A than in B for all worker types, while salaries are lower in A than in B for all manager types. Now, however, more workers and managers sort to the manager-intensive sector (1) in the manager-abundant country (A), and the quality of the match is better in A than in B for any given worker, except for those employed in different sectors in the two countries. Regarding intra-sector inequality, we see that intra-sector wage inequality is greater in A than in B in both sectors, while intra-sector salary inequality is greater in B than in A in both sectors.

[Table 16 about here.]

[Figure 12 about here.]

Finally, case 3 obtains whenever $\gamma_1 > \gamma_2$, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 > \gamma_2$, the inequalities in Propositions 5 and 6 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 17, and Figure 13 shows a typical example of differences in the matching, wage, and salary functions.[12] Here, we see that as in case 1, wages are higher in A than in B for all worker types, while salaries are lower in A than in B for all manager types. Now, however, more workers and managers sort to the manager-intensive sector (2) in the manager-abundant country (A), and the quality of the match is better in B than in A for any given worker, except for those employed in different sectors in the two countries. Regarding intra-sector inequality, we see that intra-sector wage inequality is greater in B than in A in both sectors, while intra-sector salary inequality is greater in A than in B in both sectors.

[Table 17 about here.]

[Figure 13 about here.]

2.2 Comparative Statics with Respect to S_L and S_H

Here, we examine the response of sector output levels to shifts in the supports of the worker and manager ability distributions, S_L and S_H, and interpret the comparative static

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[11]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.5, 0.5\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\bar{H}^B = 0.8$.

[12]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.7, 0.3\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.3, 0.7\}$, and $\bar{H}^B = 0.8$.

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results as a comparison of two countries A and B that differ only in S_L, S_H, or both. Specifically, we fix the supports of the distributions in countries A and B as $S_F^A = [q_{Fmin}, q_{Fmax}]$ and $S_F^B = [\lambda_F q_{Fmin}, \lambda_F q_{Fmax}]$ respectively, where $F \in \{H, L\}$. Note that this is equivalent to comparing two economies A and B in which the densities of the worker distributions satisfy $\phi_F^B(\lambda_F q_F) = \phi_F^A(q_F)/\lambda_F$, i.e. with the Jacobians of transformation accounted for. In what follows, we use parameter values listed in Table 18. The values for λ_F, $F \in \{H, L\}$ and $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but we fix $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = 1$ to keep things simple.

[Table 18 about here.]

We first study the comparative statics fixing $\lambda_H = 1$ and $\lambda_L > 1$, so that the two countries differ only in the support of the worker ability distribution, with country B having workers of higher ability than country A. Also, we suppose without loss of generality that $\gamma_1 < \gamma_2$, so that sector 2 is relatively labor intensive. The results of this analysis can then be categorized according to the following two cases:

1. Relative output of sector 2 to sector 1 is lower in country A than in country B.

2. Relative output of sector 2 to sector 1 is higher in country A than in country B.

Case 1 is more likely to obtain whenever $\frac{\alpha_2}{\beta_2}$ is large enough relative to $\frac{\alpha_1}{\beta_1}$, whereas case 2 is more likely to obtain whenever $\frac{\alpha_1}{\beta_1}$ is large enough relative to $\frac{\alpha_2}{\beta_2}$. This is true regardless of whether the equilibrium sorting pattern is of the HH/LL or HL/LH type. Examples for the two cases are listed in Tables 19 and 20 respectively, where we set $\lambda_L = 1.1$. Given the symmetry of the model with respect to the two factors, the converse comparative static results apply when $\lambda_H > 1$ and $\lambda_L = 1$.

[Table 19 about here.]

[Table 20 about here.]

How does relative sector output compare in the two countries when both λ_H and λ_L are different from 1? Here, we find that regardless of the equilibrium type, the ratio of relative sector output in the two countries depends only on the ratio λ_L/λ_H. In other words, shifting only S_L with $\lambda_L = \bar{\lambda} \neq 1$ and $\lambda_H = 1$ has the same effect on relative sector output as shifting both S_L and S_H with $\lambda_L \neq 1$, $\lambda_H \neq 1$ and $\lambda_L/\lambda_H = \bar{\lambda}$. Therefore, the comparative static results discussed above with respect to shifts in S_L alone also apply to shifts in both S_L and S_H simultaneously .

2.3 Comparative Statics with Respect to p_1/p_2

Here, we examine the responses of the matching, wage, and salary functions to a change in the relative goods price, specifically an increase in p_2. Subsection 2.3.1 considers equilibria in which the best workers and managers sort to the same sector (HH/LL sorting), while subsection 2.3.2 considers equilibria in which the best workers and the worst managers sort to the same sector (HL/LH sorting). Again, we label the sectors such that the best workers always sort to sector 1.

2.3.1 HH/LL Sorting

In this subsection, we use parameter values listed in Table 21. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that the inequalities in Proposition 11 of GHK (guaranteeing sorting of the best workers and managers to sector 1) are satisfied. Since these inequalities require $\alpha_1 + \beta_1 > \alpha_2 + \beta_2$ when $\gamma_1 = \gamma_2$, we fix $\alpha_1 + \beta_1 = 2$ and $\alpha_2 + \beta_2 = 1$.

[Table 21 about here.]

To summarize the results of this subsection, an increase in p_2 always leads more workers and managers to sort to sector 2, but in terms of the implications for (i) the quality of matches, (ii) inter-sector inequality, and (iii) intra-sector inequality, there are 5 qualitatively distinguishable sets of matching-wage-salary responses. These cases are described in Table 22. We now examine under what kinds of parameter values each case is more likely to obtain.

[Table 22 about here.]

First, case 1 is a knife-edge case that results only when $\gamma_1 = \gamma_2 = 0.5$ and $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$. Figure 14 shows the matching, wage, and salary function responses for this case.[13] Here, we see that more workers and managers sort to sector 2, but the quality of the match for a given worker or manager does not change. Regarding inter-sector inequality, workers and managers remaining in sector 2 enjoy wage and salary increases that are exactly proportional to the price increase, whereas workers and managers remaining in sector 1 see no change in their wages or salaries. Hence, real wages and salaries increase for workers and managers remaining in sector 2, but decrease for workers and managers remaining in sector 1, and change ambiguously for workers and managers that switch sectors. Furthermore, there is no change in intra-sector wage or salary inequality.

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[13]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[Figure 14 about here.]

Second, case 2 is more likely to obtain whenever $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and at least one of the following is true: (i) γ_1 and γ_2 are both small, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are large, or (iii) $\frac{\alpha_2}{\beta_2}$ is low. Examples for this case are listed in Table 23, and Figure 15 shows a typical example of the matching, wage, and salary function responses.[14] As in case 1, more workers and managers sort to sector 2, but now the quality of the match for any given worker increases and the quality of the match for any given manager decreases after the price change. Regarding inter-sector inequality, real wages increase for workers remaining in sector 2, but decrease for workers remaining in sector 1; real salaries of managers change ambiguously. Furthermore, we see that now intra-sector wage inequality increases in both sectors, whereas intra-sector salary inequality decreases in both sectors. Note that it is possible to have $\gamma_1 \neq \gamma_2$ and still have the matching-wage-salary responses characterized by case 2. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.41, 0.5, 0.5\}$, the responses are characterized by case 2.

[Table 23 about here.]

[Figure 15 about here.]

Third, case 3 is more likely to obtain whenever $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and at least one of the following is true: (i) γ_1 and γ_2 are both large, (ii) $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2}$ and both ratios are small, or (iii) $\frac{\alpha_1}{\beta_1}$ is low. Examples for this case are listed in Table 24, and Figure 16 shows a typical example of the matching, wage, and salary function responses.[15] Here, we see that the results are qualitatively the same as those for case 2, except that the roles of workers and managers are reversed. Specifically, the quality of the match for any given worker decreases and the quality of the match for any given manager increases after the price change. Regarding inter-sector wage inequality, real salaries increase for managers remaining in sector 2, but decrease for managers remaining in sector 1; real wages of workers change ambiguously. Furthermore, we see that now intra-sector wage inequality decreases in both sectors, whereas intra-sector salary inequality increases in both sectors. Note that it is possible to have $\gamma_1 \neq \gamma_2$ and still have the matching-wage-salary responses characterized by case 3. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.61, 0.5, 0.5\}$, the responses are characterized by case 3.

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[14]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[15]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[Table 24 about here.]

[Figure 16 about here.]

Fourth, case 4 is more likely to obtain whenever $\gamma_2 - \gamma_1 = \varepsilon > 0$ and ε is large enough, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 25, and Figure 17 shows a typical example of the matching, wage, and salary function responses.[16] Here, we see that the quality of the match deteriorates for a given worker remaining in sector 1, but improves for a given worker remaining in sector 2. Conversely, the quality of the match improves for a given manager remaining in sector 1, but deteriorates for a given manager remaining in sector 2. Regarding inter-sector inequality, the real wages of workers remaining in sector 2 increase, and the real salaries of managers remaining in sector 1 decrease. Real wages for workers remaining in sector 1 could either change ambiguously (as in Figure 17) or could strictly increase (not shown). Real salaries for managers remaining in sector 2 could either strictly decrease (as in Figure 17) or could change ambiguously (not shown). Regarding intra-sector inequality, wage inequality decreases in sector 1 and increases in sector 2, whereas salary inequality increases in sector 1 and decreases in sector 2. Note that it is possible to have $\gamma_2 > \gamma_1$ and yet not have the matching-wage-salary responses characterized by case 4. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.41, 0.5, 0.5\}$, the responses are characterized by case 2, and when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.61, 0.5, 0.5\}$, the responses are characterized by case 3.

[Table 25 about here.]

[Figure 17 about here.]

Finally, Case 5 is more likely to obtain whenever $\gamma_1 - \gamma_2 = \varepsilon > 0$ and ε is large enough, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 26, and Figure 18 shows a typical example of the matching, wage, and salary function responses.[17] Here, we see that the results are qualitatively the same as those for case 4, except that the roles of workers and managers are reversed. Specifically the quality of the match improves for a given worker remaining in sector 1, but deteriorates for a given worker remaining in sector 2, and conversely, the quality of the match deteriorates for a given manager remaining in sector 1, but improves for a given manager remaining in sector 2. Regarding inter-sector inequality, the real salaries of managers remaining in sector 2 increase, and the real

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[16]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

[17]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$, and $\Delta p_2 = 20\%$.

wages of workers remaining in sector 1 decrease. Real salaries for managers remaining in sector 1 could either change ambiguously (as in Figure 18) or could strictly increase (not shown). Real wages for workers remaining in sector 2 could either strictly decrease (as in Figure 18) or change ambiguously (not shown). Regarding intra-sector inequality, wage inequality increases in sector 1 and decreases in sector 2, whereas salary inequality decreases in sector 1 and increases in sector 2. Note that it is possible to have $\gamma_1 > \gamma_2$ and yet not have the matching-wage-salary responses characterized by case 4. For example, when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.39, 0.5, 0.5\}$, the responses are characterized by case 2, and when $\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.59, 0.5, 0.5\}$, the responses are characterized by case 3.

[Table 26 about here.]

[Figure 18 about here.]

2.3.2 HL/LH Sorting

In this subsection, we use parameter values listed in Table 27. The values for $\{\gamma_i, \alpha_i, \beta_i\}$, $i \in \{1, 2\}$ are varied to explore a range of qualitatively distinct cases, but always ensuring that the inequalities in Propositions 5 and 6 of GHK (guaranteeing sorting of the best workers and the worst managers to sector 1) are satisfied. Since these inequalities do not require $\alpha_1 + \beta_1 \neq \alpha_2 + \beta_2$ for particular values of γ_1 and γ_2, we fix $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = 1$ to keep things simple.

[Table 27 about here.]

To summarize the results of this subsection, an increase in p_2 always leads more workers and more managers to sort to sector 2. Furthermore, the change in the matching function is always characterized as follows: the quality of the match deteriorates for all workers that remain in their original sector, but improves for workers that switch sectors; conversely, the quality of the match improves for all managers remaining in their original sector, but deteriorates for managers that switch sectors. The implications for intra-sector inequality are also always the same: wage inequality decreases in both sectors and salary inequality increases in both sectors following the price change. The only difference in the comparative static results for this sorting pattern concerns the implications of the price change for inter-sector inequality. Here, there are 5 qualitatively distinguishable sets of responses, as described in Table 28. We now examine under what kinds of parameter values each case is more likely to obtain.

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[Table 28 about here.]

First, case 1 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are close to 0.5, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 29, and Figure 19 shows a typical example of the matching, wage, and salary function responses.[18] Here, we see that the matching function response and the implications for intra-sector inequality are as described above. With regard to inter-sector inequality, we see that real wages increase for the worst workers remaining in sector 2, change ambiguously for the best workers remaining in sector 2, and decrease for all workers remaining in sector 1. On the other hand, real salaries increase for the best managers remaining in sector 2, and change ambiguously for the worst managers remaining in sector 2 as well as for all managers remaining in sector 1. It is also possible, however, for real wages of the worst workers remaining in sector 1 to change ambiguously, and for real salaries of the worst managers managers in sector 2 to decrease instead.[19] Nonetheless, real wages of the worst workers remaining in sector 2 and real salaries of the best managers remaining in sector 2 always increase.

[Table 29 about here.]

[Figure 19 about here.]

Second, case 2 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are small, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 30, and Figure 20 shows a typical example of the matching, wage, and salary function responses.[20] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages increase for workers remaining in sector 2, but decrease for workers remaining in sector 1. Real salaries, on the other hand, change ambiguously for all managers.

[Table 30 about here.]

[Figure 20 about here.]

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[18]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.6, 0.4\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$, and $\Delta p_2 = 5\%$.

[19]For example, this happens when parameter values are the same as in Figure 19, but p_2 increases by 1% instead of 5%.

[20]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

Third, case 3 is more likely to obtain when $|\gamma_1 - \gamma_2| = \varepsilon$ for ε sufficiently small, and both γ_1 and γ_2 are large, regardless of $\frac{\alpha_1}{\beta_1}$ and $\frac{\alpha_2}{\beta_2}$. Note, however, that when $\gamma_1 = \gamma_2$, the inequalities in Proposition 10 require that $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 31, and Figure 21 shows a typical example of the matching, wage, and salary function responses.[21] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real salaries increase for managers remaining in sector 2, but decrease for managers remaining in sector 1. Real wages, on the other hand, change ambiguously for all workers.

[Table 31 about here.]

[Figure 21 about here.]

Fourth, case 4 is more likely to obtain when either (i) $\frac{\alpha_1}{\beta_1} \le \frac{\alpha_2}{\beta_2}$ or (ii) $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$ and $\gamma_2 - \gamma_1 = \varepsilon > 0$ for ε sufficiently large. Note that when $\frac{\alpha_1}{\beta_1} \le \frac{\alpha_2}{\beta_2}$, the inequalities in Proposition 10 require that $\gamma_1 < \gamma_2$ (even if we allow for $\alpha_1 + \beta_1 \ne \alpha_2 + \beta_2$). Examples for this case are listed in Table 32, and Figure 22 shows a typical example of the matching, wage, and salary function responses.[22] Here, we see that the matching function response and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages increase for all workers, while real salaries decrease for all managers. It is also possible, however, for real wages of workers remaining in sector 1 and real salaries of managers remaining in sector 2 to change ambiguously instead.[23] Nonetheless, real wages of workers remaining in sector 2 always increase, and real salaries of managers remaining in sector 1 always decrease.

[Table 32 about here.]

[Figure 22 about here.]

Finally, case 5 is more likely to obtain when $\gamma_1 - \gamma_2 = \varepsilon > 0$ for ε sufficiently large. Note that when $\gamma_1 > \gamma_2$, the inequalities in Proposition 10 also require $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$. Examples for this case are listed in Table 33, and Figure 23 shows a typical example the matching, wage, and salary function responses.[24] Here, we see that the matching function response

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[21]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

[22]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.5, 0.5\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$, and $\Delta p_2 = 10\%$.

[23]The following parameter values generate an example with these characteristics: $\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.2, 0.8\}$.

[24]Specific parameter values for this figure are $\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.9, 0.1\}$, $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$, and $\Delta p_2 = 10\%$.

and the implications for intra-sector inequality are the same as in case 1. With regard to inter-sector inequality, we see that real wages decrease for all workers, while real salaries increase for all managers. It is also possible, however, for real wages of workers remaining in sector 2 and real salaries of managers remaining in sector 1 to change ambiguously instead.[25] Nonetheless, real wages of workers remaining in sector 1 always decrease, and real salaries of managers remaining in sector 2 always increase.

[Table 33 about here.]

[Figure 23 about here.]

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[25]The following parameter values generate an example with these characteristics:$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.9, 0.1\}$ and $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.6, 0.4\}$.
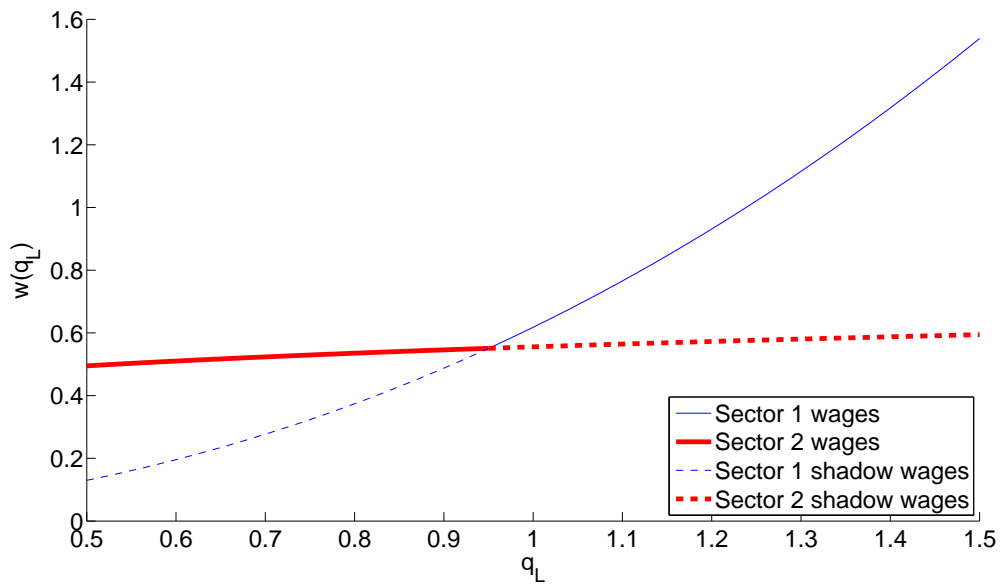
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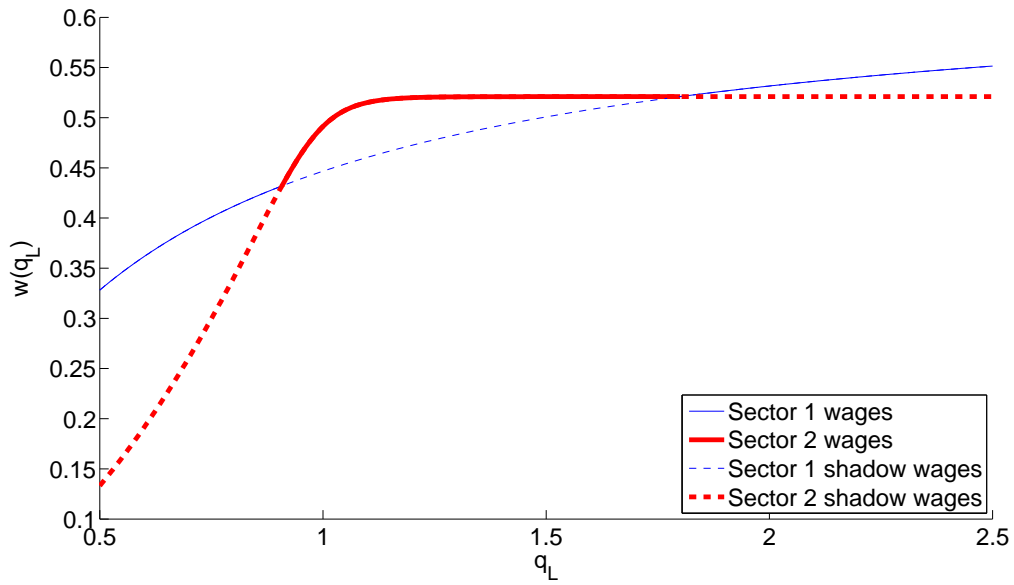
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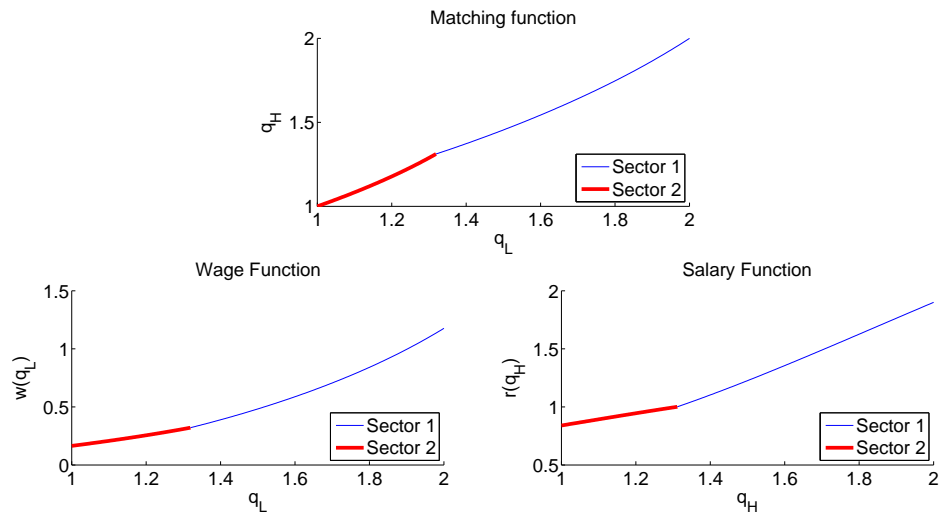
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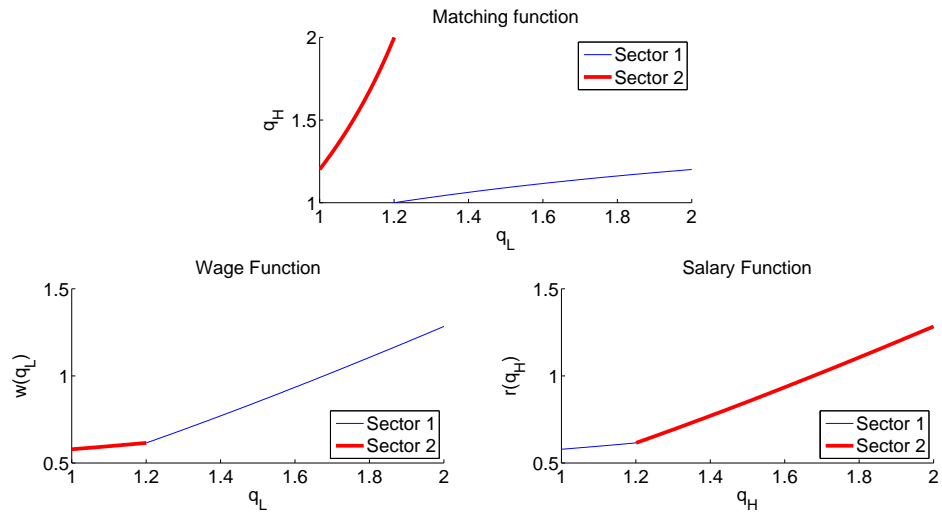
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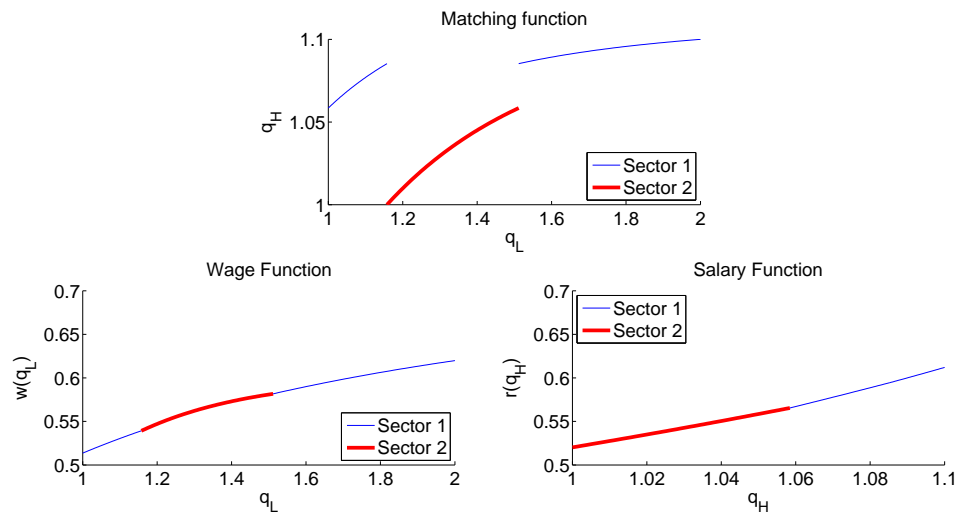
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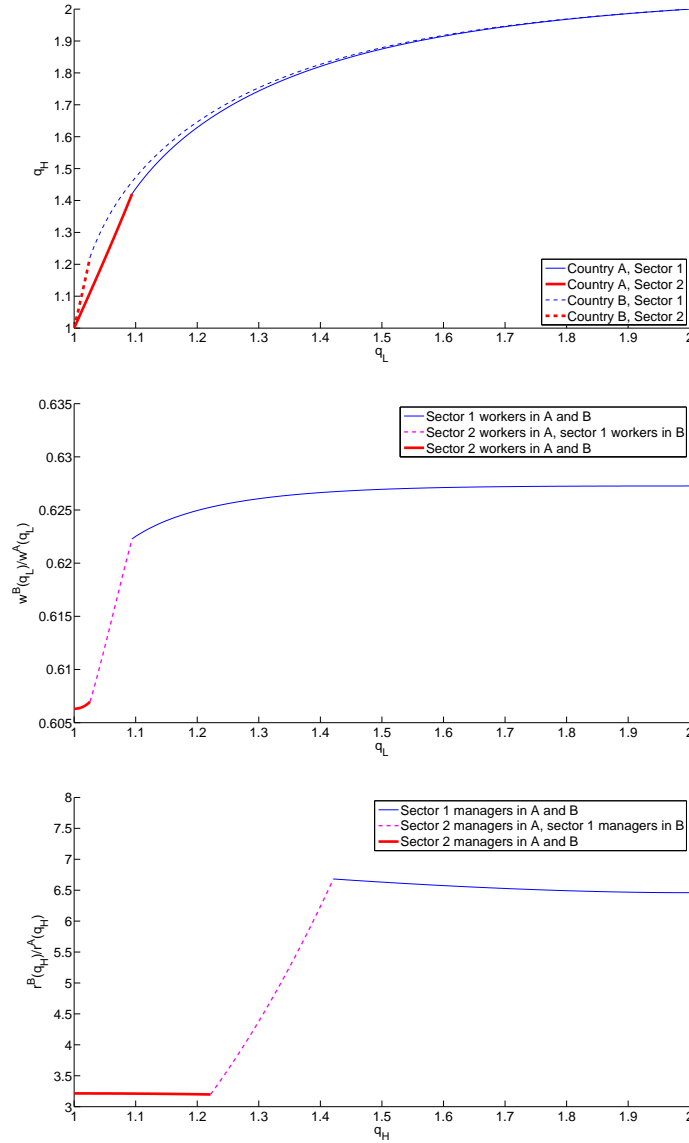
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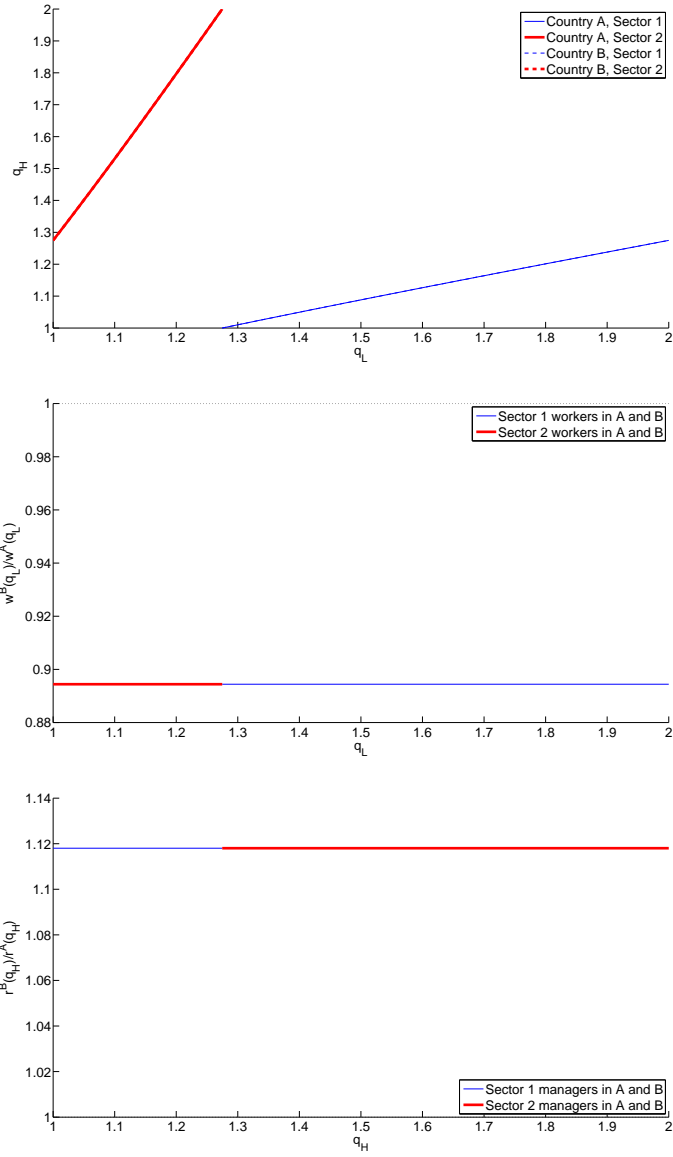
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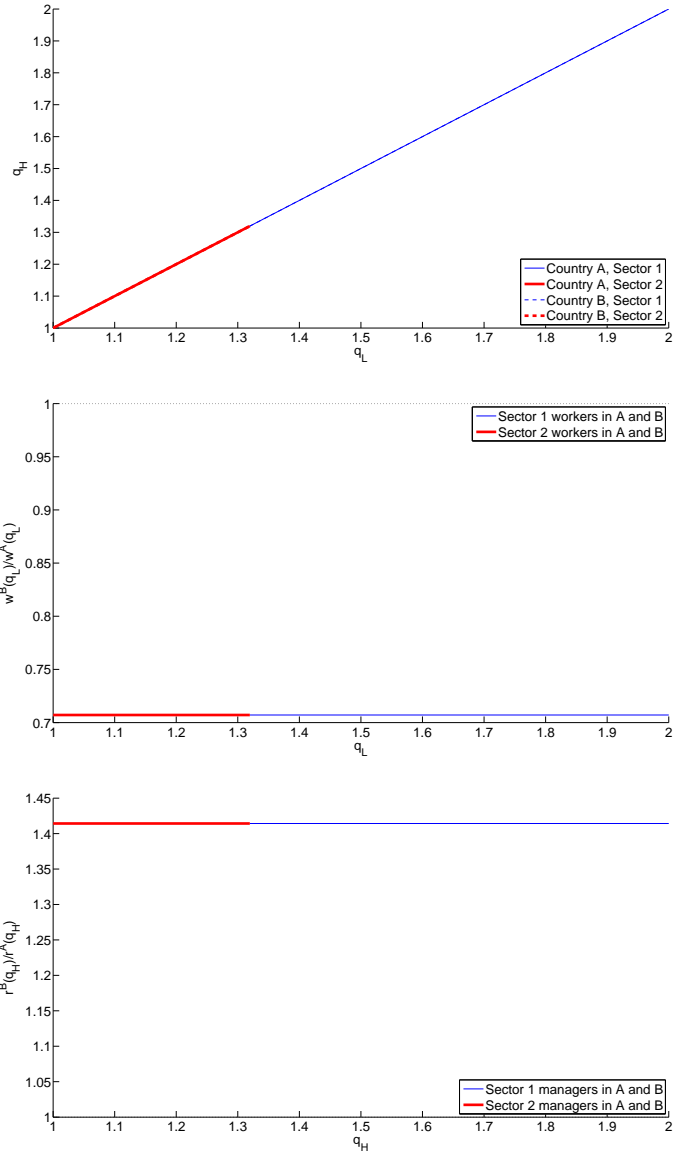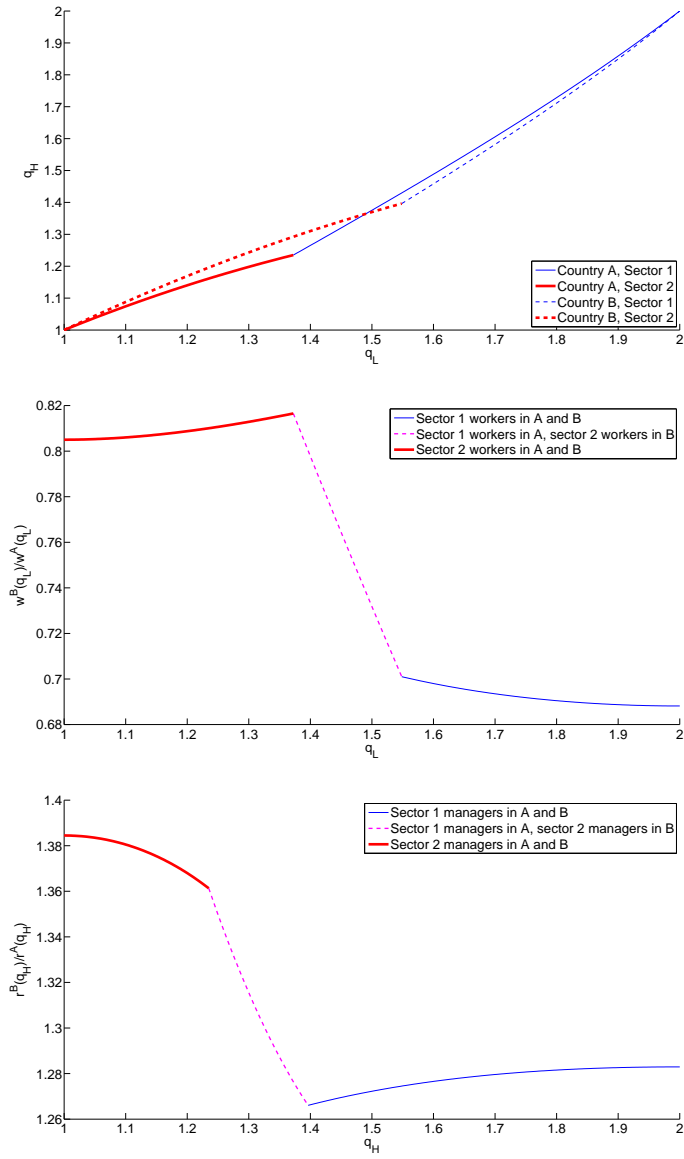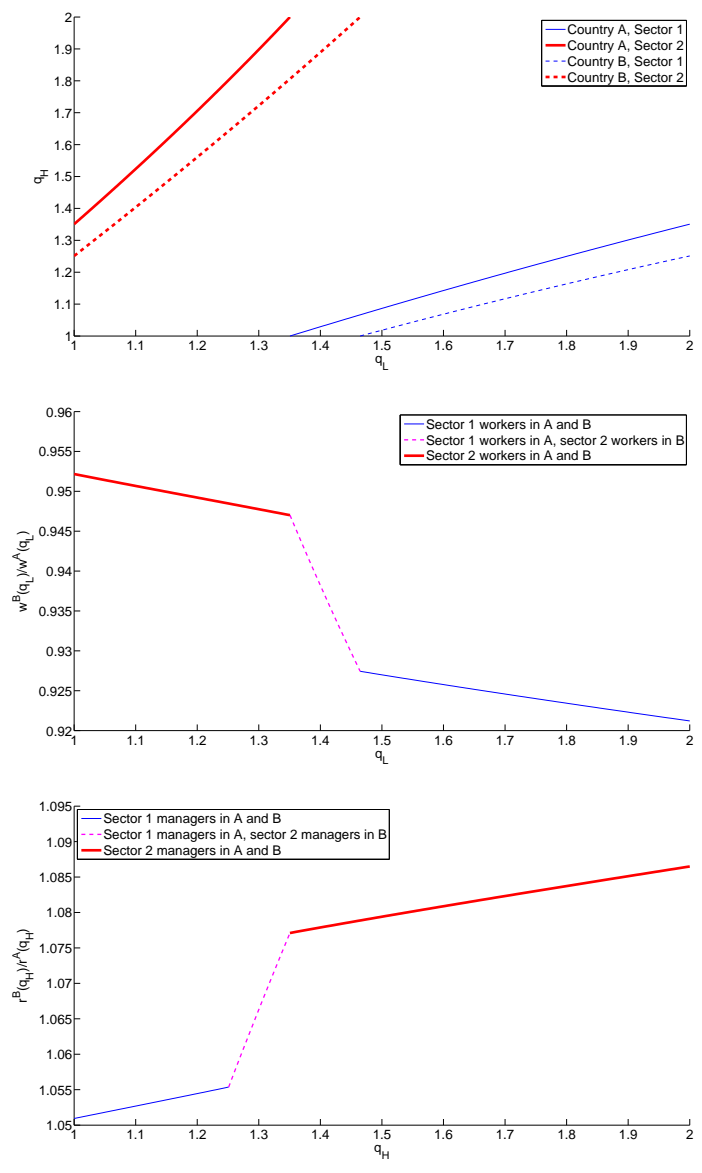
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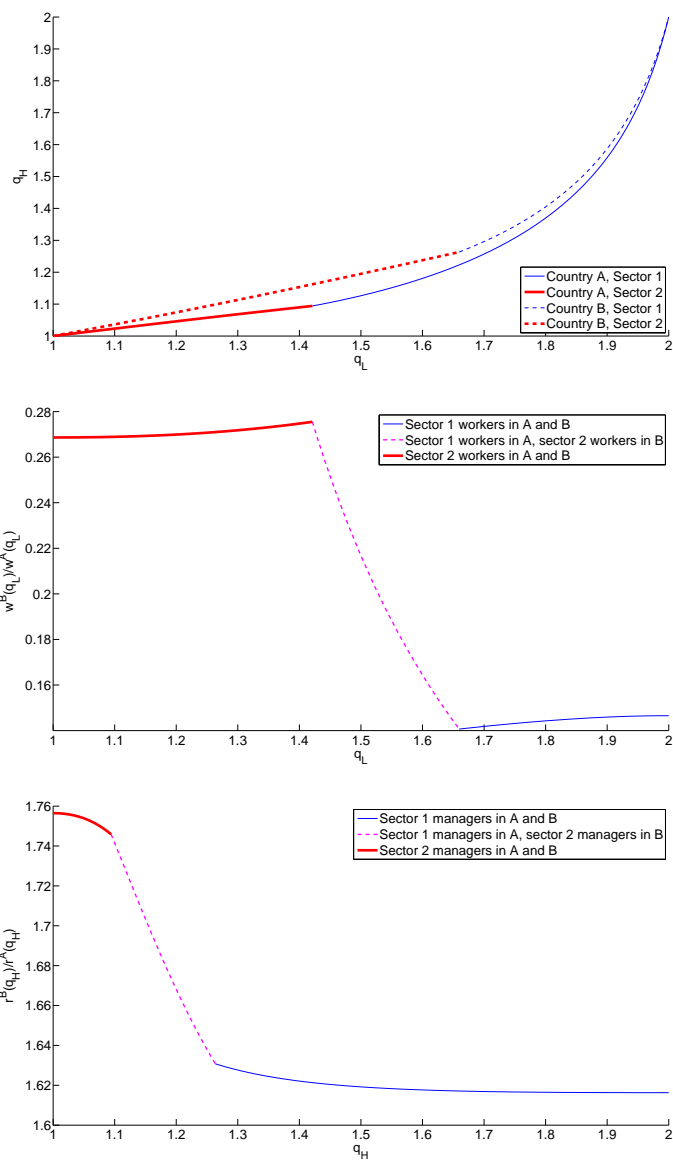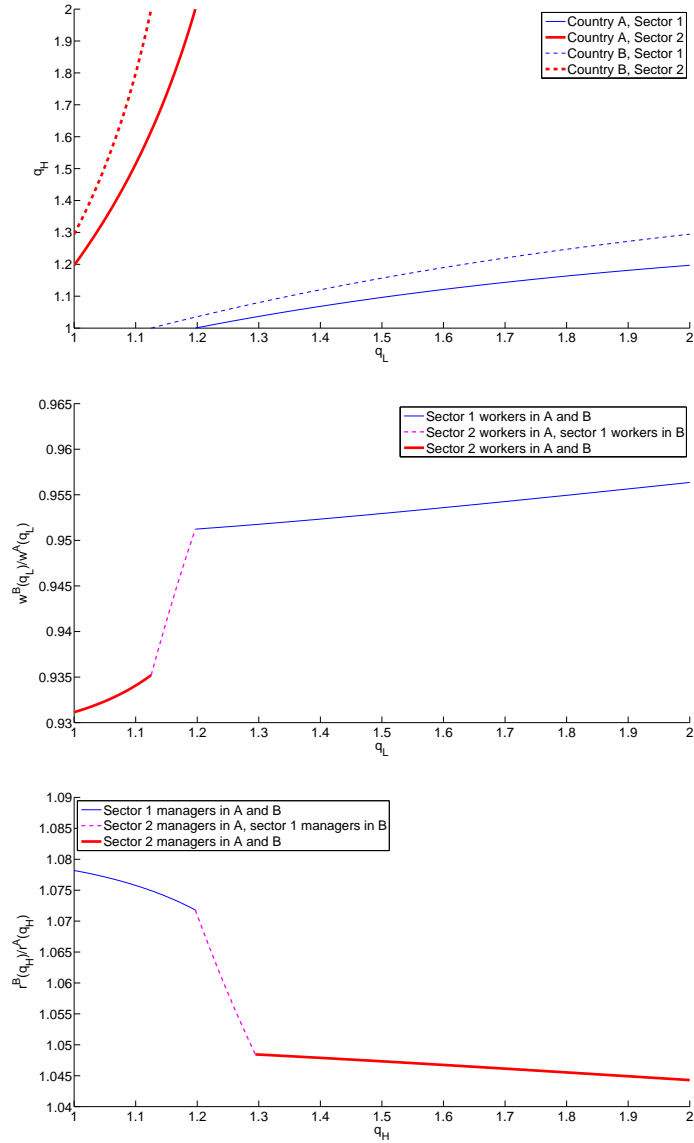
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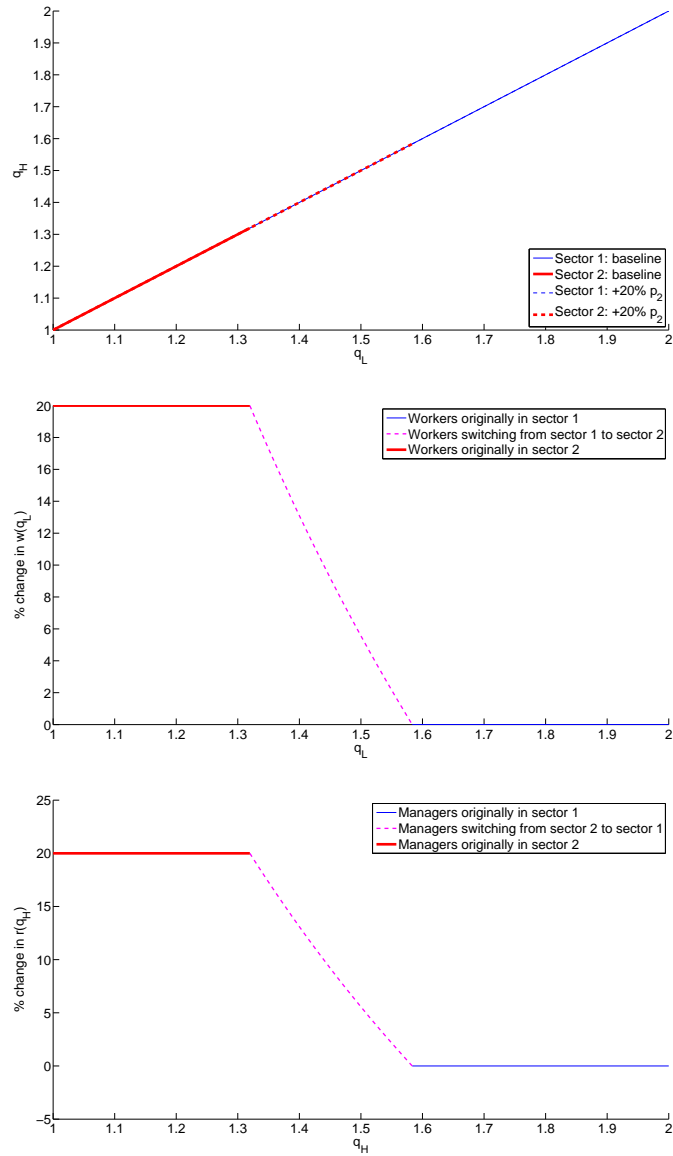
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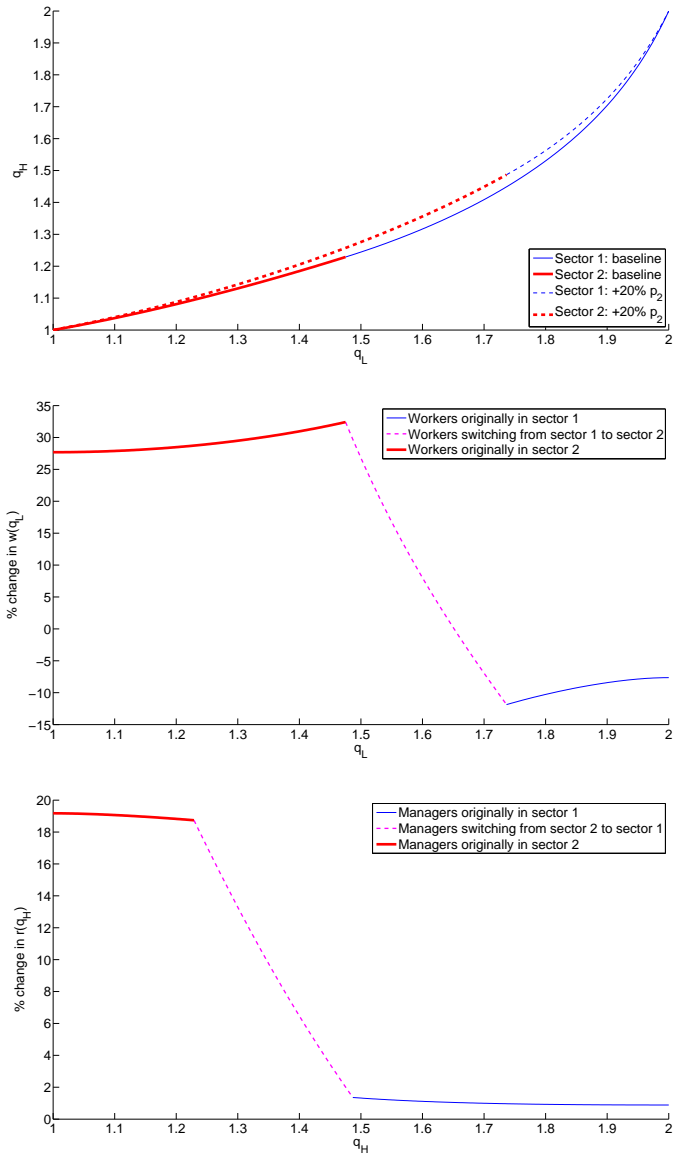
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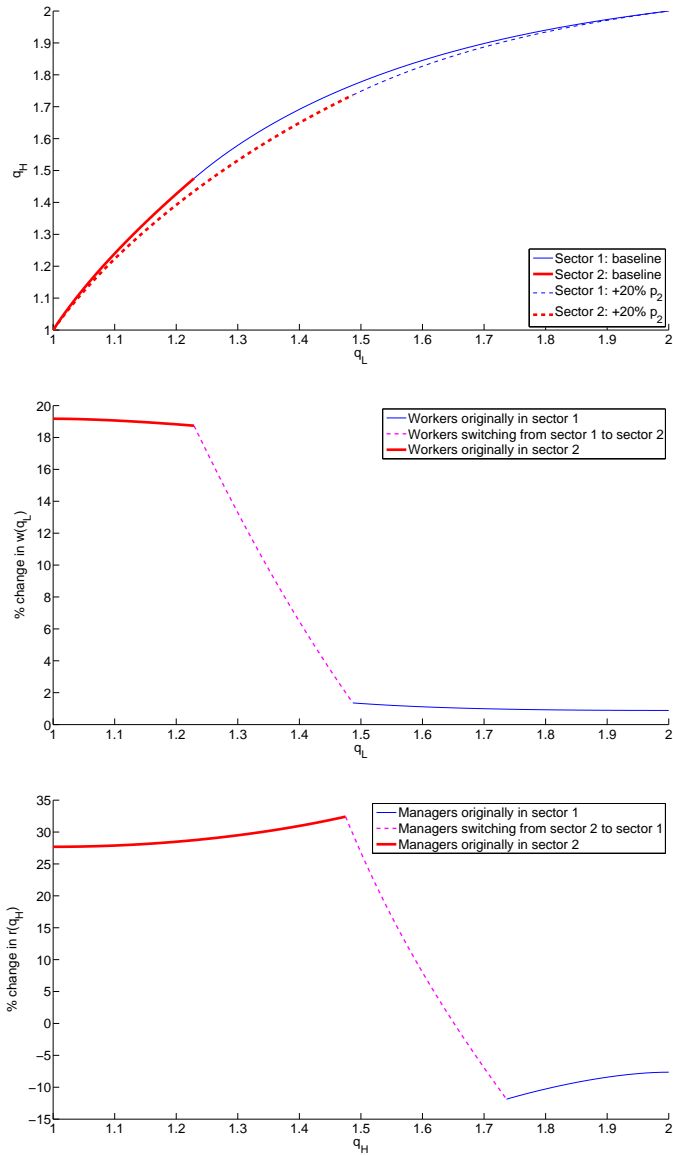
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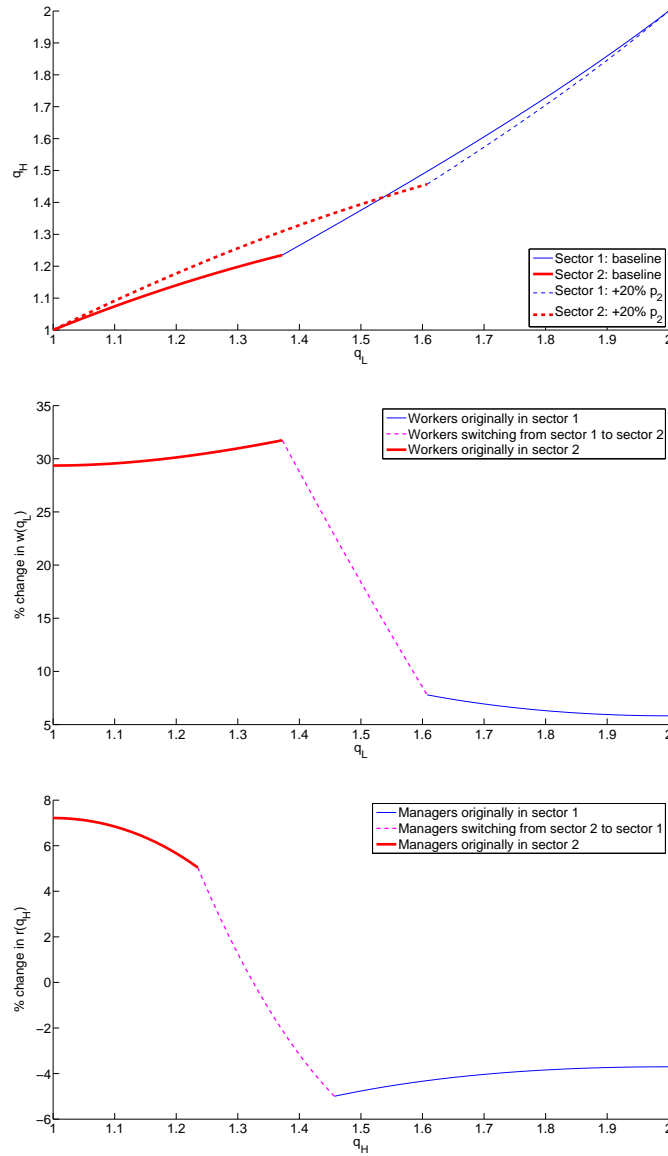
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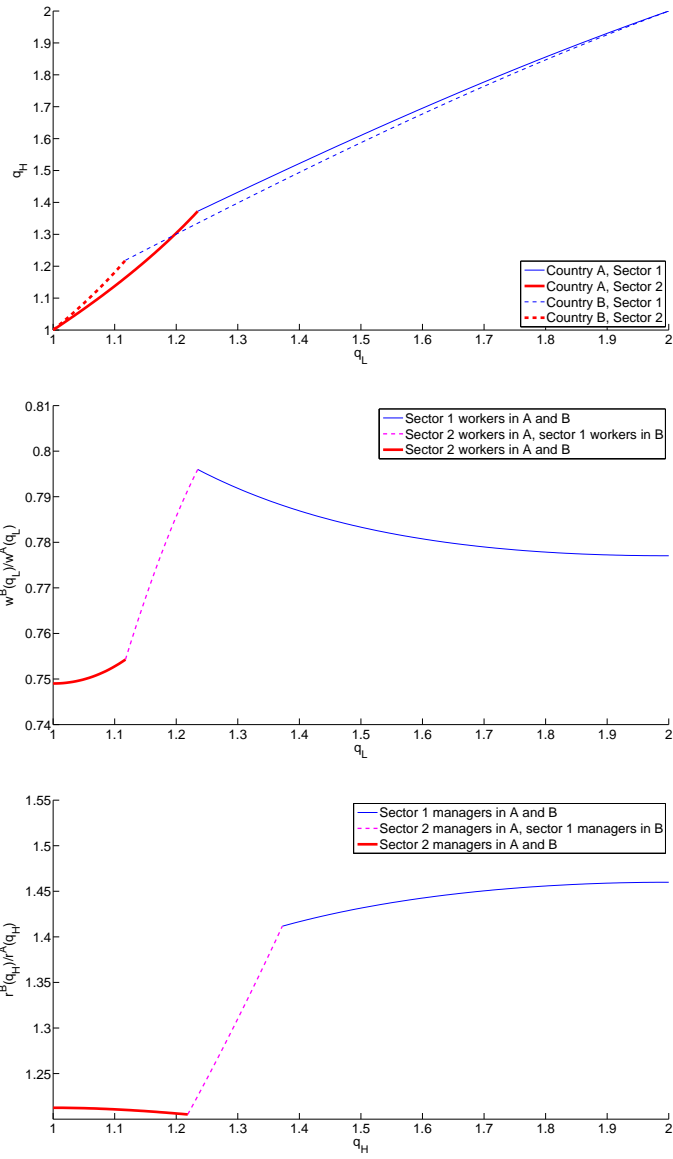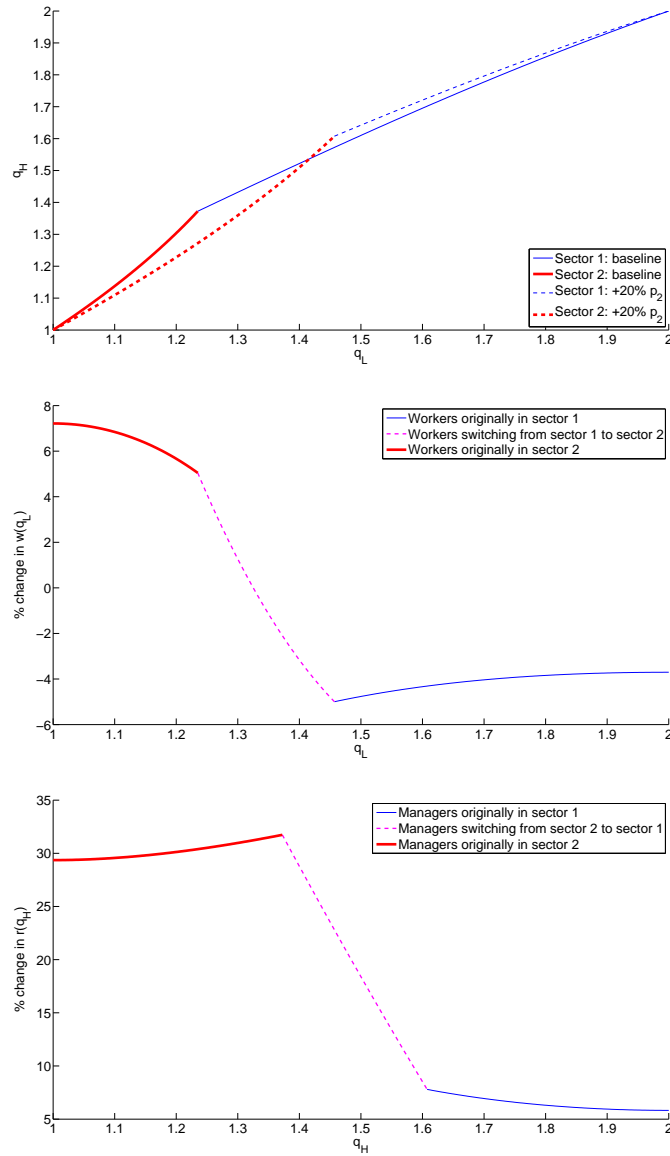Figure 18: Response of matching, wage, and salary functions for case 5, p_1/p_2 comparative statics, HH/LL sorting
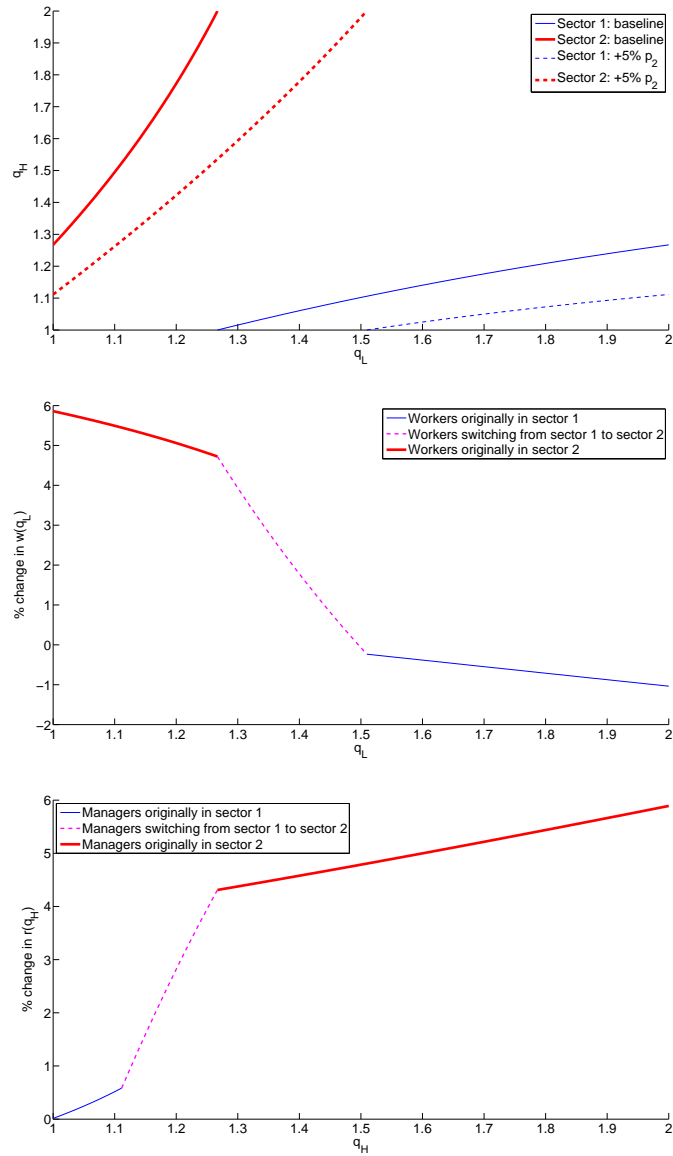
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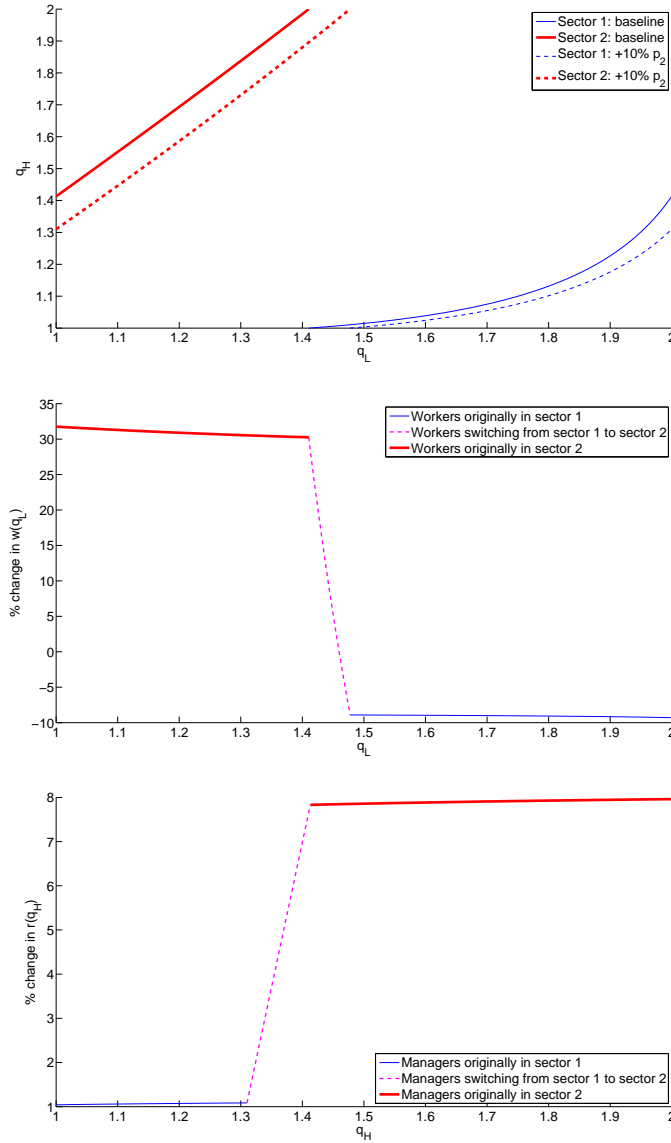
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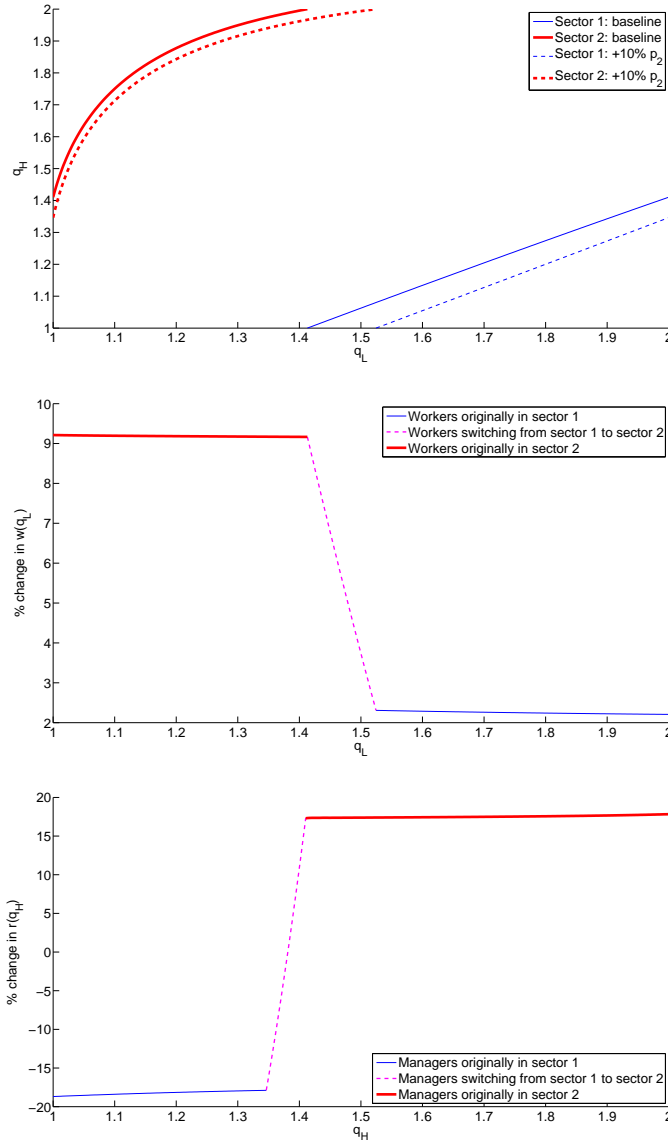
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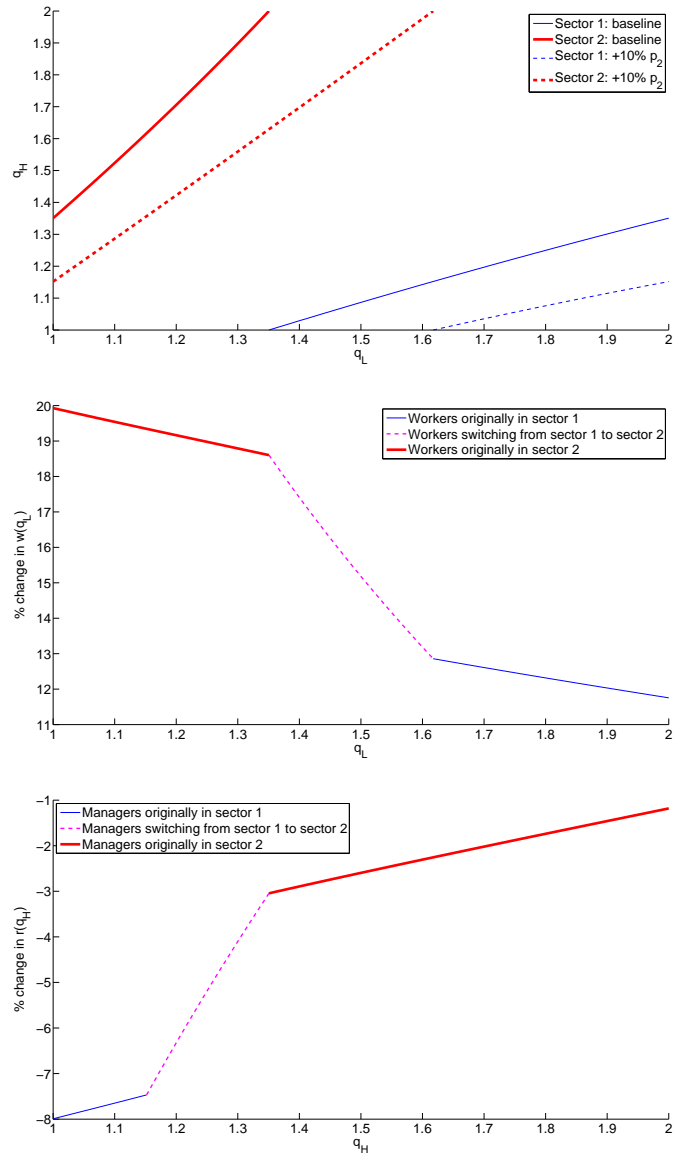
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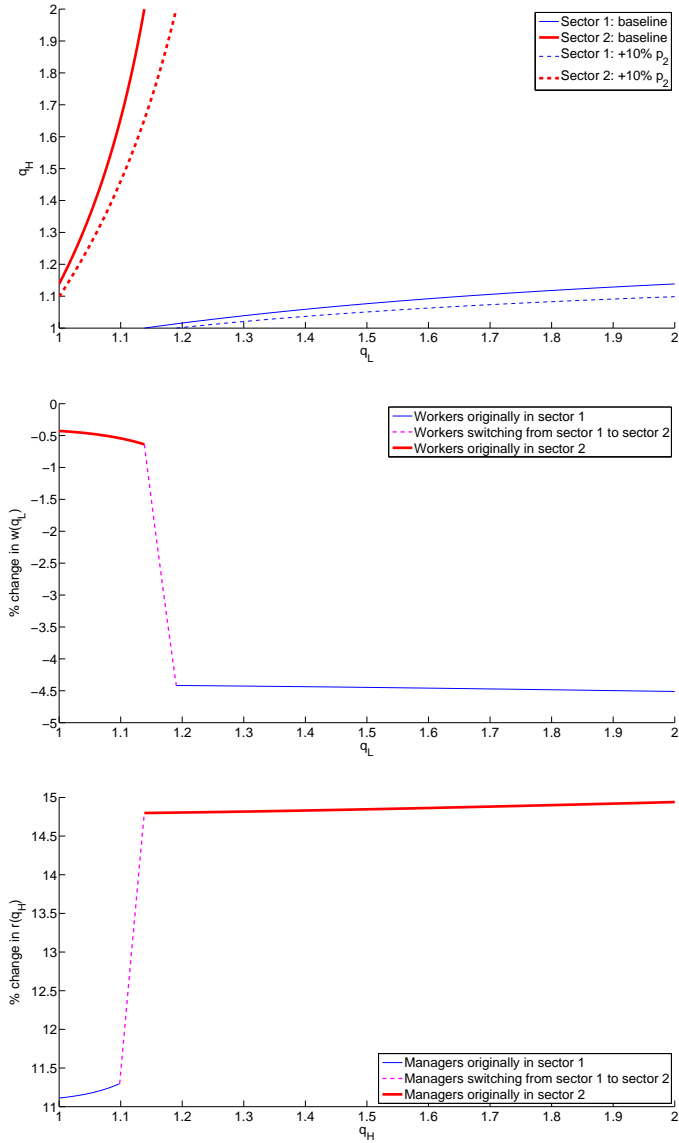
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$\bar{H}$	1
$\bar{L}$	1
S_L	$[0.5, 1.5]$
k_L	3

p_1	1
γ_1	0.4
α_1	0.9

p_2	1
γ_2	0.6
α_2	0.1

Table 1: Example of parameter values generating an equilibrium with two regions of sorting, homogeneous managers

$\bar{H}$	1	p_1	1	p_2	0.9
$\bar{L}$	1	γ_1	0.5	γ_2	0.5
S_L	$[0.5, 2.5]$	α_1	0.2	α_2	0.8
k_L	3	ρ_1	-1	ρ_2	-20

Table 2: Example of parameter values generating an equilibrium with three regions of sorting, homogeneous managers

$\bar{H}$	1		p_1	1		p_2	1
$\bar{L}$	1		γ_1	0.3		γ_2	0.2
S_H	$[1,2]$		α_1	1		α_2	0.5
S_L	$[1,2]$		β_1	1		β_2	0.5
k_H	3		ρ_1	-5		ρ_2	-5
k_L	3						

Table 3: Example of parameter values generating an equilibrium with the HH/LL sorting pattern, heterogeneous managers

$\bar{H}$	1		p_1	1		p_2	1
$\bar{L}$	1		γ_1	0.6		γ_2	0.4
S_H	$[1,2]$		α_1	0.9		α_2	0.1
S_L	$[1,2]$		β_1	0.1		β_2	0.9
k_H	3		ρ_1	-1		ρ_2	-1
k_L	3						

Table 4: Example of parameter values generating an equilibrium with the HL/LH sorting pattern, heterogeneous managers

$\bar{H}$	1		p_1	1		p_2	1
$\bar{L}$	1		γ_1	0.6		γ_2	0.4
S_H	$[1, 1.1]$		α_1	0.2		α_2	0.3
S_L	$[1, 2]$		β_1	0.8		β_2	0.7
k_H	3		ρ_1	-1		ρ_2	-5
k_L	3						

Table 5: Example of parameter values generating an equilibrium with three regions of sorting, heterogeneous managers

$\bar{H}^A$	1	S_H	$[1,2]$	p_1	1	p_2	1
$\bar{L}^A$	1	S_L	$[1,2]$	γ_1	varied	γ_2	varied
$\bar{H}^B$	varied, < 1	k_H	3	α_1	varied	α_2	varied
$\bar{L}^B$	1	k_L	3	β_1	varied	β_2	varied
				ρ_1	-5	ρ_2	-5

Table 6: Parameter values used for studying comparative statics with respect to $\bar{H}/\bar{L}$, HH/LL sorting

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	same sorting in A and B	same matching function in A and B	$w^B(q_L) < w^A(q_L)$ for all $q_L \in S_L$ $r^B(q_H) > r^A(q_H)$ for all $q_H \in S_H$	same intra-sector inequality in A and B
(2)	more Ws and Ms sort to S2 in B than in A	quality of match for a given W in S2 better in B than in A, quality of match for a given W in S1 better in A than in B	same as (1)	w inequality greater in A than in B for S1, greater in B than in A for S2; r inequality greater in B than in A for S1, greater in A than in B for S2
(3)	same as (2)	quality of match for any given W better in B than in A	same as (1)	w inequality greater in B than in A for both S1 and S2; r inequality greater in A than in B for both S1 and S2
(4)	more Ws and Ms sort to S1 in B than in A	quality of match for a given W in S2 better in B than in A, quality of match for a given W in S1 better in A than in B	same as (1)	same as (2)
(5)	same as (4)	quality of match for any given W better in B than in A	same as (1)	same as (3)

Table 7: Possible cases for $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
1.1	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$
1.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.3	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.9, 0.1\}$
1.4	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.5, 0.5\}$
1.5	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
1.6	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.9, 0.1\}$
1.7	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
1.8	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
1.9	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
1.10	$\gamma_1 = \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.8, 1.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$
1.11	$\gamma_1 = \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$

Table 8: Examples for case 1, $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
2.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
2.2	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$
2.3	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.5, 0.5\}$
2.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
2.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
2.6	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.1, 0.9\}$
2.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.9, 0.1\}$
2.8	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.9, 0.1\}$

Table 9: Examples for case 2, $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
3.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.9, 0.1\}$
3.2	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
3.3	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.4, 0.6\}$

Table 10: Examples for case 3, $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
4.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.3, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.32, 0.5, 0.5\}$
4.2	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$
4.3	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.5, 0.5\}$
4.4	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.3, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.1, 0.9\}$
4.5	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.1, 0.9\}$
4.6	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.3, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.9, 0.1\}$
4.7	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
4.8	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.9, 0.1\}$

Table 11: Examples for case 4, $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
5.1	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.2, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.1, 0.9\}$
5.2	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.7, 0.3\}$
5.3	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.6, 1.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.5, 0.5\}$

Table 12: Examples for case 5, $\bar{H}/\bar{L}$ comparative statics, HH/LL sorting

$\bar{H}^A$	1
$\bar{L}^A$	1
$\bar{H}^B$	varied, < 1
$\bar{L}^B$	1

S_H	$[1,2]$
S_L	$[1,2]$
k_H	3
k_L	3

p_1	1
γ_1	varied
α_1	varied
β_1	varied
ρ_1	-0.5

p_2	1
γ_2	varied
α_2	varied
β_2	varied
ρ_2	-0.5

Table 13: Parameter values used for studying comparative statics with respect to $\bar{H}/\bar{L}$, HL/LH sorting

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	same sorting in A and B	same matching function in A and B	$w^B(q_L) < w^A(q_L)$ for all $q_L \in S_L$ $r^B(q_H) > r^A(q_H)$ for all $q_H \in S_H$	same intra-sector inequality in A and B
(2)	more Ws and Ms sort to S2 in B than in A	quality of match for all Ws better in A than in B, except for workers employed in different sectors in A and B	same as (1)	w inequality greater in A than in B for both S1 and S2; r inequality greater in B than in A for both S1 and S2
(3)	more Ws and Ms sort to S1 in B than in A	quality of match for all Ws better in B than in A, except for workers employed in different sectors in A and B	same as (1)	w inequality greater in B than in A for both S1 and S2; r inequality greater in A than in B for both S1 and S2

Table 14: Possible cases for $\bar{H}/\bar{L}$ comparative statics, HL/LH sorting (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
1.1	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.3	$\gamma_1 = \gamma_2 = 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.8, 0.2\}$
1.4	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
1.5	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
1.6	$\gamma_1 = \gamma_2 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.8, 0.2\}$
1.7	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
1.8	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
1.9	$\gamma_1 = \gamma_2 > 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.8, 0.2\}$

Table 15: Examples for case 1, $\bar{H}/\bar{L}$ comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
2.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$
2.2	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.1, 0.9\}$
2.3	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
2.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.3, 0.7\}$
2.5	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.1, 0.9\}$
2.6	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
2.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
2.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
2.9	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.7, 0.3\}$
2.10	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
2.11	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
2.12	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
2.13	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
2.14	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.7, 0.3\}$
2.15	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 16: Examples for case 2, $\bar{H}/\bar{L}$ comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
3.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
3.2	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
3.3	$\gamma_2 < \gamma_1 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.55, 0.45\}$
3.4	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.1, 0.9\}$
3.5	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
3.6	$\gamma_2 < 0.5 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.6, 0.4\}$
3.7	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.45, 0.55\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
3.8	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
3.9	$0.5 < \gamma_2 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.7, 0.3\}$

Table 17: Examples for case 3, $\bar{H}/\bar{L}$ comparative statics, HL/LH sorting

$\bar{H}^A = \bar{H}^B$	1	p_1	1	p_2	1
$\bar{L}^A = \bar{L}^B$	1	γ_1	varied	γ_2	varied
S_H^A	$[1,2]$	α_1	varied	α_2	varied
S_L^A	$[1,2]$	β_1	varied	β_2	varied
S_H^B	$\lambda_H S_H^A$	ρ_1	-1	ρ_2	-1
S_L^B	$\lambda_L S_L^A$				
k_H	3				
k_L	3				

Table 18: Parameter values used for studying comparative statics with respect to S_L and S_H

Example	Qualitative Type	Specific values
1.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.9, 0.1\}$
1.2	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} > 1 > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.6, 0.4\}$
1.3	$\gamma_1 < \gamma_2 < 0.5$ $1 > \frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.4, 0.6\}$
1.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
1.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > 1 > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.6, 0.4\}$
1.6	$\gamma_1 < 0.5 < \gamma_2$ $1 > \frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.2, 0.8\}$
1.7	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.7, 0.3\}$
1.8	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > 1 > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.6, 0.4\}$
1.9	$0.5 < \gamma_1 < \gamma_2$ $1 > \frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.2, 0.8\}$

Table 19: Examples for case 1, S_L comparative statics

Example	Qualitative Type	Specific values
1.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.8, 0.2\}$
1.2	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} > 1 > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.4, 0.6\}$
1.3	$\gamma_1 < \gamma_2 < 0.5$ $1 > \frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.3, 0.7\}$
1.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.8, 0.2\}$
1.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > 1 > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.4, 0.6\}$
1.6	$\gamma_1 < 0.5 < \gamma_2$ $1 > \frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
1.7	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.7, 0.3\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.6, 0.4\}$
1.8	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > 1 > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.4, 0.6\}$
1.9	$0.5 < \gamma_1 < \gamma_2$ $1 > \frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 20: Examples for case 2, S_L comparative statics

$\bar{H}^A = \bar{H}^B$	1	p_1	1	p_2	1
$\bar{L}^A = \bar{L}^B$	1	γ_1	varied	γ_2	varied
S_H	$[1,2]$	α_1	varied	α_2	varied
S_L	$[1,2]$	β_1	varied	β_2	varied
k_H	3	ρ_1	-5	ρ_2	-5
k_L	3				

Table 21: Parameter values used for studying comparative statics with respect to p_1/p_2, HH/LL sorting

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	more Ws and Ms sort to S2	no change in match quality for a given W or M	real w and r increase for Ws and Ms in S2, decrease for Ws and Ms in S1, and change ambiguously for Ws and Ms that switch sectors	no change in w or r inequality
(2)	same as (1)	quality of match for a given W increases	real w increases for Ws in S2, and decreases for Ws in S1; ambiguous change in real r for Ms	w inequality increases in both S1 and S2, r inequality decreases in both S1 and S2
(3)	same as (1)	quality of match for a given W decreases	real r increases for Ms in S2, and decreases for Ms in S1; ambiguous change in real w for Ws	w inequality decreases in both S1 and S2, r inequality increases in both S1 and S2
(4)	same as (1)	quality of match for a given W in S1 decreases, quality of match for a given W in S2 increases	real w increases for Ws in S2, and either increases or changes ambiguously for Ws in S1; real r decreases for Ms in S1, and either decreases or changes ambiguously for Ms in S2	w inequality decreases in S1 and increases in S2, r inequality increases in S1 and decreases in S2
(5)	same as (1)	quality of match for a given W in S1 increases, quality of match for a given W in S2 decreases	real r increases for Ms in S2, and either increases or changes ambiguously for Ms in S1; real w decreases for Ws in S1, and either decreases or changes ambiguously for Ws in S2	w inequality increases in S1 and decreases in S2, r inequality decreases in S1 and increases in S2

Table 22: Possible cases for p_1/p_2 comparative statics, HH/LL sorting (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
2.1	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.5, 0.5\}$
2.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.9, 0.1\}$
2.3	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$

Table 23: Examples for case 2, p_1/p_2 comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
3.1	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
3.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
3.3	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.8, 1.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.5, 0.5\}$

Table 24: Examples for case 3, p_1/p_2 comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
4.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
4.2	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.5, 0.5\}$
4.3	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.5, 0.5\}$
4.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
4.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.9, 0.1\}$
4.6	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.3, 1.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.2, 0.8\}$
4.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.5, 1.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.2, 0.8\}$
4.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1.7, 0.3\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.8, 0.2\}$
4.9	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 1.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.8, 0.2\}$

Table 25: Examples for case 4, p_1/p_2 comparative statics, HH/LL sorting

Example	Qualitative Type	Specific values
5.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.3, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.2, 0.5, 0.5\}$
5.2	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.5, 0.5\}$
5.3	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 1, 1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.7, 0.5, 0.5\}$
5.4	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.2, 1.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.1, 0.9\}$
5.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
5.6	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.3, 1.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.2, 0.8\}$
5.7	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.5, 1.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.2, 0.8\}$
5.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.7, 0.3\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.8, 0.2\}$
5.9	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_2}{\beta_2} > \frac{\alpha_1}{\beta_1} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 1.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.8, 0.2\}$

Table 26: Examples for case 5, p_1/p_2 comparative statics, HH/LL sorting

$\bar{H}^A = \bar{H}^B$	1		p_1	1		p_2	1
$\bar{L}^A = \bar{L}^B$	1		γ_1	varied		γ_2	varied
S_H	$[1,2]$		α_1	varied		α_2	varied
S_L	$[1,2]$		β_1	varied		β_2	varied
k_H	3		ρ_1	-0.5		ρ_2	-0.5
k_L	3						

Table 27: Parameter values used for studying comparative statics with respect to p_1/p_2, HL/LH sorting

Case	Sorting	Matching	Inter-sector Inequality	Intra-sector Inequality
(1)	more Ws and Ms sort to S2	quality of match for a given W decreases	real w increases for worst Ws in S2, changes ambiguously for best Ws in S2, and decreases or changes ambiguously for Ws in S1; real r increases for best Ms in S2, changes ambiguously for worst Ms in S2, and decreases or changes ambiguously for Ms in S1	w inequality decreases in both S1 and S2, r inequality increases in both S1 and S2
(2)	same as (1)	same as (1)	real w increases for Ws in S2, and either increases or changes ambiguously for Ws in S1; real r decreases for Ms in S1, and either decreases or changes ambiguously for Ms in S2	same as (1)
(3)	same as (1)	same as (1)	real r increases for Ms in S2, and either increases or changes ambiguously for Ms in S1; real w decreases for Ws in S1, and either decreases or changes ambiguously for Ws in S2	same as (1)
(4)	same as (1)	same as (1)	real w increases for Ws in S2, and decreases for Ws in S1; ambiguous change in real r for Ms	same as (1)
(5)	same as (1)	same as (1)	real r increases for Ms in S2, and decreases for Ms in S1; ambiguous change in real w for Ws	same as (1)

Table 28: Possible cases for p_1/p_2 comparative statics, HL/LH sorting (W: worker, M: manager, S: sector)

Example	Qualitative Type	Specific values
1.1	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.2	$\gamma_1 = \gamma_2 = 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.1, 0.9\}$
1.3	$\gamma_1 = \gamma_2 = 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.8, 0.2\}$
1.4	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.45, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.4, 0.6\}$
1.5	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.55, 0.4, 0.6\}$
1.6	$\gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.5, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.505, 0.4, 0.6\}$
1.7	$\gamma_1 > \gamma_2$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.505, 0.6, 0.4\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.5, 0.4, 0.6\}$

Table 29: Examples for case 1, p_1/p_2 comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
2.1	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
2.2	$\gamma_1 = \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
2.3	$\gamma_1 = \gamma_2 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.8, 0.2\}$
2.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.15, 0.1, 0.9\}$
2.5	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.15, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$

Table 30: Examples for case 2, p_1/p_2 comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
3.1	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.2, 0.8\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
3.2	$\gamma_1 = \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
3.3	$\gamma_1 = \gamma_2 > 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.8, 0.2\}$
3.4	$\gamma_1 > \gamma_2 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.85, 0.1, 0.9\}$
3.5	$\gamma_2 > \gamma_1 > 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.85, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 31: Examples for case 3, p_1/p_2 comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
4.1	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.5, 0.5\}$
4.2	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
4.3	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_2}{\beta_2} = \frac{\alpha_1}{\beta_1} = 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.5, 0.5\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.5, 0.5\}$
4.4	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
4.5	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
4.6	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.6, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$
4.7	$\gamma_1 < \gamma_2 < 0.5$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.4, 0.9, 0.1\}$
4.8	$\gamma_1 < 0.5 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.9	$0.5 < \gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} > 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.8, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.10	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.1, 0.9\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.2, 0.8\}$
4.11	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} < 1 < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.7, 0.3\}$
4.12	$\gamma_1 < \gamma_2$ $1 < \frac{\alpha_1}{\beta_1} < \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.8, 0.2\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.9, 0.1\}$
4.13	$\gamma_1 < \gamma_2$ $\frac{\alpha_1}{\beta_1} > \frac{\alpha_2}{\beta_2}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.1, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.9, 0.1, 0.9\}$

Table 32: Examples for case 4, p_1/p_2 comparative statics, HL/LH sorting

Example	Qualitative Type	Specific values
5.1	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.2, 0.3, 0.7\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
5.2	$\gamma_2 < \gamma_1 < 0.5$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.4, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.1, 0.1, 0.9\}$
5.3	$\gamma_2 < \gamma_1 < 0.5$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.45, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.35, 0.6, 0.4\}$
5.4	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.1, 0.9\}$
5.5	$\gamma_2 < 0.5 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.7, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.3, 0.1, 0.9\}$
5.6	$\gamma_2 < 0.5 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.55, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.45, 0.6, 0.4\}$
5.7	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1} < 1$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.65, 0.4, 0.6\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.55, 0.1, 0.9\}$
5.8	$0.5 < \gamma_2 < \gamma_1$ $\frac{\alpha_2}{\beta_2} < 1 < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.6, 0.1, 0.9\}$
5.9	$0.5 < \gamma_2 < \gamma_1$ $1 < \frac{\alpha_2}{\beta_2} < \frac{\alpha_1}{\beta_1}$	$\{\gamma_1, \alpha_1, \beta_1\} = \{0.9, 0.9, 0.1\}$ $\{\gamma_2, \alpha_2, \beta_2\} = \{0.8, 0.7, 0.3\}$

Table 33: Examples for case 5, p_1/p_2 comparative statics, HL/LH sorting