Appendix for "The Lucas Orchard"—not for publication

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Figure 1 shows the functions $\mathscr{F}_\gamma(z)$, scaled by 2^γ so that they integrate to 1.

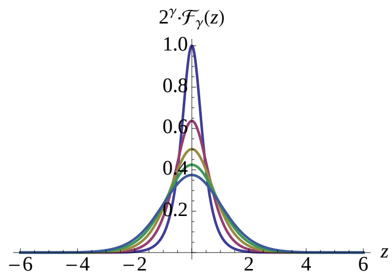
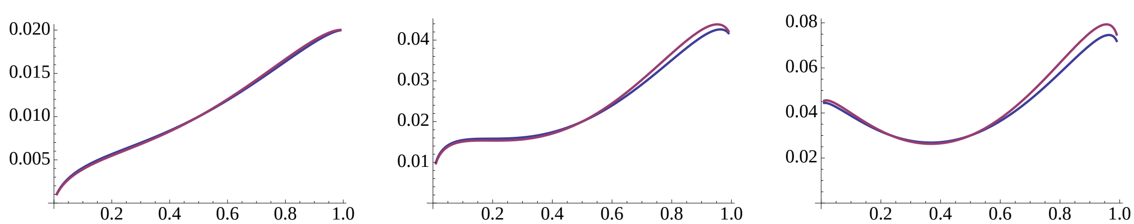


Figure 1: The functions $2^\gamma \cdot \mathscr{F}_\gamma(z)$ for $\gamma = 1$ (most peaked), 2, 3, 4, 5 (least peaked).

Figure 2 documents the claim that equation (14) of the paper (corresponding to equation (8) of Campbell and Vuolteenaho (2004)) holds to a good approximation in the conditionally lognormal calibration.



(a) $\gamma = 2$ (b) $\gamma = 4$ (c) $\gamma = 6$

Figure 2: The exact excess return on asset 1 plotted against its share s (blue); and the excess return predicted by the continuous time analog of Campbell and Vuolteenaho's equation (8) (red).

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0.1 Size vs. value

We can also use the N-tree solution to explore the distinction between size and value. Previous examples have featured symmetrically distributed, independent assets. Larger assets had higher risk premia and lower price-dividend ratios—they were *value* assets—while smaller assets had lower risk premia and higher price-dividend ratios, so were *growth* assets. (I follow the standard practice of using the terms value and growth to refer to assets with low or high price-dividend ratios, respectively. Both terms are relative, and within the model there is no sense in which either is "better".) To generate a spread on the value dimension that does not line up perfectly with size, I now consider a series of examples that break the symmetry at the level of fundamentals. In each case, I set $\gamma = 4$ and choose ρ so that the long rate is 7%. I arrange things so that in each calibration, assets 1 through 4 are small-growth (SG), small-value (SV), large-growth (LG), and large-value (LV) respectively.

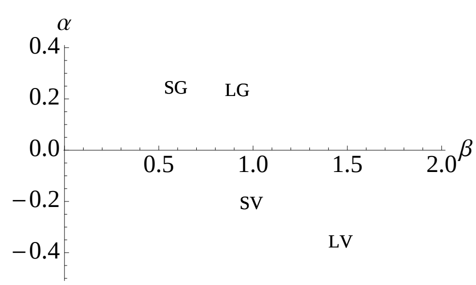
There are several natural ways of doing so. I consider each in turn, since the various possibilities turn out to have very different implications. We can introduce a spread in the mean dividend growth rate of assets. Value assets will then be those with lower mean dividend growth. Or we can introduce a spread in dividend volatility: value assets are those with more unstable cashflows. A third possibility is that value assets are more severely affected at times of economic distress as measured by declines in, say, labor income.

To illustrate the first possibility, consider an example in which all four assets have independent dividend growth, with volatility of 10%. Assets 1 and 2 have dividend shares $s_1 = s_2 = 0.2$, while assets 3 and 4 have dividend shares $s_3 = s_4 = 0.3$. The value assets 2 and 4 have lower mean log dividend growth, at 1%, while the growth assets 1 and 3 have mean log dividend growth of 3%. Figure 3a plots CAPM alpha against CAPM beta. Within size categories, growth and value stocks earn almost identical risk premia (not shown), but value stocks have considerably higher betas. As a result, growth assets earn positive alphas, while value assets earn negative alphas. This 'growth premium' pattern was also studied by Santos and Veronesi (2009). It is the opposite of what is observed in the recent data, namely that value assets have lower betas and higher alphas than growth assets (Fama and French (2004)). See also Lettau and Wachter (2007).

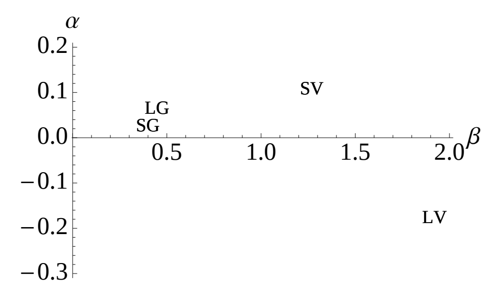
In the second example, the variation is in dividend volatility rather than in expected dividend growth: all four assets have independent dividend growth with mean log dividend growth of 2%, and the two value assets have 15% volatility, while the two growth assets have 5% volatility. Figure 3b shows that the calibration again generates extremely high betas for value assets, and the large-value asset counterfactually earns a negative alpha.

To model the third possibility, I add a fifth asset that is not represented in the econometrician's measure of "the market". All five assets have mean dividend growth of 2%, and dividend volatility of 10%, and the first four assets are uncorrelated with each other; but the dividends of the two value assets are positively correlated with the fifth asset's dividend (with correlation of 0.5). Bearing in mind that labor's share of income is roughly two thirds, I let the fifth asset have share $s_5 = 0.67$. The dividend shares of the other four assets are in the same proportions as before. Figure 3c shows the results. Value assets have betas that are moderately higher than growth assets. These betas line up with excess returns in the manner predicted by the CAPM for small-growth and large-value assets, while the small-value and large-growth assets earn positive and negative alphas, respectively.

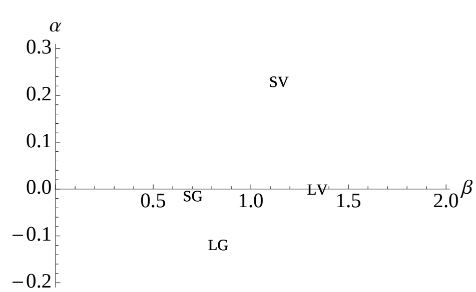
Figure 3d carries out a very similar exercise, but now value assets experience disasters at the same time as the fifth asset, while having independent Brownian motion components. Disasters arrive at rate 0.017 and affect the value assets and the fifth asset identically. As usual, the mean jump size is -0.38 (in logs) and the standard deviation is 0.25. Betas are conditional on no disaster occurring, i.e. they are the betas that would be computed by an econometrician who did not observe a disaster in sample. The result looks qualitatively (though not quantitatively—the size of the effect is too small) like the data: value and growth assets have betas close to one, and value assets have positive alpha while growth assets have negative alpha.



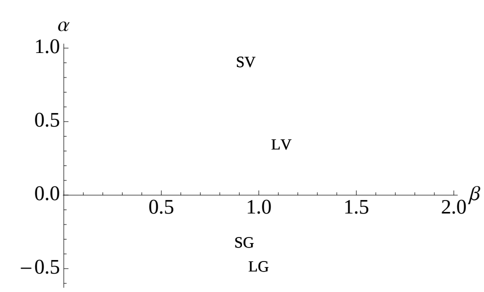
(a) Value assets have low mean dividend growth



(b) Value assets have high dividend volatility



(c) Value assets are correlated with background risk

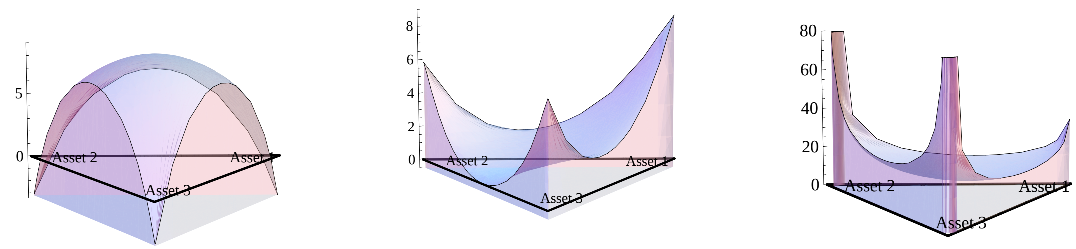


(d) Value assets and background risk are exposed to jumps

Figure 3: Alphas against betas, for various characterizations of value assets.

0.2 3D plots

Figure 4 shows 3D plots corresponding to the contour plots on page 33. They are in lower resolution than the plots in the main paper and, I think, convey less information, but are interesting in any case.



(a) Riskless rate (b) Asset 1's excess return (c) Asset 1's P/D

Figure 4: The riskless rate, and asset 1's excess return and price-dividend ratio.

1 References

Fama, E. F., and K. R. French (2004), "The Capital Asset Pricing Model: Theory and Evidence," *Journal of Economic Perspectives*, 18:3:25–46.

Lettau, M., and J. A. Wachter (2007), "Why is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium," *Journal of Finance*, 62:1:55–92.

Santos, T., and P. Veronesi (2009), "Habit Formation, the Cross Section of Stock Returns and the Cash-Flow Risk Puzzle," working paper.