Appendix I (for publication online as supplementary material)

Proofs

Proposition 2. *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, agents are risk neutral, and $R2$ is the rationing rule. Then there exist $\overline{n}$ and a finite threshold $k(n) \geq 1$ such that for all $n \leq \overline{n}$ and $v_{n-1} \geq k(n)v_{n-2}$ the following set of price and actions constitute an Ex Ante Vote-Trading Equilibrium:*

1. *Price $p^* = \frac{v_{n-1}}{2(n-1)}$.*
2. *Voters 1 to $n-2$ offer to sell their vote with probability 1.*
3. *Voter n demands $\frac{n-1}{2}$ votes with probability 1.*
4. *Voter $n-1$ offers his vote with probability $\frac{2}{n+1}$, and demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$.*

In particular, 1–4 constitute an equilibrium if $n \leq 9$ and $v_{n-1} \geq 1.15v_{n-2}$. The latter condition is satisfied in our experimental treatments; it is a sufficient condition for $n < 9$, and it is necessary and sufficient for $n = 9$.

Proof. If $n = 3$, in equilibrium $R2$ is identical to $R1$, and Theorem 1 1 applies here. immediately. The proof considers $n > 3$.

Voter $\mathbf{n-1}$. In the candidate equilibrium, he has expected utility $U^{n-1}(-1) = 1/2(p + v_{n-1}/2) + 1/2(v_{n-1}/2) = v_{n-1}/2 + p/2$. (a) Demanding a number of votes $x \in (0, (n-1)/2)$ cannot be a profitable deviation. Any such demand is satisfied with probability 1, causing an expenditure of $px > 0$ while leaving the probability of obtaining the desired outcome at $1/2$, and thus results in expected utility $U^{n-1}(x) = v_{n-1}/2 - px$. (b) Demanding more than $(n-1)/2$ votes cannot be a profitable deviation: as long as voter $n-1$ has received less than $(n-1)/2$ votes, $n-1$'s order is outstanding whether his demand is $(n-1)/2$ or higher - and thus the deviation does not affect the probability of individual n being rationed; once voter $n-1$ has received $(n-1)/2$ votes, he controls the final outcome, and any further expenditure is wasted. (c) Finally, doing nothing ($U^{n-1}(0) = v_{n-1}/2$) is dominated by offering to sell.

Voter n. (a) Doing nothing is again dominated by selling: it is identical to selling if $n-1$ sells, and it is strictly dominated if $n-1$ buys. (b) Selling is dominated by demanding

$(n-1)/2$ votes. If $n-1$ offers to buy, then n must prefer demanding $(n-1)/2$ to selling because in the identical circumstance, $n-1$, with smaller valuation, is indifferent between the two options. If $n-1$ offers to sell, again n must prefer to buy $(n-1)/2$: when $n-1$ sells, buying yields expected utility $v_n - (n-1)/2p$, while offering to sell means that no trade takes place (all voters try to sell) and n wins with probability $\mu \equiv \sum_{k=(n-1)/2}^{n-1} \binom{n-1}{k}(1/2)^{n-1}$ (the probability that at least $(n-1)/2$ of the other voters agree with him). But μ is declining in n, and thus is maximal at $n=3$, where it equals $3/4$. Hence when $n-1$ sells, n's expected utility from offering to sell has upper bound $(3/4)v_n$. But $v_n - (n-1)/2p = v_n - v_{n-1}/4 > (3/4)v_n$ for all $v_n > v_{n-1}$. Hence the only deviation to consider is demanding a quantity of votes x different from $(n-1)/2$. (c) Demanding a quantity x larger than $(n-1)/2$ cannot be profitable. If voter $n-1$ is selling, the order will be filled and is less profitable than demanding $(n-1)/2$; if voter $n-1$ is buying, the argument is identical to point 1b above. (d) Demanding a quantity x smaller than $(n-1)/2$ is not a profitable deviation either. In the candidate equilibrium n has expected utility equal to:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right) &= \left(\frac{2}{n+1}\right)(v_n - p\frac{n-1}{2}) + \\ &+ \left(\frac{n-1}{n+1}\right)\left[\frac{1}{2}\left(v_n/2 - p\frac{n-3}{2}\right) + \frac{1}{2}\left(v_n - p\frac{n-1}{2}\right)\right] = \\ &= \frac{v_n(5+3n) - nv_{n-1}}{4(n+1)} \end{aligned}$$

where the second expression is obtained by substituting for p. If n offers to buy $x < (n-1)/2$, his demand is always satisfied. His expected utility is $v_n/2 - px$ if $n-1$ is buying, and $\gamma(x)v_n - px$ if $n-1$ is offering to sell, where $\gamma(x) \equiv \sum_{k=(n-1)/2-x}^{n-1-x} \binom{n-1-x}{k}(1/2)^{n-1-x}$ is the probability that n obtains his preferred outcome when owning $x+1$ votes (while everyone else has one vote). Thus:

$$U^n(x) = \left(\frac{2}{n+1}\right)(\gamma(x)v_n - px) + \left(\frac{n-1}{n+1}\right)(v_n/2 - px)$$

With $x < (n-1)/2$, the difference $U^n\left(\frac{n-1}{2}\right) - U^n(x)$ is minimal when p is highest, i.e when $v_{n-1} = v_n$. But:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} &= \frac{5+2n}{4(n+1)} \\ U^n(x)|_{v_{n-1}=v_n} &= (1/2)\left(\frac{n-1+4\gamma(x)}{n+1} - \frac{x}{n-1}\right) \end{aligned}$$

It then follows immediately that $U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} > U^n(x)|_{v_{n-1}=v_n}$ for all $x > 0$ and all $\gamma(x) \leq 1$. Hence $U^n\left(\frac{n-1}{2}\right) > U^n(x)$: deviation is not advantageous.

Voters $\mathbf{1, 2, ..., n-2}$. (a) One possible deviation is for voter $i \in \{1, .., n-2\}$ to do nothing. When voter $n-1$ demands $(n-1)/2$ votes, supply is $n-3$ and it is then possible for neither $n-1$ nor n to be dictator. Call $r_e(n)$ the probability that votes supplied are allocated equally to n and $n-1$, when $n-1$ demands $(n-1)/2$ votes. I.e.:

$$r_e(n) = \binom{n-3}{(n-3)/2}(1/2)^{n-3}. \tag{8}$$

Then i's expected utility from doing nothing, U_i^0, is given by:

$$U^i(0) = \frac{2}{n+1}\left(\frac{v_i}{2}\right) + \frac{n-1}{n+1}\left[(1-r_e(n))\frac{v_i}{2} + r_e(n)\frac{3}{4}v_i\right]. \tag{9}$$

Voter i's expected utility from selling, $U^i(-1)$, is:

$$U^i(-1) = \frac{2}{n+1}\left(\frac{v_i}{2} + \frac{p}{2}\right) + \frac{n-1}{n+1}\left(\frac{v_i}{2} + p\right) \tag{10}$$

Comparing (9) to (10) and substituting (8), we derive;

$$\begin{aligned} U^i(-1) &\geq U^i(0) \Longleftrightarrow \\ \frac{v_{n-1}}{v_i} &\geq \frac{(n-1)^2}{n}\binom{n-3}{(n-3)/2}(1/2)^{n-2} \end{aligned} \tag{11}$$

The right hand side of equation (11) is smaller than 1 for all $n < 9$, and equals 10/9 at $n = 9$. Thus deviation to doing nothing is never advantageous for any i at $n = 5$ or 7; at $n = 9$, we need to impose $v_{n-1} \geq (10/9)v_{n-2}$. It will be shown below that this is not the binding restriction. However:

$$\lim_{n->\infty} \frac{(n-1)^2}{n} \binom{n-3}{(n-3)/2} (1/2)^{n-2} = \infty$$

Thus unless $v_i = 0$ for all $i < n-1$, there must always exist a number $\overline{n}$ such that for all $n > \overline{n}$ voter $n-2$ prefers to do nothing than selling. The actions and price described in the Proposition can only be an equilibrium for $n \leq \overline{n}$.

(b) The other possible deviation for voter i is demanding a positive number of votes x, with $x \in \{1, 2, .., (n-1)/2\}$. As before, demanding more than $(n-1)/2$ votes is never advantageous. Consider first i's expected utility from demanding $(n-1)/2$ votes. Call $\delta_i(n, x_{n-1})$ the probability that i becomes the dictator, i.e. the probability that he obtains $(n-1)/2$ votes, as function of n and of voter $n-1$'s demand, x_{n-1}. When $n-1$ demands $(n-1)/2$ votes, the total supply of votes is $n-3$, and $\delta_i(n, (n-1)/2)$ is the probability that at least $(n-1)/2$ votes are randomly allocated to voter i:

$$\delta_i(n, (n-1)/2) = \sum_{i=(n-1)/2}^{n-3} \sum_{z=0}^{n-3-i} \frac{(n-3)!}{i!z!(n-3-z-i)!} (1/3)^{n-3}$$

Similarly, $\delta_{-i}(n, (n-1)/2))$ is the probability that either n or $n-1$ become dictator, i.e. the probability that at least $(n-1)/2$ votes are randomly allocated to one of them:

$$\delta_{-i}(n, (n-1)/2) = 2 \sum_{z=(N-1)/2}^{n-3} \sum_{y=0}^{n-3-z} \frac{(n-3)!}{z!y!(n-3-z-y)!} (1/3)^{n-3}$$

Thus i's expected utility, when $n-1$ demands $(n-1)/2$ votes, either n or $n-1$ become dictator and i demands $x_i = (n-1)/2$ votes is given by:

$$Z^i_{d(-i)} \left(\frac{n-1}{2} \right) = 2 \sum_{z=(n-1)/2}^{n-3} \sum_{y=0}^{n-3-z} \frac{(n-3)!}{z!y!(n-3-z-y)!} (1/3)^{n-3} \left[\frac{v}{2} - p \left(n-3-z-y + \sum_{i=1}^{z-(n-1)/2} \binom{z-(n-1)/2}{i} i (1/2)^{z-(n-1)/2} \right) \right]$$

Finally, there is the probability that no dictator arises:

$$1-\delta_i-\delta_{-i}=\sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$

and the corresponding expected utility:

$$Z^i_{nod}\left(\frac{n-1}{2}\right)=\sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}\left[\frac{3v}{4}-p(n-3-z-y)\right]$$

We can then write:

$$\begin{aligned}U^i\left(\frac{n-1}{2}\right) &= \frac{n-1}{n+1}\left[Z^i_{d(-i)}\left(\frac{n-1}{2}\right)+Z^i_{nod}\left(\frac{n-1}{2}\right)+\delta_i\left(v-\frac{n-1}{2}p\right)\right]+\\ &\quad \frac{2}{n+1}\left[\frac{1}{2}\left(v-\frac{n-1}{2}p\right)+\frac{1}{2}\left(\frac{v}{2}-\frac{n-3}{2}p\right)\right]\end{aligned}$$

Comparing $U^i\left(\frac{n-1}{2}\right)$ to i's expected utility from selling, and substituting p, we find that $U^i(-1)\geq U^i\left(\frac{n-1}{2}\right)$ for all $i<n-1$ if and only if: $v_{n-1}\geq(25/24)v_{n-2}$, if $n=5$; $v_{n-1}\geq(11/10)v_{n-2}$, if $n=7$; and $v_{n-1}\geq 1.15v_{n-2}$, if $n=9$.

(c) For $n>3$, demanding less than $(n-1)/2$ votes can in principle be advantageous if $n-1$ demands $(n-1)/2$ votes, and neither $n-1$ nor n emerge as dictators. The calculations are somewhat cumbersome, but follow the logic just described, and we do not report them here (they are available from the authors upon demand). They show: (1) $U^i(-1)\geq U^i\left(\frac{n-3}{2}\right)$ for all i as long as $v_{n-1}\geq v_{n-2}$, satisfied by definition. For $n=5$, this concludes the proof. (2) $U^i(-1)\geq U^i\left(\frac{n-5}{2}\right)$ for all i as long as $v_{n-1}\geq v_{n-2}$, satisfied by definition. For $n=7$, this completes the proof. (3) For $n=9$, $U^i(-1)\geq U^i\left(\frac{n-7}{2}\right)$ for all i as long as $v_{n-1}\geq 1.04v_{n-2}$. The condition is not binding, because $v_{n-1}\geq 1.15v_{n-2}$ is required to prevent $n-2$ to deviate to doing nothing.

Summarizing the conditions derived in (a), (b) and (c), we conclude that the actions and price described in the proposition are an equilibrium for $n=5$ if and only if $v_{n-1}\geq(25/24)v_{n-2}$; for $n=7$, if and only if $v_{n-1}\geq(11/10)v_{n-2}$; and for $n=9$, if and only if $v_{n-1}\geq 1.15v_{n-2}$. With $1.15>11/10>25/24$, the latter condition is sufficient for $n=3,5,7$ and necessary and sufficient for $n=9$. This is the statement in the Proposition. ■

Proposition 3. *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, $u(\cdot) = -e^{-\rho(\cdot)}$* with $\rho > 0$, *and R1 is the rationing rule. Then for all n there exists a finite threshold $\mu_n \geq 1$ such that if $v_n \geq \mu_n v_{n-1}$, the set of actions presented in Theorem 1 together with the price $p = \frac{2}{r(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{rv_{n-1}}\right)$ constitute an Ex Ante Vote-Trading Equilibrium.*

Proof. Voter $\mathbf{n-1}$. As in Theorem 1, p must be such that individual $n-1$ is indifferent between selling his vote or demanding a majority of votes. If voter $n-1$ offers to sell his vote, he is rationed with probability 1/2; whether he is rationed or not, the decision is made by voter n, who owns a majority of votes and agrees with voter $n-1$ with probability 1/2. Thus:

$$U^{n-1}(-1) = \frac{1}{4}\left(u(0) + u(v_{n-1}) + u(p) + u(v_{n-1} + p)\right)$$

If voter $n-1$ demands $\frac{n-1}{2}$ votes, he is again rationed withe probability 1/2; if he is not rationed, he is dictator, if he is rationed, the dictator is voter n who agrees with $n-1$ with probability 1/2. Hence:

$$U^{n-1}\left(\frac{n-1}{2}\right) = \frac{1}{4}\left(u(0) + u(v_{n-1})\right) + \frac{1}{2}u\left(v_{n-1} - \frac{n-1}{2}p\right).$$

Thus the price at which $n-1$ is indifferent must solve:

$$u\left(p\right) + u\left(v_{n-1} + p\right) = 2 \cdot u\left(v_{n-1} - \frac{n-1}{2}p\right) \tag{12}$$

In the case of a CARA utility, the price that makes voter $n-1$ indifferent is computable and equal to $p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{\rho v_{n-1}}\right)$.

As in Theorem 1, demanding other quantities is strictly dominated because it is either equivalent to demanding $\left(\frac{n-1}{2}\right)$ if voter $n-1$ is rationed, or strictly worse, if he is not.

Voter n. In equilibrium, voter n's expected utility from demanding $\left(\frac{n-1}{2}\right)$ votes, is given by:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right) &= \frac{2}{n+1}u\left(v_n - \frac{n-1}{2}p\right) + \\ &+\frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + u\left(v_n - \frac{n-1}{2}p\right)\right] \end{aligned}$$

If voter n deviates and offers his vote for sale, his expected utility is

$$\begin{aligned} U^n(-1) &= \frac{2}{n+1}\left[\phi^n\left(\frac{n-1}{2}\right)u(v_n) + \left(1-\phi^n\left(\frac{n-1}{2}\right)\right)u(0)\right] + \\ &+ \frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + \frac{1}{2}u(p) + u(v_n+p)\right] \end{aligned}$$

where, as defined earlier, $\phi^n\left(\frac{n-1}{2}\right) = \sum_{i=(n-1)/2}^{n-1}\binom{n-1}{i}\cdot\left(\frac{1}{2}\right)^{n-1}$ is the probability that at least $\frac{n-1}{2}$ other voters agree with him, in the event that no trade has occurred.

Finally, if voter n deviates and demands $\left(\frac{n-1}{2}-g\right)$ votes, his expected utility is:

$$\begin{aligned} U^n\left(\frac{n-1}{2}-g\right) &= \left(\frac{2}{n+1}\phi^n(g) + \frac{n-1}{2(n+1)}\right)u\left(v_n - \left(\frac{n-1}{2}-g\right)p\right) + \\ &+ \left(\frac{2}{n+1}(1-\phi^n(g)) + \frac{n-1}{2(n+1)}\right)u\left(-\left(\frac{n-1}{2}-g\right)p\right) \end{aligned}$$

where again $\phi^n(g) = \sum_{i\geq g}^{\frac{n-1}{2}+g}\binom{\frac{n-1}{2}+g}{i}\cdot\left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$ is the probability that at least g other voters agree with him, when voter $n-1$ has offered his vote for sale and $\frac{n-1}{2}+g$ voters in all retain their vote.

All three expected utilities are continuous in p; $U^n\left(\frac{n-1}{2}\right)$ and $U^n\left(\frac{n-1}{2}-g\right)$ are everywhere strictly decreasing in p, $U^n(-1)$ is everywhere strictly increasing in p Notice that at $p=0$ (and thus $v_{n-1}=0$), $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n(-1)|_{p=0}$, because $\phi^n\left(\frac{n-1}{2}\right) < 1$, and $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n\left(\frac{n-1}{2}-g\right)|_{p=0}$, because $U^n\left(\frac{n-1}{2}-g\right)|_{p=0}$ is increasing in $\phi^n(g)$ and $\phi^n(g)$ is maximal at $g=0$. Thus for any v_n $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2}-g\right)$ if v_{n-1} (and thus p) is sufficiently low. Equivalently there must exist a value μ_n such that if $v_n \geq \mu_n v_{n-1}$, $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2}-g\right)$. This is the gap identified in the Proposition.

Voters $\mathbf{1,2,...,n-2}$. As in Theorem 1, for voter $i\in\{1,...,n-2\}$ deviation can be profitable only if he demands $\frac{n-1}{2}-1$ or $\frac{n-1}{2}$ votes. (1) Consider first deviation to demanding $\frac{n-1}{2}$ votes. If $n>3$, the possible outcomes and their probabilities are represented in the following Table:

Offer		**Demand $\frac{n-1}{2}$ Votes**	
Outcome	Prob	Outcome	Prob
0	$\frac{1-\delta}{2}$	0	$\frac{1-\varepsilon}{2}$
p	$\frac{1}{2}\delta$		
v_i	$\frac{1-\delta}{2}$	$v_i - \frac{n-1}{2}p$	ε
$v_i + p$	$\frac{1}{2}\delta$	v_i	$\frac{1-\varepsilon}{2}$

where we define $\delta = \frac{1}{n+1} + \frac{(n-1)^2}{2(n+1)(n-2)}$ and $\varepsilon = \frac{n+2}{3(n+1)}$. Thus:

$$\begin{aligned} U^i(-1) &> U^i\left(\frac{n-1}{2}\right) \Longleftrightarrow \\ \delta \cdot [u(p) + u(v_i + p)] &\geq 2\varepsilon \cdot u\left(v_i - \frac{n-1}{2}p\right) \\ &\quad + (\delta - \varepsilon) \cdot [u(v_i) + u(0)] \end{aligned} \tag{13}$$

Note that $v_i \leq v_{n-1}$. At $v_i = v_{n-1}$, given equation (12), the fact that $u(\cdot)$ is increasing and the fact that $\delta > \varepsilon$, equation (13) holds with strict inequality. We can show that if equation (13) holds at $v_i = v_{n-1}$, it must hold at all $v_i < v_{n-1}$. Denote:

$$\Delta = \delta \cdot [u(p) + u(v_i + p)] - 2\varepsilon \cdot u\left(v_i - \frac{n-1}{2}p\right) - (\delta - \varepsilon) \cdot [u(v_i) + u(0)]$$

Then

$$\frac{\partial \Delta}{\partial v} = \delta \cdot [u'(v_i + p) - u'(v_i)] - \varepsilon \cdot \left[2 \cdot u'\left(v_i - \frac{n-1}{2}p\right) - u'(v_i)\right]$$

But the concavity of u then implies $\frac{\partial \Delta}{\partial v} < 0$, and the result is established.

If $n = 3$, selling is preferred to buying one vote if $2u(p) + 2u(v+p) \geq 3u(v) + u(0)$. Note first that because $2u'(v+p) - 3u'(v) < 0$ by concavity, we only need to check the condition at $v = v_{n-1}$. Using the specific functional form of CARA utility simplifies the rest of the proof. Recall that the price is given by $p = \frac{1}{2\rho} ln\left(\frac{1+e^{\rho v_{n-1}}}{2}\right)$. Hence, $e^{-\rho p} = \left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}}$ and $e^{-\rho(v+p)} = \left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}} e^{-\rho v}$. The inequality that we need to verify reduces to

$$2\sqrt{2}\left(1 + e^{\rho v_{n-1}}\right)^{\frac{1}{2}} \leq e^{\rho v_{n-1}} + 3$$

Define $x = 1 + e^{\rho v_{n-1}}$. Then, we want to show that $2\sqrt{2}\sqrt{x} \leq 2 + x$. But $2 + x - 2\sqrt{2}\sqrt{x}$ has a minimum at $x = 2$, which is 0. Hence, the condition is always satisfied.

(2) We now show that if $n > 3$, selling one's vote dominates demanding $\frac{n-3}{2}$ votes. If $n > 3$, the difference of utilities is given by:

$$\begin{aligned} U^k(-1) - U^k(\frac{n-3}{2}) &= -\frac{n^2 - 6n + 11}{12(n+1)(n-2)}(u(v_k) + u(0)) \\ &\quad + \frac{n^2 - 3}{4(n+1)(n-2)}(u(p) + u(v_k + p)) \\ &- \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right] u\left(-\frac{n-3}{2}p\right) \\ &- \left[\frac{n-1}{3(n+1)}\left(1 - \left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right] u\left(v_k - \frac{n-3}{2}p\right) \\ &- \frac{1}{n+1}\left[u\left(v_k - \frac{n-3}{2}p\right) + u\left(-\frac{n-3}{2}p\right)\right] \end{aligned} \tag{14}$$

It is somewhat cumbersome but not difficult to show that the expression is decreasing in v.[26] Hence it is minimal at $v = v_{n-1}$; if 14 is positive then, it is positive for all $v_k \leq v_{n-1}$. Again, we make use of the CARA functional form. Define $\lambda = \frac{1+e^{\rho v_{n-1}}}{2}$ so that $p = \frac{2}{(n+1)\rho} ln\left(\lambda\right)$. Thus:

$$\begin{array}{ll} e^{-\rho p} = \lambda^{-\frac{2}{n+1}} & e^{-\rho(v+p)} = e^{-\rho v}\lambda^{-\frac{2}{n+1}} \\ e^{\rho\frac{n-3}{2}p} = \lambda^{\frac{n-3}{n+1}} & e^{-\rho\left(v - \frac{n-3}{2}p\right)} = e^{-\rho v}\lambda^{\frac{n-3}{n+1}} \end{array}$$

[26]The proof is available upon request.

Substituting in equation 14, we can write:

$$\begin{aligned}
e^{\rho v_{n-1}}(U^k(-1) - U^k(\frac{n-3}{2})) &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) \\
&\quad -\frac{n^2-3}{4(n+1)(n-2)}\lambda^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&\quad + \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right] e^{\rho v_{n-1}}\lambda^{\frac{n-3}{n+1}} \\
&\quad + \left[\frac{n-1}{3(n+1)}\left(1-\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right]\lambda^{\frac{n-3}{n+1}} \\
&\quad + \frac{1}{n+1}\lambda^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

First, because $e^{\rho v_{n-1}} \geq 1$, it is sufficient to show that

$$\begin{aligned}
\gamma &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) - \frac{n^2-3}{4(n+1)(n-2)}\lambda^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&+ \frac{n-1}{3(n+1)}\lambda^{\frac{n-3}{n+1}} \\
&+ \frac{1}{n+1}\lambda^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

is positive. Note that $1 + e^{\rho v_{n-1}} = 2\lambda$. Hence, we can write

$$\gamma = \frac{\lambda^{\frac{n-3}{n+1}}}{n+1}\left[\frac{n^2-6n+11}{6(n-2)}\lambda^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}\lambda^{\frac{2}{n+1}} + \frac{n-1}{3} + 2\lambda\right]$$

Denote $\Gamma = \frac{n^2-6n+11}{6(n-2)}\lambda^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}\lambda^{\frac{2}{n+1}} + \frac{n-1}{3} + 2\lambda$. We want to show that $\Gamma \geq 0$. Note that

$$\begin{aligned}
\frac{\partial \Gamma}{\partial \lambda} &= 2(\frac{n^2-6n+11}{3(n+1)(n-2)})\lambda^{\frac{3-n}{n+1}} - \frac{n^2-3}{(n+1)(n-2)}\lambda^{\frac{1-n}{n+1}} + 2 \\
\frac{\partial^2 \Gamma}{\partial \lambda^2} &= 2(\frac{(3-n)(n^2-6n+11)}{3(n+1)^2(n-2)})\lambda^{\frac{2(1-n)}{n+1}} - \frac{(1-n)(n^2-3)}{(n+1)^2(n-2)}\lambda^{\frac{-2n}{n+1}}
\end{aligned}$$

Therefore,

$$\frac{\partial^2\Gamma}{\partial\lambda^2} \geq 0 \;\Leftrightarrow\; 2(n^2-6n+11)\lambda^{\frac{2}{n+1}} \leq 3(n-1)(n+3)$$
$$\Leftrightarrow\; \lambda \leq \lambda^* = \left(\frac{3(n-1)(n+3)}{2(n^2-6n+11)}\right)^{\frac{n+1}{2}} \quad (>1)$$

Note that by construction, $\lambda \in [1,+\infty]$. Hence, $\frac{\partial\Gamma}{\partial\lambda}$ has a maximum at λ^*. But:

$$\frac{\partial\Gamma}{\partial\lambda}_{|\lambda=1} = \frac{5n^2-18n+19}{3(n+1)(n-2)}$$

which is always positive for $n \geq 3$. Moreover, as $\lambda \to \infty$, we can see that for $n > 3$,

$$\frac{\partial\Gamma}{\partial\lambda} \sim_{\lambda\to\infty} \frac{2}{n+1} > 0$$

Therefore, $\frac{\partial\Gamma}{\partial\lambda} \geq 0$ for any n and ρ. Hence, we only need to show that at $\lambda = 1$, $\Gamma \geq 0$. But $\Gamma_{|\lambda=1} = 0$. Thus: $\Gamma \geq 0$, which implies $\gamma \geq 0$, which implies $U^k(-1) - U^k(\frac{n-3}{2}) \geq 0$, concluding the proof for $n > 3$.

Finally, we need to show that demanding $\frac{n-3}{2}$ is also dominated when $n = 3$, a condition that amounts to showing $3u(v+p) - 4u(v) - 2u(0) + 3u(p) \geq 0$. But $3u'(v+p) - 4u'(v) \leq 0$, and thus we only need to check the inequality at $v = v_{n-1}$. Redefining $\lambda = \frac{1+e^{rv_{n-1}}}{2}$, the condition becomes $-6\lambda + 2\sqrt{\lambda} + 4\lambda\sqrt{\lambda} \geq 0$. The RHS is increasing in λ and is 0 at $\lambda = 1$. Hence, it is always satisfied. ■

Statistical Analysis of the Final Vote Allocation' s Distance to Equilibrium

	Variable		Coef		
$n = 5$	Match		-0.025	Obs	80
	Round		-0.184	Pr > chi2	0.029
	Dummies:	H	0.501^{*}	PseudoR^2	0.114
		T	0.948^{*}	LpL	-74.64
$n = 9$	Match		-0.141	Obs	80
	Round		0.010	Pr > chi2	0.065
	Dummies:	H	-0.006	PseudoR^2	0.062
		T	0.862^{**}	LpL	-102.69

Table 10: Ordered probit regression of distance to equilibrium d as a function of the number of a match, the number of a round, a dummy that takes value one if there are high values, H, and a dummy that takes value one if values are concentrated on the top, T. Data is clustered by session and standard errors are robust. * significant at 10%; ** significant at 5%; *** significant at 1%.

Appendix II (for publication online as supplementary material)

Sample Instructions

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc.

You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments.

During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

The experiment you are participating in is a committee voting experiment, where you will have an opportunity to buy and sell votes before voting on an outcome.

At the end of the experiment you will be paid the sum of what you have earned, plus a show-up fee of $10.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in POINTS. Your DOLLAR earnings are determined by dividing your earnings in POINTS by 350, that is, for every 350 POINTS you get 1 DOLLAR.

In this experiment you will be in a 5 member committee to decide on an outcome, X or Y. Each of you will be randomly assigned with probability 1/2 to be either in favor of X or in favor of Y. You will be told which outcome you favor, but will not be told the outcome favored by anyone else. You will also be assigned a Value, which you will earn if and only if your preferred outcome is the committee decision. If the opposite outcome is the committee decision you do not earn your value. Values will be different for different members. All values are integers between 1 and 1000 points.

Committee decisions are made by majority rule. Outcome X is the committee decision if there are more votes for X than for Y and vice versa.

Every round consists of two stages. Each committee member starts the round with one vote. After being told your value, but before voting, there will be a 2 minute trading stage, during which you and the other members of your committee will have an opportunity to buy or sell votes. We will describe how trading occurs momentarily.

After the trading stage ends, we proceed to the voting stage. In this stage you do not really have any choice. You will simply be asked to click a button to cast all your votes, if you have any, for your preferred outcome.

We will repeat this procedure for a set of 5 rounds, each consisting of the same two stages, trading and voting, described above. This set of five rounds is called a match. During each round of the match each of you keep the same Value you were assigned in round 1 of the match, but you will be randomly assigned to be in favor of X or Y (with each equally likely). Therefore, your preferred outcome can change from round to round. At the end of the fifth round of the match, a second match of 5 rounds will begin. In this new match, and you will be assigned a different Value, which you will keep for each of the 5 rounds in the second match. The experiment consists of 4 matches of 5 rounds each.

[SCREEN 1]

When we begin the experiment, you will see a screen like this. Your Subject ID# is printed at the very top left of your screen, and remains the same throughout the whole experiment.

The current match number, round number, your value, and your preferred outcome are displayed below your subject ID in the left part of the screen. The match number and round number are both equal to 1 now, indicating that this is the first election in your committee. Notice that this is an example where member 3's preferred outcome is X and his value 13. The committee number will identify you during the trading stage, and will be the same for the different rounds of a same match, and different between matches.

The middle panel is the trading window. Just above the panel, there is your cash holdings. At the beginning of the experiment, you will be loaned an initial amount of cash of 10.000 points, which will not be included in your final earnings. In the right part of the panel there is a table that clarifies how many votes each member of the committee currently has. Your information is highlighted and the other members' information is not. Notice that you do not see the values of the other members.

As the experiment proceeds, your cash holdings will be updated to reflect any earnings you make. It increases when you sell votes or when you earn your value as a result of the voting. It decreases when you buy votes. At the top of the panel, there is a countdown timer that tells you how much time is left in the trading period. The timer will turn red when there are 10 seconds left in the trading period, as you can see in the screen. There is a history panel in the lower part of the screen which will keep track of the history of the current and all past rounds and matches.

Trading occurs in the following way. At any time during this trading period, any member may post a bid to buy or an offer to sell one or multiple votes. At the bottom of the middle trading panel there is an area where you can type in your bid or your offer. When you do so, it will look like this: [SCREEN 2, 999 entered]. You also have to choose the amount of units you want to buy or sell [SCREEN 3]. Your bids or offers must always be between 1 and 1000. After you type in a price and a quantity, click the "bid" or "offer" button just to the right, and your bid or offer (price and quantity) will be posted on the trading board on the computer screens of all committee members, as you can see in this screen [SCREEN 4]. In this case, the column Bidder ID indicates that the member who made the bid was member 3; the bid price indicates that the price is 999. In the Bidder's Fulfilled columns you can see two numbers: the one on the right indicates the number of units he bid for, and the first number indicates the number of partial acceptances. In our example, he made a bid for one unit and nobody accepted so far. Whenever a new bid or offer is entered, it is added to the board, and does not cancel any outstanding bids or offer if there are any. When other members make bids or offers, you will also see the additions to the table as you can see now in the screen [SCREEN 5]. In this case member 2 made an offer for 201 for one unit, and member 1 made a bid for two units at price 3 (the price indicates the price paid per unit). All members in your committee see this information. The numbers on this slide are for illustration only.

If another member has an active bid or offer, then you may accept it. In order to accept a bid or an offer you just have to click on it and it will become highlighted in yellow [SCREEN 6]. In this case member 3 clicked the offer. At that point, a button below the table becomes active. If there is only one unit to be transacted, as in this case, by clicking the button the unit will be transacted and the transaction is highlighted in green [SCREEN 7].

If you accept an offer, as in this case, you will have an extra vote, and in exchange you will pay the other member the price of his offer. This information is immediately updated on your screens. See that the table on the right has been updated: now member 3 has 2 votes and member 2 has none and the cash holdings have also been updated. Similarly, if you have an active offer for exactly one unit, and another member accepts it, he will own your vote and he will pay you the amount of your offer. The same goes for bids that are accepted, except the transaction is a buy, by the person who posted the bid, rather than a sell. It's very important to remember that you post a Bid if you want to buy and post an offer if you want to sell!

If you accept a bid or offer and the order is for more than one unit, the transaction does not take place until the whole order has been filled. Thus, if someone submitted a bid to buy two votes, and you accept their bid, nothing happens yet because their order has not yet been filled. [SCREEN 8] As you can see now, some member accepted member 1's bid. It will be filled only after a second acceptance has been made, at which time both transactions will be executed simultaneously and the transaction is highlighted in green [SCREEN 9].

If you have an active bid or offer that has not been transacted you can cancel it. To do so, you need to click on it. [SCREEN 10] By doing so, the bid or offer will be highlighted in yellow and cancel button will become active. Clicking the cancel button you will cancel the bid or offer that you clicked on. It will then disappear from the screen [SCREEN 11]. If you have accepted a bid or offer for multiple units which has not been transacted, you can also cancel your partial acceptance. In that case, the bid or offer will remain in the screen but the number of partial acceptances will be updated. You may also cancel all your untransacted market activities at any time by clicking "Cancel All" button, located on the right hand side of the panel, below the table. See that, as the remaining time is less than 10 seconds, the remaining time is red.

The trading period ends after 2 minutes. There are two additional trading rules. First, if your cash holdings ever become 0 or negative, you may not place any bid nor accept any offer until it becomes positive again. Second, you may not sell votes if you do not have any or if all the votes you currently own are committed.

After 2 minutes, the trading stage of the round is over and we proceed to the voting stage. Your screen would now look like [SCREEN 12]. At this stage, you simply cast your votes by clicking on the vote button. These votes are automatically cast as votes for X if your preferred outcome is X, and are automatically cast as votes for Y if your preferred outcome is Y.

After you and the other members of the committee have voted, the results are displayed in the right hand panel, and summarized in the history screen. [SCREEN 13] We will then proceed to the next round. [SCREEN 14] In the next round, as you can see in the right table, all members' votes will be reinitialized to one and your preferred outcome will be randomly assigned. Because this round belongs to the same match, you will be able to see the bids offers and transactions of the previous rounds of the same match. [SCREEN 15].