Technical Appendix for

“Self-Fulfilling Risk Panics”

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The Appendix is organized as follows. In section I we provide all the algebra associated with the solution of the model with financial shocks. We consider a generalized version of the model that also encompasses two extensions discussed in section 6.3 of the paper. In section II we provide the algebra associated with a cubic solution of the model with financial shocks (also discussed in section 6.3). Finally, in section III we report all results from sensitivity analysis discussed in section 6.3 of the paper.

I Solution of Model with Financial Shocks

I.A Building blocks

We consider a generalized setting of the model with financial shocks that also allows some of the endowment of investors to be in the form of trees and allows the financial shock to include an aggregate dimension in addition to a redistributive one. The baseline setting presented in section 5 of the paper is a particular case of this general one. This generalized setting also encompasses two extensions discussed in section 6.3 of the paper. The first extension introduces a feedback effect from asset prices to wealth by allowing some of the wealth of investors to be in the form of trees. The second extension considers wealth shocks that only affect investors (no redistribution between households and investors).

At the beginning of period t the newborn investors receive an endowment of $W_{I,t} - \delta K\tilde{Q}$ units of the consumption good and δK trees, so the value of their endowment in terms of the consumption good is:

$$W_{I,t} + \delta K \left(Q_t - \tilde{Q} \right)$$

The baseline model in the text is the case where the endowment consists solely of consumption good, i.e. $\delta = 0$.

The newborn households receive an endowment of $W_{H,t}$ units of the consumption good. $W_{I,t}$ and $W_{H,t}$ are stochastic and given by:

$$\begin{aligned} W_{I,t} &= \bar{W}_I \exp\left[-m\theta_t - \frac{1}{2}m^2\theta_t^2\right] \\ W_{H,t} &= W - \bar{W}_I \exp\left[-n\theta_t - \frac{1}{2}n^2\theta_t^2\right] \end{aligned}$$

where

$$\theta_{t+1} = \rho_\theta \theta_t + \epsilon_{t+1}^\theta \tag{1}$$

and $\epsilon_{t+1}^\theta \sim N(0, \sigma_\theta^2)$ and $\rho_\theta \in (0, 1]$. A rise in θ_t represents a redistribution away from investors towards households. A quadratic approximation of endowments around $\theta_t = 0$ implies that endowments are linear in θ_t and thus have a constant variance:

$$W_{I,t} = \bar{W}_I \left(1 - m\theta_t\right) \quad , \quad W_{H,t} = \left(W - \bar{W}_I\right) + n\bar{W}_I\theta_t$$

Up to a quadratic approximation, the aggregate endowment is:

$$W_t = W_{I,t} + W_{H,t} = W + \bar{W}_I \left(n - m\right) \theta_t \tag{2}$$

The baseline model in the text is the case where the aggregate endowment is constant ($m = n$). The case where endowment shocks only affect investors corresponds to $n = 0$.

The return on equity consists of the dividend yield and capital gain, with trees depreciating at a rate δ to exactly offset the endowment in trees of the newborn agents. The rate of return on equity is then:

$$R_{K,t+1} = \frac{A_{t+1} + \left(1 - \delta\right) Q_{t+1}}{Q_t} \tag{3}$$

The dividend is an exogenous process:

$$A_{t+1} = \bar{A} \exp\left[a_{t+1} - \frac{1}{2} a_{t+1}^2\right]$$

where $a_{t+1} = \epsilon_{t+1}^a$ has a symmetric distribution with mean zero and variance σ_a^2 and is independent from ϵ_{t+1}^θ. A quadratic approximation of A_{t+1} is $\bar{A}(1 + a_{t+1})$, which has a constant variance $\bar{A}^2\sigma_a^2$. As the dividend is iid it does not affect the equity price.

Newborn households allocate their endowments between bonds and a riskfree technology with decreasing returns to scale. Investing $K_{H,t+1}$ units of the good in the technology at time t yields an output of $Y_{t+1} = \frac{1}{\eta}\left[\nu K_{H,t+1} - \frac{1}{2}\left(K_{H,t+1}\right)^2\right]$ goods at time $t + 1$. Only households can invest in the technology. Household maximize their future consumption, given by $Y_{t+1} + R_{t+1}\left(W_{H,t} - K_{H,t+1}\right)$, which implies that $K_{H,t+1} = \nu - \eta R_{t+1}$. The amount invested in bonds is then:

$$W_{H,t} - K_{H,t+1} = W_{H,t} - \nu + \eta R_{t+1}$$

Bonds are in zero net supply. The clearing of the bond and equity markets requires:

$$KQ_t = \alpha_t \left(W_{I,t} + \delta K \left(Q_t - \tilde{Q}\right)\right) \tag{4}$$

$$0 = (1-\alpha_t)\left(W_{I,t} + \delta K \left(Q_t - \tilde{Q}\right)\right) + W_{H,t} - \nu + \eta R_{t+1} \tag{5}$$

Taking the sum of (4)-(5) and using (2) yields a positive relation between the equity price and the interest rate:

$$R_{t+1} = \frac{1}{\eta}\left[\nu - W + \bar{W}_I (m-n)\theta_t + (1-\delta)KQ_t + \delta K\tilde{Q}\right] \tag{6}$$

I.B Approximation equity market clearing condition

The optimal portfolio is given by the mean-variance relation:

$$\alpha_t = \frac{E_t R_{K,t+1} - R}{\gamma var_t(R_{K,t+1})} \tag{7}$$

Using (7) the equity market clearing condition (4) becomes:

$$\frac{E_t R_{K,t+1} - R_{t+1}}{\gamma var(R_{K,t+1})}\left(W_{I,t} + \delta K\left(Q_t - \tilde{Q}\right)\right) = KQ_t \tag{8}$$

(3) implies that the expected rate of return on equity and its variance are (using the fact that the dividend and asset price are uncorrelated):

$$E_t R_{K,t+1} = \frac{\bar{A} + (1-\delta) E_t Q_{t+1}}{Q_t}$$

$$var_t(R_{K,t+1}) = \frac{\bar{A}^2\sigma_a^2}{Q_t^2} + \frac{(1-\delta)^2}{Q_t^2} var_t(Q_{t+1})$$

(8) is then rewritten as:

$$\left(\bar{A} + (1-\delta)E_t Q_{t+1} - Q_t R_{t+1}\right)\frac{W_{I,t} + \delta K\left(Q_t - \tilde{Q}\right)}{\gamma} \tag{9}$$

$$= K\left(\bar{A}^2\sigma_a^2 + (1-\delta)^2 var_t(Q_{t+1})\right)$$

We conjecture the log equity price is a linear-quadratic function of the financial shock:

$$Q_t = \tilde{Q}\exp\left[-\upsilon\theta_t - V\theta_t^2\right] \tag{10}$$

Taking a quadratic expansion of Q_t around $\theta_t = 0$ gives:

$$Q_t = \tilde{Q}\left[1 - v\theta_t + \left(\frac{1}{2}v^2 - V\right)\theta_t^2\right] \tag{11}$$

This implies

$$Q_{t+1} = \tilde{Q}\left[1 - v\theta_{t+1} + \left(\frac{1}{2}v^2 - V\right)\theta_{t+1}^2\right] \tag{12}$$

Using (1) we can write:

$$Q_{t+1} = \tilde{Q}\left(1 - v\rho_\theta\theta_t - v\epsilon_{t+1}^\theta + (-V + 0.5v^2)(\rho_\theta^2\theta_t^2 + 2\rho\theta_t\epsilon_{t+1}^\theta + \sigma_\theta^2)\right) \tag{13}$$

where we used the continuous time approximation $(\epsilon_{t+1}^\theta)^2 = \sigma_\theta^2$. The expectation and variance of the equity price next period are then:

$$\begin{aligned} E_t Q_{t+1} &= \tilde{Q}\left[1 + \left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2 - v\rho_\theta\theta_t + \left(\frac{1}{2}v^2 - V\right)\rho_\theta^2\theta_t^2\right] && (14) \\ var(Q_{t+1}) &= \tilde{Q}^2\left[-v + 2\left(\frac{1}{2}v^2 - V\right)\rho_\theta\theta_t\right]^2 \sigma_\theta^2 && (15) \end{aligned}$$

Using (6) we have

$$Q_t R_{t+1} = \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)Q_t + \frac{1-\delta}{\eta}KQ_t^2 + \frac{m-n}{\eta}\bar{W}_I\theta_t Q_t \tag{16}$$

Taking a quadratic approximation of Q_t^2 gives

$$Q_t^2 = \tilde{Q}^2[1 - 2v\theta_t + 2(v^2 - V)\theta_t^2] \tag{17}$$

Substituting (11) and (17) into (16), and omitting cubic and higher order terms in θ_t, gives

$$\begin{aligned} Q_t R_{t+1} = {} & \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)\tilde{Q}\left[1 - v\theta_t + \left(\frac{1}{2}v^2 - V\right)\theta_t^2\right] && (18) \\ & + \frac{1-\delta}{\eta}K\tilde{Q}^2\left[1 - 2v\theta_t + 2\left(v^2 - V\right)\theta_t^2\right] \\ & + \frac{m-n}{\eta}\bar{W}_I\tilde{Q}\left[\theta_t - v\theta_t^2\right] \end{aligned}$$

Combining these approximations (9) is rewritten as:

$$X\left[\bar{W} - \bar{m}\theta_t + k\tilde{Q}\left[-v\theta_t + \left(\frac{1}{2}v^2 - V\right)\theta_t^2\right]\right] = Y \tag{19}$$

where:

$$\bar{W} = \frac{\bar{W}_I}{\gamma} \quad ; \quad \bar{m} = \frac{\bar{W}_I}{\gamma} m \quad ; \quad k = \frac{\delta K}{\gamma}$$

and:

$$\begin{aligned} X &= \bar{A} + (1-\delta)\tilde{Q}\left[1 - v\rho_\theta\theta_t + \left(\frac{1}{2}v^2 - V\right)\left(\rho_\theta^2\theta_t^2 + \sigma_\theta^2\right)\right] \\ &\quad -\frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)\tilde{Q}\left[1 - v\theta_t + \left(\frac{1}{2}v^2 - V\right)\theta_t^2\right] \\ &\quad -\frac{1-\delta}{\eta}K\tilde{Q}^2\left[1 - 2v\theta_t + 2\left(v^2 - V\right)\theta_t^2\right] \\ &\quad -\frac{m-n}{\eta}\bar{W}_I\tilde{Q}\left[\theta_t - v\theta_t^2\right] \end{aligned}$$

and:

$$Y = K\bar{A}^2\sigma_a^2 + K(1-\delta)^2\tilde{Q}^2\left[-v + 2\left(\frac{1}{2}v^2 - V\right)\rho_\theta\theta_t\right]^2\sigma_\theta^2$$

I.C Solution without switching

We restrict ourselves to the constant, linear, and quadratic components of (19) in terms of θ_t:

$$Z_0 + Z_1\theta_t + Z_2\theta_t^2 = 0$$

where the coefficient on the constant is:

$$\begin{aligned} Z_0 &= -K\bar{A}^2\sigma_a^2 - (1-\delta)^2\tilde{Q}^2Kv^2\sigma_\theta^2 \\ &\quad +\bar{W}\left[\begin{array}{c} \bar{A} + (1-\delta)\tilde{Q} + (1-\delta)\tilde{Q}\left(-V + \frac{1}{2}v^2\right)\sigma_\theta^2 \\ -\frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2 \end{array}\right] \end{aligned} \tag{20}$$

the coefficient on θ_t is:

$$\begin{aligned} Z_1 &= -(\bar{m} + kv\tilde{Q})\left[\begin{array}{c} \bar{A} + (1-\delta)\tilde{Q}\left[1 + \left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2\right] \\ -\frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2 \end{array}\right] \\ &\quad -\bar{W}\tilde{Q}\left[(1-\delta)v\rho_\theta - \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)v - \frac{1-\delta}{\eta}K\tilde{Q}2v + \frac{m-n}{\eta}\bar{W}_I\right] \\ &\quad +4(1-\delta)^2K\tilde{Q}^2v\left(-V + \frac{1}{2}v^2\right)\rho_\theta\sigma_\theta^2 \end{aligned} \tag{21}$$

and the coefficient on θ_t^2 is:

$$\begin{aligned}
Z_2 &= (\bar{m} + kv\tilde{Q})\tilde{Q}\left[(1-\delta)v\rho_\theta - v\frac{1}{\eta}(\nu - W + \delta K\tilde{Q}) - 2\frac{1-\delta}{\eta}K\tilde{Q}v + \frac{m-n}{\eta}\bar{W}_I\right] \\
&\quad + k\tilde{Q}\left(\frac{1}{2}v^2 - V\right)\left[\begin{array}{c} \bar{A} + (1-\delta)\tilde{Q} + (1-\delta)\tilde{Q}\left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2 \\ -\frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2 \end{array}\right] \\
&\quad + \bar{W}\tilde{Q}\left[\begin{array}{c} (1-\delta)\left(\frac{1}{2}v^2 - V\right)\rho_\theta^2 - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\left(\frac{1}{2}v^2 - V\right) \\ -\frac{1-\delta}{\eta}2K\tilde{Q}\left(v^2 - V\right) + v\frac{m-n}{\eta}\bar{W}_I \end{array}\right] \\
&\quad - 4(1-\delta)^2 K\tilde{Q}^2\left(\frac{1}{2}v^2 - V\right)^2\rho_\theta^2\sigma_\theta^2 \qquad (22)
\end{aligned}$$

The method of undetermined coefficients implies $Z_0 = Z_1 = Z_2 = 0$. Setting $Z_0 = 0$ in (20) implies:

$$\tilde{Q}V = \alpha_1 + \alpha_2 v^2 \qquad (23)$$

where:

$$\begin{aligned}
\alpha_1 &= \frac{1}{(1-\delta)\sigma_\theta^2}\left[\bar{A} + (1-\delta)\tilde{Q} - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2\right] \\
&\quad - \frac{K\bar{A}^2\sigma_a^2}{(1-\delta)\bar{W}\sigma_\theta^2} \\
\alpha_2 &= \frac{1}{2}\tilde{Q} - \frac{(1-\delta)\tilde{Q}^2 K}{\bar{W}}
\end{aligned}$$

Setting $Z_1 = 0$ in (21) implies:

$$\beta_1 + \beta_2 v + \beta_3 v^2 + \beta_4 v^3 + \beta_5\left[\tilde{Q}V\right] + \beta_6\left[\tilde{Q}V\right]v = 0 \qquad (24)$$

where:

$$\begin{aligned}
\beta_1 &= \bar{m}\left[\bar{A} + (1-\delta)\tilde{Q} - \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2\right] + \bar{W}\tilde{Q}\frac{m-n}{\eta}\bar{W}_I \\
\beta_2 &= \bar{W}\tilde{Q}\left[(1-\delta)\rho_\theta - \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right) - \frac{1-\delta}{\eta}2K\tilde{Q}\right] \\
&\quad + k\tilde{Q}\left[\bar{A} + (1-\delta)\tilde{Q} - \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}\right)\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2\right] \\
\beta_3 &= \bar{m}(1-\delta)\tilde{Q}\frac{1}{2}\sigma_\theta^2 \\
\beta_4 &= -2(1-\delta)^2 K\tilde{Q}^2\rho_\theta\sigma_\theta^2 + k\frac{1}{2}\tilde{Q}^2(1-\delta)\sigma_\theta^2 \\
\beta_5 &= -(1-\delta)\bar{m}\sigma_\theta^2 \\
\beta_6 &= 4(1-\delta)^2 K\tilde{Q}\rho_\theta\sigma_\theta^2 - k\tilde{Q}(1-\delta)\sigma_\theta^2
\end{aligned}$$

Setting $Z_2 = 0$ in (22) implies:

$$\lambda_0 + \lambda_1 v + \lambda_2 v^2 + \lambda_3 v^4 + \lambda_4 \left[\tilde{Q}V\right] + \lambda_5 \left[\tilde{Q}V\right]^2 + \lambda_6 \left[\tilde{Q}V\right] v^2 = 0 \qquad (25)$$

where:

$$\begin{aligned}
\lambda_0 &= \bar{m}\tilde{Q}\frac{m-n}{\eta}\bar{W}_I \\
\lambda_1 &= \bar{m}\tilde{Q}\left[(1-\delta)\rho_\theta - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q}) - 2\frac{1-\delta}{\eta}K\tilde{Q}\right] \\
&\quad + \left(k\tilde{Q} + \bar{W}\right)\tilde{Q}\frac{m-n}{\eta}\bar{W}_I \\
\lambda_2 &= \bar{W}\tilde{Q}\left[(1-\delta)\frac{1}{2}\rho_\theta^2 - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\frac{1}{2} - \frac{1-\delta}{\eta}2K\tilde{Q}\right] \\
&\quad + k\tilde{Q}^2\left[(1-\delta)\rho_\theta - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q}) - 2\frac{1-\delta}{\eta}K\tilde{Q}\right] \\
&\quad + k\tilde{Q}\frac{1}{2}\left[\bar{A} + (1-\delta)\tilde{Q} - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2\right] \\
\lambda_3 &= -(1-\delta)^2 K\tilde{Q}^2\rho_\theta^2\sigma_\theta^2 + \frac{1}{4}k\tilde{Q}^2(1-\delta)\sigma_\theta^2 \\
\lambda_4 &= -\bar{W}\left[(1-\delta)\rho_\theta^2 - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q}) - \frac{1-\delta}{\eta}2K\tilde{Q}\right] \\
&\quad - k\left[\bar{A} + (1-\delta)\tilde{Q} - \frac{1}{\eta}(\nu - W + \delta K\tilde{Q})\tilde{Q} - \frac{1-\delta}{\eta}K\tilde{Q}^2\right] \\
\lambda_5 &= -4(1-\delta)^2 K\rho_\theta^2\sigma_\theta^2 + k(1-\delta)\sigma_\theta^2 \\
\lambda_6 &= 4(1-\delta)^2 K\tilde{Q}\rho_\theta^2\sigma_\theta^2 - k(1-\delta)\tilde{Q}\sigma_\theta^2
\end{aligned}$$

Substituting (23) in (24) we get:

$$h_1 + h_2 v + h_3 v^2 + h_4 v^3 = 0 \qquad (26)$$

where

$$\begin{aligned}
h_1 &= \beta_1 + \beta_5\alpha_1 \\
h_2 &= \beta_2 + \beta_6\alpha_1 \\
h_3 &= \beta_3 + \beta_5\alpha_2 \\
h_4 &= \beta_4 + \beta_6\alpha_2
\end{aligned}$$

Substituting (23) in (25), we have:

$$g_1 + g_2 v + g_3 v^2 + g_4 v^4 = 0 \tag{27}$$

where

$$\begin{aligned} g_1 &= \lambda_0 + \lambda_4 \alpha_1 + \lambda_5 \alpha_1^2 \\ g_2 &= \lambda_1 \\ g_3 &= \lambda_2 + \lambda_4 \alpha_2 + 2\lambda_5 \alpha_1 \alpha_2 + \lambda_6 \alpha_1 \\ g_4 &= \lambda_3 + \lambda_5 \alpha_2^2 + \lambda_6 \alpha_2 \end{aligned}$$

(26)-(27) can be represented graphically in a $\tilde{Q}$ - v space, with the intersections of the schedules giving the various equilibria. Given $\tilde{Q}$ and v, V follows from (23).

I.D Solution with switching equilibria

We now consider equilibria that allow for the possibility of switching between high risk and low risk states. Define state 1 as the low risk state and let p_1 be the probability that next period we will be in the low risk state when this period we are in the low risk state. Similarly, let p_2 be the probability that next period we are in the high risk state if this period we are in the high risk state. The equity prices in the low and high risk states are state-specific functions of θ_t of the same form as (10):

$$\begin{aligned} Q_t^{\text{low risk}} &= \tilde{Q}_1 \exp\left[-v_1\theta_t - V_1\theta_t^2\right] \\ Q_t^{\text{high risk}} &= \tilde{Q}_2 \exp\left[-v_2\theta_t - V_2\theta_t^2\right] \end{aligned}$$

Using the same approximations as above, we write:

$$\begin{aligned} Q_{t+1}^{\text{low risk}} &= \tilde{Q}_1 \left(1 - v_1\rho_\theta\theta_t - v_1\epsilon_{t+1}^\theta + \omega_1\left(\rho_\theta^2\theta_t^2 + 2\rho_\theta\theta_t\epsilon_{t+1}^\theta + \sigma_\theta^2\right)\right) \\ Q_{t+1}^{\text{high risk}} &= \tilde{Q}_2 \left(1 - v_2\rho_\theta\theta_t - v_2\epsilon_{t+1}^\theta + \omega_2\left(\rho_\theta^2\theta_t^2 + 2\rho_\theta\theta_t\epsilon_{t+1}^\theta + \sigma_\theta^2\right)\right) \end{aligned}$$

where $\omega_1 = -V_1 + 0.5v_1^2$ and $\omega_2 = -V_2 + 0.5v_2^2$. We need to compute the expectation and variance of the equity price in period $t+1$ from the point of view of period t, as well as the product of the equity prices and the interest rate.

We start be computing the expectation of the equity price in period $t+1$ from the point of view of period t. The expectation of Q_{t+1}, conditional on being in a low risk state in $t+1$, is

$$E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) = a_{1,low} + a_{2,low}\theta_t + a_{3,low}\theta_t^2$$

where

$$\begin{aligned} a_{1,low} &= \tilde{Q}_1\left(1+\omega_1\sigma_\theta^2\right) \\ a_{2,low} &= -\tilde{Q}_1 \upsilon_1 \rho_\theta \\ a_{3,low} &= \tilde{Q}_1 \omega_1 \rho_\theta^2 \end{aligned}$$

Analogously, the expectation of Q_{t+1}, conditional on being in a high risk state in $t+1$, is

$$E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) = a_{1,high} + a_{2,high}\theta_t + a_{3,high}\theta_t^2$$

where

$$\begin{aligned} a_{1,high} &= \tilde{Q}_2\left(1+\omega_2\sigma_\theta^2\right) \\ a_{2,high} &= -\tilde{Q}_2 \upsilon_2 \rho_\theta \\ a_{3,high} &= \tilde{Q}_2 \omega_2 \rho_\theta^2 \end{aligned}$$

If the economy is in the low risk state at time t, the expectation of Q_{t+1} is:

$$\begin{aligned} E_t\left(Q_{t+1}|t \text{ } is \text{ } low\right) &= p_1 E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) + \left(1-p_1\right) E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) \\ &= d_{1,low} + d_{2,low}\theta_t + d_{3,low}\theta_t^2 \qquad (28) \end{aligned}$$

where

$$\begin{aligned} d_{1,low} &= p_1 a_{1,low} + \left(1-p_1\right) a_{1,high} \\ d_{2,low} &= p_1 a_{2,low} + \left(1-p_1\right) a_{2,high} \\ d_{3,low} &= p_1 a_{3,low} + \left(1-p_1\right) a_{3,high} \end{aligned}$$

Similarly, if the economy is in the high risk state at time t, the expectation of Q_{t+1} is:

$$\begin{aligned} E_t\left(Q_{t+1}|t \text{ } is \text{ } high\right) &= \left(1-p_2\right) E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) + p_2 E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) \\ &= d_{1,high} + d_{2,high}\theta_t + d_{3,high}\theta_t^2 \qquad (29) \end{aligned}$$

where

$$
\begin{aligned}
d_{1,high} &= (1-p_2)\, a_{1,low} + p_2 a_{1,high} \\
d_{2,high} &= (1-p_2)\, a_{2,low} + p_2 a_{2,high} \\
d_{3,high} &= (1-p_2)\, a_{3,low} + p_2 a_{3,high}
\end{aligned}
$$

Next, we compute the variance of the equity price in period $t+1$ from the point of view of period t. The variance of Q_{t+1} is

$$var\left(Q_{t+1}\right) = E_t Q_{t+1}^2 - \left(E_t Q_{t+1}\right)^2$$

In a state $i = 1, 2$ we have:

$$Q_{i,t+1} = \tilde{Q}_i \left(1 + \omega_i \sigma_\theta^2 - v_i \rho_\theta \theta_t + \omega_i \rho_\theta^2 \theta_t^2 + \left(-v_i + 2\omega_i \rho_\theta \theta_t\right) \epsilon_{t+1}\right)$$

It follows that in a specific $t+1$ state (dropping terms in θ_t^3 and above):

$$
\begin{aligned}
E_t\left(Q_{t+1}^2 | t+1 \; is \; i\right) &= \tilde{Q}_i^2 \left[\left(1+\omega_i\sigma_\theta^2\right)^2 + v_i^2 \rho_\theta^2 \theta_t^2 + 2\left(1+\omega_i\sigma_\theta^2\right)\omega_i \rho_\theta^2 \theta_t^2 - 2\left(1+\omega_i\sigma_\theta^2\right) v_i \rho_\theta \theta_t\right] \\
&\quad + \tilde{Q}_i^2 \left(-v_i + 2\omega_i \rho_\theta \theta_t\right)^2 \sigma_\theta^2
\end{aligned}
$$

We therefore write:

$$E_t\left(Q_{t+1}^2 | t+1 \; is \; low\right) = b_{1,low} + b_{2,low}\theta_t + b_{3,low}\theta_t^2$$

where:

$$
\begin{aligned}
b_{1,low} &= \tilde{Q}_1^2 \left[\left(1+\omega_1\sigma_\theta^2\right)^2 + v_1^2 \sigma_\theta^2\right] \\
b_{2,low} &= -\tilde{Q}_1^2 \rho_\theta \left(4 v_1 \omega_1 \sigma_\theta^2 + 2\left(1+\omega_1\sigma_\theta^2\right) v_1\right) \\
b_{3,low} &= \tilde{Q}_1^2 \rho_\theta^2 \left(v_1^2 + 2\left(1+\omega_1\sigma_\theta^2\right)\omega_1 + 4\omega_1^2 \sigma_\theta^2\right)
\end{aligned}
$$

Similarly:

$$E_t\left(Q_{t+1}^2 | t+1 \; is \; high\right) = b_{1,high} + b_{2,high}\theta_t + b_{3,high}\theta_t^2$$

where:

$$
\begin{aligned}
b_{1,high} &= \tilde{Q}_2^2 \left[\left(1+\omega_2\sigma_\theta^2\right)^2 + v_2^2 \sigma_\theta^2\right] \\
b_{2,high} &= -\tilde{Q}_2^2 \rho_\theta \left(4 v_2 \omega_2 \sigma_\theta^2 + 2\left(1+\omega_2\sigma_\theta^2\right) v_2\right) \\
b_{3,high} &= \tilde{Q}_2^2 \rho_\theta^2 \left(v_2^2 + 2\left(1+\omega_2\sigma_\theta^2\right)\omega_2 + 4\omega_2^2 \sigma_\theta^2\right)
\end{aligned}
$$

Consider that the economy is in the low risk state at time t. We drop terms where θ_t is of cubic or higher power. The expectation of Q_{t+1}^2 is then:

$$\begin{aligned} E_t\left(Q_{t+1}^2|t \text{ is low}\right) &= p_1 E_{t+1}\left(Q_{t+1}^2|t+1 \text{ is low}\right) + (1-p_1)\, E_{t+1}\left(Q_{t+1}^2|t+1 \text{ is high}\right) \\ &= c_{1,low} + c_{2,low}\theta_t + c_{3,low}\theta_t^2 \end{aligned}$$

where

$$\begin{aligned} c_{1,low} &= p_1 b_{1,low} + (1-p_1)\, b_{1,high} \\ c_{2,low} &= p_1 b_{2,low} + (1-p_1)\, b_{2,high} \\ c_{3,low} &= p_1 b_{3,low} + (1-p_1)\, b_{3,high} \end{aligned}$$

Using (28) we write:

$$\left(E_t\left(Q_{t+1}|t \text{ is low}\right)\right)^2 = d_{1,low}^2 + 2d_{1,low}d_{2,low}\theta_t + \left(d_{2,low}^2 + 2d_{1,low}d_{3,low}\right)\theta_t^2$$

The variance of the asset price in period $t+1$ from the point of view of period t, when the economy is in the low risk state in period t, is thus:

$$\begin{aligned} var\left(Q_{t+1}|t \text{ is low}\right) &= \left(c_{1,low} - d_{1,low}^2\right) + \left(c_{2,low} - 2d_{1,low}d_{2,low}\right)\theta_t \\ &\quad + \left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right)\theta_t^2 \end{aligned} \tag{30}$$

Similarly, the variance of the asset price in period $t+1$ from the point of view of period t, when the economy is in the high risk state in period t, is:

$$\begin{aligned} var\left(Q_{t+1}|t \text{ is high}\right) &= c_{1,high} - d_{1,high}^2 + (c_{2,high} - 2d_{1,high}d_{2,high})\theta_t \\ &\quad + (c_{3,high} - d_{2,high}^2 - 2d_{1,high}d_{3,high})\theta_t^2 \end{aligned} \tag{31}$$

where:

$$\begin{aligned} c_{1,high} &= (1-p_2)\, b_{1,low} + p_2 b_{1,high} \\ c_{2,high} &= (1-p_2)\, b_{2,low} + p_2 b_{2,high} \\ c_{3,high} &= (1-p_2)\, b_{3,low} + p_2 b_{3,high} \end{aligned}$$

We now compute the product of the interest rate and the equity price in period $t+1$ from the point of view of period t. The interest rate in any state is given by (6):

$$R_{i,t+1} = \frac{1}{\eta}\left(\nu - W + \delta K\tilde{Q}_i + \bar{W}_I\,(m-n)\,\theta_t + (1-\delta)KQ_{i,t}\right)$$

If the economy is in the low risk state at time t, we get:

$$(Q_t R_{t+1}|t\ is\ low) = e_{1,low} + e_{2,low}\theta_t + e_{3,low}\theta_t^2 \tag{32}$$

where:

$$\begin{aligned}
e_{1,low} &= \frac{1}{\eta}\left[\nu - W + K\tilde{Q}_1\right]\tilde{Q}_1 \\
e_{2,low} &= -\frac{1}{\eta}\left[\left(\nu - W + \delta K\tilde{Q}_1\right) + (1-\delta)\,2K\tilde{Q}_1\right]\tilde{Q}_1 v_1 + \frac{m-n}{\eta}\bar{W}_I\tilde{Q}_1 \\
e_{3,low} &= \frac{1}{\eta}\left[\left(\nu - W + \delta K\tilde{Q}_1\right)\omega_1 + (1-\delta)\,2K\tilde{Q}_1\left(v_1^2 - V_1\right)\right]\tilde{Q}_1 - \frac{m-n}{\eta}\bar{W}_I\tilde{Q}_1 v_1
\end{aligned}$$

Similarly, if the economy is in the high risk state at time t, we have:

$$(Q_t R_{t+1}|t\ is\ high) = e_{1,high} + e_{2,high}\theta_t + e_{3,high}\theta_t^2 \tag{33}$$

where:

$$\begin{aligned}
e_{1,high} &= \frac{1}{\eta}\left[\nu - W + K\tilde{Q}_2\right]\tilde{Q}_2 \\
e_{2,high} &= -\frac{1}{\eta}\left[\left(\nu - W + \delta K\tilde{Q}_2\right) + (1-\delta)\,2K\tilde{Q}_2\right]\tilde{Q}_2 v_2 + \frac{m-n}{\eta}\bar{W}_I\tilde{Q}_2 \\
e_{3,high} &= \frac{1}{\eta}\left[\left(\nu - W + \delta K\tilde{Q}_2\right)\omega_2 + (1-\delta)\,2K\tilde{Q}_2\left(v_2^2 - V_2\right)\right]\tilde{Q}_2 - \frac{m-n}{\eta}\bar{W}_I\tilde{Q}_2 v_2
\end{aligned}$$

We can now compute the equity market clearing condition. Consider that the economy is in the low risk state at time t. Using (28), (30) and (32), (9) is written as:

$$X_{t=low}\left[\bar{W} - \left(\bar{m} + k\tilde{Q}_1 v_1\right)\theta_t + k\tilde{Q}_1\omega_1\theta_t^2\right] = Y_{t=low} \tag{34}$$

where

$$\begin{aligned}
X_{t=low} &= \bar{A} + (1-\delta)\,d_{1,low} - e_{1,low} \\
&\quad + \left[(1-\delta)\,d_{2,low} - e_{2,low}\right]\theta_t \\
&\quad + \left[(1-\delta)\,d_{3,low} - e_{3,low}\right]\theta_t^2
\end{aligned}$$

and

$$\begin{aligned}
Y_{t=low} &= K\bar{A}^2\sigma_a^2 + (1-\delta)^2 K\left(c_{1,low} - d_{1,low}^2\right) \\
&\quad + (1-\delta)^2 K\left(c_{2,low} - 2d_{1,low}d_{2,low}\right)\theta_t \\
&\quad + (1-\delta)^2 K\left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right)\theta_t^2
\end{aligned}$$

We focus on the terms in (34) that are constant, proportional to θ_t, and proportional to θ_t^2. The constant terms in (34) are:

$$\begin{aligned} & \bar{W}\left[\bar{A}+(1-\delta)\,d_{1,low}-e_{1,low}\right] \\ = \; & K\bar{A}^2\sigma_a^2+(1-\delta)^2 K\left(c_{1,low}-d_{1,low}^2\right) \end{aligned} \tag{35}$$

The linear terms in (34) are:

$$\begin{aligned} & \bar{W}\left[(1-\delta)\,d_{2,low}-e_{2,low}\right] \\ & -\left[\bar{m}+k\tilde{Q}_1 v_1\right]\left[\bar{A}+(1-\delta)\,d_{1,low}-e_{1,low}\right] \\ = \; & (1-\delta)^2 K\left(c_{2,low}-2d_{1,low}d_{2,low}\right) \end{aligned} \tag{36}$$

The quadratic terms in (34) are:

$$\begin{aligned} & \bar{W}\left[(1-\delta)\,d_{3,low}-e_{3,low}\right] \\ & -\left[\bar{m}+k\tilde{Q}_1 v_1\right]\left[(1-\delta)\,d_{2,low}-e_{2,low}\right] \\ & +k\tilde{Q}_1\omega_1\left[\bar{A}+(1-\delta)\,d_{1,low}-e_{1,low}\right] \\ = \; & (1-\delta)^2 K\left(c_{3,low}-d_{2,low}^2-2d_{1,low}d_{3,low}\right) \end{aligned} \tag{37}$$

If the economy is in the high risk state at time t, we use (29), (31) and (33), to write (9) as:

$$X_{t=high}\left[\bar{W}-\left(\bar{m}+k\tilde{Q}_2 v_2\right)\theta_t+k\tilde{Q}_2\omega_2\theta_t^2\right]=Y_{t=high} \tag{38}$$

where

$$\begin{aligned} X_{t=high} \; = \; & \bar{A}+(1-\delta)\,d_{1,high}-e_{1,high} \\ & +\left[(1-\delta)\,d_{2,high}-e_{2,high}\right]\theta_t \\ & +\left[(1-\delta)\,d_{3,high}-e_{3,high}\right]\theta_t^2 \end{aligned}$$

and

$$\begin{aligned} Y_{t=high} \; = \; & K\bar{A}^2\sigma_a^2+(1-\delta)^2 K\left(c_{1,high}-d_{1,high}^2\right) \\ & +(1-\delta)^2 K\left(c_{2,high}-2d_{1,high}d_{2,high}\right)\theta_t \\ & +(1-\delta)^2 K\left(c_{3,high}-d_{2,high}^2-2d_{1,high}d_{3,high}\right)\theta_t^2 \end{aligned}$$

We focus on the terms in (38) that are constant, proportional to θ_t, and proportional to θ_t^2. The constant terms in (38) are:

$$\begin{aligned} & \bar{W}\left[\bar{A}+(1-\delta)\, d_{1,high}-e_{1,high}\right] \\ = \;& K\bar{A}^2\sigma_a^2+(1-\delta)^2 K\left(c_{1,high}-d_{1,high}^2\right) \end{aligned} \tag{39}$$

The linear terms in (38) are:

$$\begin{aligned} & \bar{W}\left[(1-\delta)\, d_{2,high}-e_{2,high}\right] \\ & -\left[\bar{m}+k\tilde{Q}_2 v_2\right]\left[\bar{A}+(1-\delta)\, d_{1,high}-e_{1,high}\right] \\ = \;& (1-\delta)^2 K\left(c_{2,high}-2d_{1,high}d_{2,high}\right) \end{aligned} \tag{40}$$

The quadratic terms in (38) are:

$$\begin{aligned} & \bar{W}\left[(1-\delta)\, d_{3,high}-e_{3,high}\right] \\ & -\left[\bar{m}+k\tilde{Q}_2 v_2\right]\left[(1-\delta)\, d_{2,high}-e_{2,high}\right] \\ & +k\tilde{Q}_2\omega_2\left[\bar{A}+(1-\delta)\, d_{1,high}-e_{1,high}\right] \\ = \;& (1-\delta)^2 K\left(c_{3,high}-d_{2,high}^2-2d_{1,high}d_{3,high}\right) \end{aligned} \tag{41}$$

The system of six equations (35)-(37) and (39)-(41) can be solved numerically for the six unknowns $\tilde{Q}_1$, v_1, V_1, $\tilde{Q}_2$, v_2, V_2.

I.E Computation of the Volatility of Risk

Equity price risk is the standard deviation of Q_{t+1}/Q_t. Using the expression (30) we write the standard deviation of Q_{t+1}/Q_t in the low risk state (at time t) as:

$$\begin{aligned} risk_t\,(t\ is\ low) &= \frac{\sqrt{var\,(Q_{t+1}|t\ is\ low)}}{\tilde{Q}_1 \exp\left[-v_1\theta_t - V_1\theta_t^2\right]} \\ &= \frac{\exp\left[v_1\theta_t + V_1\theta_t^2\right]}{\tilde{Q}_1}\sqrt{f_{1,low}+f_{2,low}\theta_t+f_{3,low}\theta_t^2} \end{aligned} \tag{42}$$

where

$$\begin{aligned} f_{1,low} &= c_{1,low}-d_{1,low}^2 \\ f_{2,low} &= c_{2,low}-2d_{1,low}d_{2,low} \\ f_{3,low} &= c_{3,low}-d_{2,low}^2-2d_{1,low}d_{3,low} \end{aligned}$$

Taking a quadratic approximation of (42), we write:

$$risk_t\,(t\ is\ low) = g_{1,low} + g_{2,low}\theta_t + g_{3,low}\theta_t^2$$

where:

$$\begin{aligned} g_{1,low} &= \frac{1}{\tilde{Q}_1}\left(f_{1,low}\right)^{0.5} \\ g_{2,low} &= \frac{1}{\tilde{Q}_1}\left[\frac{1}{2}\left(f_{1,low}\right)^{-0.5} f_{2,low} + \left(f_{1,low}\right)^{0.5} v_1\right] \\ g_{3,low} &= \frac{1}{2\tilde{Q}_1}\left[\begin{array}{c} -\frac{1}{4}\left(f_{1,low}\right)^{-1.5}\left(f_{2,low}\right)^2 + \left(f_{1,low}\right)^{-0.5} f_{3,low} \\ +\left(f_{1,low}\right)^{-0.5} f_{2,low}v_1 + \left(f_{1,low}\right)^{0.5}\left(v_1^2 + 2V_1\right) \end{array}\right] \end{aligned}$$

Conditional on being in future low states, the volatility of risk is the standard deviation of:

$$\begin{aligned} & risk_{t+1}\,(t+1\ is\ low) \\ = \; & g_{1,low} + g_{2,low}\left(\rho_\theta\theta_t + \epsilon_{t+1}^\theta\right) + g_{3,low}\left(\rho_\theta\theta_t + \epsilon_{t+1}^\theta\right)^2 \\ = \; & g_{1,low} + g_{3,low}\sigma_\theta^2 + g_{2,low}\rho_\theta\theta_t + g_{3,low}\left(\rho_\theta\theta_t\right)^2 + \left(g_{2,low} + 2g_{3,low}\rho_\theta\theta_t\right)\epsilon_{t+1}^\theta \end{aligned}$$

From the point of view of period t the standard deviation of risk is then:

$$SD_t\left(risk_{t+1}\,(t+1\ is\ low)\right) = \left(g_{2,low} + 2g_{3,low}\rho_\theta\theta_t\right)\sigma_\theta \tag{43}$$

Results are analogous if we are in the high risk state at time t. The standard deviation of Q_{t+1}/Q_t in the high risk state as

$$risk_t\,(t\ is\ high) = \frac{\exp\left[v_2\theta_t + V_2\theta_t^2\right]}{\tilde{Q}_2}\sqrt{f_{1,high} + f_{2,high}\theta_t + f_{3,high}\theta_t^2}$$

where

$$\begin{aligned} f_{1,high} &= c_{1,high} - d_{1,high}^2 \\ f_{2,high} &= c_{2,high} - 2d_{1,high}d_{2,high} \\ f_{3,high} &= c_{3,high} - d_{2,high}^2 - 2d_{1,high}d_{3,high} \end{aligned}$$

Taking a quadratic approximation of this, the standard deviation of Q_{t+1}/Q_t in the high risk state becomes

$$risk_t\,(t\ is\ high) = g_{1,high} + g_{2,high}\theta_t + g_{3,high}\theta_t^2$$

were

$$g_{1,high} = \frac{1}{\tilde{Q}_2} \left(f_{1,high}\right)^{0.5}$$
$$g_{2,high} = \frac{1}{\tilde{Q}_2} \left[\frac{1}{2} \left(f_{1,high}\right)^{-0.5} f_{2,high} + \left(f_{1,high}\right)^{0.5} v_2\right]$$
$$g_{3,high} = \frac{1}{2\tilde{Q}_2} \begin{bmatrix} -\frac{1}{4} \left(f_{1,high}\right)^{-1.5} \left(f_{2,high}\right)^2 + \left(f_{1,high}\right)^{-0.5} f_{3,high} \\ + \left(f_{1,high}\right)^{-0.5} f_{2,high} v_2 + \left(f_{1,high}\right)^{0.5} \left(v_2^2 + 2V_2\right) \end{bmatrix}$$

The volatility of risk is then

$$SD_t \left(risk_{t+1} \left(t + 1 \text{ } is \text{ } high\right)\right) = \left(g_{2,high} + 2g_{3,high}\rho_\theta\theta_t\right) \sigma_\theta \tag{44}$$

II Cubic Solution

This section presents the model solution if we take cubic expansions. For brevity we focus on the case where the endowment does not entail any trees and shocks are solely redistributive and do not affect aggregate endowments.

II.A Building blocks

At the beginning of period t the newborn investors receive an endowment of $W_{I,t}$ units of the consumption good. The newborn households receive an endowment of $W_{H,t} = W - W_{I,t}$ units of the consumption good. $W_{I,t}$ is given by:

$$W_{I,t} = \bar{W}_I \exp\left[-m\theta_t - \frac{1}{2}m^2\theta_t^2 - \frac{1}{3}m^3\theta_t^3\right]$$

where

$$\theta_{t+1} = \rho_\theta\theta_t + \epsilon_{t+1}^\theta \tag{45}$$

and ϵ_{t+1}^θ follows a symmetric distribution with mean zero and variance σ_θ^2 and $\rho_\theta \in (0, 1]$. A rise in θ_t represents a redistribution away from investors towards households. A cubic approximation of endowments around $\theta_t = 0$ implies that endowments are linear in θ_t and thus have a constant variance:

$$W_{I,t} = \bar{W}_I \left(1 - m\theta_t\right)$$

The return on equity consists of the dividend yield and capital gain, with trees depreciating at a rate δ to exactly offset the endowment in trees of the newborn agents. The rate of return on equity is then:

$$R_{K,t+1} = \frac{A_{t+1} + (1-\delta)\, Q_{t+1}}{Q_t} \tag{46}$$

The dividend is an exogenous process:

$$A_{t+1} = \bar{A} \exp\left[a_{t+1} - \frac{1}{2}a_{t+1}^2 + \frac{1}{3}a_{t+1}^3\right]$$

where $a_{t+1} = \epsilon_{t+1}^a$ follows a symmetric distribution with mean 0 and variance σ_a^2 and is independent from ϵ_{t+1}^θ. A cubic approximation of A_{t+1} is $\bar{A}(1 + a_{t+1})$, which has a constant variance $\bar{A}^2\sigma_a^2$. As the dividend is iid it does not affect the equity price.

Newborn households allocate their endowments between bonds and a riskfree technology with decreasing returns to scale. Investing $K_{H,t+1}$ units of the good in the technology at time t yields and output of $Y_{t+1} = \frac{1}{\eta}\left[\nu K_{H,t+1} - \frac{1}{2}\left(K_{H,t+1}\right)^2\right]$ goods at time $t+1$. Only households can invest in the technology. Household maximize their future consumption given by $Y_{t+1} + R_{t+1}\left(W_{H,t} - K_{H,t+1}\right)$, which implies that $K_{H,t+1} = \nu - \eta R_{t+1}$. The amount invested in bonds is then:

$$W_{H,t} - K_{H,t+1} = W_{H,t} - \nu + \eta R_{t+1}$$

Bonds are in zero net supply. The clearing of the bond and equity markets requires:

$$KQ_t = \alpha_t W_{I,t} \tag{47}$$

$$0 = (1-\alpha_t)\, W_{I,t} + W_{H,t} - \nu + \eta R_{t+1} \tag{48}$$

Taking the sum of (47)-(48) yields a positive relation between the equity price and the interest rate:

$$R_{t+1} = \frac{1}{\eta}\left[\nu - W + KQ_t\right] \tag{49}$$

II.B Approximation equity market clearing condition

The optimal portfolio is given by the mean-variance relation:

$$\alpha_t = \frac{E_t R_{K,t+1} - R}{\gamma var_t(R_{K,t+1})} \tag{50}$$

Using (50) the equity market clearing condition (47) becomes:

$$\frac{E_t R_{K,t+1} - R_{t+1}}{\gamma var(R_{K,t+1})} W_{I,t} = K Q_t \tag{51}$$

(46) implies that the expected rate of return on equity and its variance are (using the fact that the dividend and asset price are uncorrelated):

$$\begin{aligned} E_t R_{K,t+1} &= \frac{\bar{A} + E_t Q_{t+1}}{Q_t} \\ var(R_{K,t+1}) &= \frac{\bar{A}^2 \sigma_a^2}{Q_t^2} + \frac{1}{Q_t^2} var(Q_{t+1}) \end{aligned}$$

(51) is then rewritten as:

$$\left(\bar{A} + E_t Q_{t+1} - Q_t R_{t+1}\right) \frac{W_{I,t}}{\gamma} = K \left(\bar{A}^2 \sigma_a^2 + var(Q_{t+1})\right) \tag{52}$$

We conjecture the log equity price is a cubic function of the financial shock:

$$Q_t = \tilde{Q} \exp\left[-v\theta_t - V\theta_t^2 - Z\theta_t^3\right] \tag{53}$$

For any coefficient $\varkappa$, a cubic expansion of $Q_t^{\varkappa}$ around $\theta_t = 0$ gives:

$$Q_t^{\varkappa} = \tilde{Q}^{\varkappa} \left[1 - v\varkappa\theta_t + \left(\frac{1}{2}v^2\varkappa^2 - V\varkappa\right)\theta_t^2 + \left(-\frac{1}{6}v^3\varkappa^3 + vV\varkappa^2 - Z\varkappa\right)\theta_t^3\right] \tag{54}$$

Using (45) we can write:

$$\begin{aligned} Q_{t+1}^{\varkappa} &= \tilde{Q}^{\varkappa}\left[1 + \left(\frac{1}{2}v^2\varkappa^2 - V\varkappa\right)\sigma_\theta^2\right] \\ &+ \tilde{Q}^{\varkappa}\left[-v\varkappa + \left(-\frac{1}{6}v^3\varkappa^3 + vV\varkappa^2 - Z\varkappa\right)3\sigma_\theta^2\right]\rho_\theta\theta_t \\ &+ \tilde{Q}^{\varkappa}\left(\frac{1}{2}v^2\varkappa^2 - V\varkappa\right)\rho_\theta^2\theta_t^2 + \tilde{Q}^{\varkappa}\left(-\frac{1}{6}v^3\varkappa^3 + vV\varkappa^2 - Z\varkappa\right)\rho_\theta^3\theta_t^3 \\ &+ \tilde{Q}^{\varkappa}\left[\begin{array}{c} -v\varkappa + \left(\frac{1}{2}v^2\varkappa^2 - V\varkappa\right)2\rho_\theta\theta_t \\ + \left(-\frac{1}{6}v^3\varkappa^3 + vV\varkappa^2 - Z\varkappa\right)3\rho_\theta^2\theta_t^2 \end{array}\right]\epsilon_{t+1}^\theta \end{aligned} \tag{55}$$

where we used the continuous time approximations $(\epsilon_{t+1}^\theta)^2 = \sigma_\theta^2$ and $(\epsilon_{t+1}^\theta)^3 = 0$. The expectation and variance of the equity price next period are then:

$$E_t Q_{t+1} = \tilde{Q}\left[1 + \left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2\right] \tag{56}$$

$$+\tilde{Q}\left[-v+\left(-\frac{1}{6}v^3+vV-Z\right)3\sigma_\theta^2\right]\rho_\theta\theta_t$$
$$+\tilde{Q}\left(\frac{1}{2}v^2-V\right)\rho_\theta^2\theta_t^2+\tilde{Q}\left(-\frac{1}{6}v^3+vV-Z\right)\rho_\theta^3\theta_t^3$$
$$var(Q_{t+1}) = \tilde{Q}^2\left[\begin{array}{c} -v+2\left(\frac{1}{2}v^2-V\right)\rho_\theta\theta_t \\ +\left(-\frac{1}{6}v^3+vV-Z\right)3\rho_\theta^2\theta_t^2 \end{array}\right]^2\sigma_\theta^2 \quad (57)$$

Using (49) and (54), and omitting terms in θ_t^4 or above, we write:

$$Q_tR_{t+1} = \frac{1}{\eta}\left[\nu-W\right]\tilde{Q}\left[1-v\theta_t+\left(\frac{1}{2}v^2-V\right)\theta_t^2+\left(-\frac{1}{6}v^3+vV-Z\right)\theta_t^3\right]$$
$$+\frac{1}{\eta}K\tilde{Q}^2\left[1-2v\theta_t+2\left(v^2-V\right)\theta_t^2+\left(-\frac{4}{3}v^3+4vV-2Z\right)\theta_t^3\right] \quad (58)$$

Combining these approximations (52) is rewritten as:

$$X\left[\bar{W}-\bar{m}\theta_t\right]=Y \quad (59)$$

where:

$$\bar{W}=\frac{\bar{W}_I}{\gamma} \quad ; \quad \bar{m}=\frac{\bar{W}_I}{\gamma}m$$

and:

$$X = \bar{A}+\tilde{Q}\left[1+\left(\frac{1}{2}v^2-V\right)\sigma_\theta^2\right]+\tilde{Q}\left[-v+\left(-\frac{1}{6}v^3+vV-Z\right)3\sigma_\theta^2\right]\rho_\theta\theta_t$$
$$+\tilde{Q}\left(\frac{1}{2}v^2-V\right)\rho_\theta^2\theta_t^2+\tilde{Q}\left(-\frac{1}{6}v^3+vV-Z\right)\rho_\theta^3\theta_t^3$$
$$-\frac{1}{\eta}\left[\nu-W\right]\tilde{Q}\left[1-v\theta_t+\left(\frac{1}{2}v^2-V\right)\theta_t^2+\left(-\frac{1}{6}v^3+vV-Z\right)\theta_t^3\right]$$
$$-\frac{1}{\eta}K\tilde{Q}^2\left[1-2v\theta_t+2\left(v^2-V\right)\theta_t^2+\left(-\frac{4}{3}v^3+4vV-2Z\right)\theta_t^3\right]$$

and:

$$Y=K\bar{A}^2\sigma_a^2+K\tilde{Q}^2\left[\begin{array}{c} -v+2\left(\frac{1}{2}v^2-V\right)\rho_\theta\theta_t \\ +\left(-\frac{1}{6}v^3+vV-Z\right)3\rho_\theta^2\theta_t^2 \end{array}\right]^2\sigma_\theta^2$$

II.C Solution without switching

We restrict ourselves to the constant, linear, and quadratic components of (59) in terms of θ_t:

$$Z_0+Z_1\theta_t+Z_2\theta_t^2+Z_3\theta_t^3=0$$

where the coefficient on the constant is:

$$\begin{aligned} Z_0 &= -K\bar{A}^2\sigma_a^2 - K\tilde{Q}^2 v^2 \sigma_\theta^2 \\ &\quad + \bar{W}\left[\bar{A} + \tilde{Q}\left[1 + \left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2\right] - \frac{1}{\eta}\left[\nu - W\right]\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right] \end{aligned} \tag{60}$$

the coefficient on θ_t is:

$$\begin{aligned} Z_1 &= -\bar{W}\tilde{Q}\left[\left[v - \left(-\frac{1}{6}v^3 + vV - Z\right)3\sigma_\theta^2\right]\rho_\theta - \frac{1}{\eta}\left[\nu - W\right]v - 2\frac{1}{\eta}K\tilde{Q}v\right] \\ &\quad - \bar{m}\left[\bar{A} + \tilde{Q}\left[1 + \left(\frac{1}{2}v^2 - V\right)\sigma_\theta^2\right] - \frac{1}{\eta}\left[\nu - W\right]\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right] \\ &\quad + 4K\tilde{Q}^2 v\left(\frac{1}{2}v^2 - V\right)\rho_\theta\sigma_\theta^2 \end{aligned} \tag{61}$$

the coefficient on θ_t^2 is:

$$\begin{aligned} Z_2 &= \bar{W}\tilde{Q}\left[\left(\frac{1}{2}v^2 - V\right)\rho_\theta^2 - \frac{1}{\eta}\left[\nu - W\right]\left(\frac{1}{2}v^2 - V\right) - \frac{1}{\eta}K\tilde{Q}2\left(v^2 - V\right)\right] \\ &\quad + \bar{m}\tilde{Q}\left[\left[v - \left(-\frac{1}{6}v^3 + vV - Z\right)3\sigma_\theta^2\right]\rho_\theta - \frac{1}{\eta}\left[\nu - W\right]v - 2\frac{1}{\eta}K\tilde{Q}v\right] \\ &\quad - 4K\tilde{Q}^2\left(\frac{1}{2}v^2 - V\right)^2\rho_\theta^2\sigma_\theta^2 + 6K\tilde{Q}^2 v\left(-\frac{1}{6}v^3 + vV - Z\right)\rho_\theta^2\sigma_\theta^2 \end{aligned} \tag{62}$$

and the coefficient on θ_t^3 is:

$$\begin{aligned} Z_3 &= \bar{W}\tilde{Q}\left[\begin{array}{c} \left(-\frac{1}{6}v^3 + vV - Z\right)\rho_\theta^3 - \frac{1}{\eta}\left[\nu - W\right]\left(-\frac{1}{6}v^3 + vV - Z\right) \\ -\frac{1}{\eta}2K\tilde{Q}\left(-\frac{2}{3}v^3 + 2vV - Z\right) \end{array}\right] \\ &\quad - \bar{m}\tilde{Q}\left[\left(\frac{1}{2}v^2 - V\right)\rho_\theta^2 - \frac{1}{\eta}\left[\nu - W\right]\left(\frac{1}{2}v^2 - V\right) - \frac{1}{\eta}K\tilde{Q}2\left(v^2 - V\right)\right] \\ &\quad - 12K\tilde{Q}^2\left(\frac{1}{2}v^2 - V\right)\left(-\frac{1}{6}v^3 + vV - Z\right)\rho_\theta^3\sigma_\theta^2 \end{aligned} \tag{63}$$

The method of undetermined coefficients implies $Z_0 = Z_1 = Z_2 = Z_3 = 0$. Setting $Z_0 = 0$ in (60) implies:

$$\tilde{Q}V = \alpha_1 + \alpha_2 v^2 \tag{64}$$

where:

$$\alpha_1 = \frac{1}{\sigma_\theta^2}\left[\bar{A} + \tilde{Q} - \frac{1}{\eta}(\nu - W)\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right] - \frac{K\bar{A}^2\sigma_a^2}{\bar{W}\sigma_\theta^2}$$

$$\alpha_2 = \frac{1}{2}\tilde{Q} - \frac{\tilde{Q}^2 K}{\bar{W}}$$

Setting $Z_1 = 0$ in (61) implies:

$$\tilde{Q}Z = \beta_1 + \beta_2 v + \beta_3 v^2 + \beta_4 v^3 + \beta_5 \left[\tilde{Q}V\right] + \beta_6 \left[\tilde{Q}V\right] v \tag{65}$$

where:

$$\begin{aligned}
\beta_1 &= -\frac{\bar{m}}{3\bar{W}\sigma_\theta^2\rho_\theta}\left[\bar{A} + \tilde{Q} - \frac{1}{\eta}\left[\nu - W\right]\tilde{Q} - \frac{1}{\eta}K\tilde{Q}^2\right] \\
\beta_2 &= \frac{1}{3\sigma_\theta^2\rho_\theta}\tilde{Q}\left[-\rho_\theta + \frac{1}{\eta}\left[\nu - W\right] + 2\frac{1}{\eta}K\tilde{Q}\right] \\
\beta_3 &= -\frac{\bar{m}\tilde{Q}}{6\bar{W}\rho_\theta} \\
\beta_4 &= -\frac{\tilde{Q}}{6} + \frac{2K\tilde{Q}^2}{3\bar{W}} \\
\beta_5 &= \frac{\bar{m}}{3\bar{W}\rho_\theta} \\
\beta_6 &= 1 - \frac{4K\tilde{Q}}{3\bar{W}}
\end{aligned}$$

Setting $Z_2 = 0$ in (62) implies:

$$\begin{aligned}
& \lambda_1 v + \lambda_2 v^2 + \lambda_3 v^3 + \lambda_4 v^4 \\
& +\lambda_5 \left[\tilde{Q}V\right] + \lambda_6 \left[\tilde{Q}V\right]^2 + \lambda_7 \left[\tilde{Q}V\right] v + \lambda_8 \left[\tilde{Q}V\right] v^2 \\
& +\lambda_9 \left[\tilde{Q}Z\right] + \lambda_{10} \left[\tilde{Q}Z\right] v \\
= \; & 0
\end{aligned} \tag{66}$$

where:

$$\begin{aligned}
\lambda_1 &= \bar{m}\tilde{Q}\left[\rho_\theta - \frac{1}{\eta}\left[\nu - W\right] - 2\frac{1}{\eta}K\tilde{Q}\right] \\
\lambda_2 &= \bar{W}\tilde{Q}\left[\frac{1}{2}\rho_\theta^2 - \frac{1}{\eta}\left[\nu - W\right]\frac{1}{2} - \frac{1}{\eta}2K\tilde{Q}\right] \\
\lambda_3 &= \bar{m}\tilde{Q}\frac{1}{2}\sigma_\theta^2\rho_\theta \\
\lambda_4 &= -2K\tilde{Q}^2\rho_\theta^2\sigma_\theta^2 \\
\lambda_5 &= \bar{W}\left[-\rho_\theta^2 + \frac{1}{\eta}\left[\nu - W\right] + \frac{1}{\eta}2K\tilde{Q}\right] \\
\lambda_6 &= -4K\rho_\theta^2\sigma_\theta^2
\end{aligned}$$

$$
\begin{aligned}
\lambda_7 &= -3\bar{m}\sigma_\theta^2\rho_\theta \\
\lambda_8 &= 10K\tilde{Q}\rho_\theta^2\sigma_\theta^2 \\
\lambda_9 &= 3\bar{m}\sigma_\theta^2\rho_\theta \\
\lambda_{10} &= -6K\tilde{Q}\rho_\theta^2\sigma_\theta^2
\end{aligned}
$$

Setting $Z_3 = 0$ in (63) implies:

$$
\begin{aligned}
& \kappa_1 v^2 + \kappa_2 v^3 + \kappa_3 v^5 \\
& +\kappa_4 \left[\tilde{Q}V\right] + \kappa_5 \left[\tilde{Q}V\right] v + \kappa_6 \left[\tilde{Q}V\right]^2 v + \kappa_7 \left[\tilde{Q}V\right] v^3 \\
& +\kappa_8 \left[\tilde{Q}Z\right] + \kappa_9 \left[\tilde{Q}Z\right] v^2 + \kappa_{10} \left[\tilde{Q}V\right] \left[\tilde{Q}Z\right] \\
= & \; 0
\end{aligned}
\tag{67}
$$

where:

$$
\begin{aligned}
\kappa_1 &= -\bar{m}\tilde{Q}\left[\frac{1}{2}\rho_\theta^2 - \frac{1}{\eta}\left[\nu - W\right]\frac{1}{2} - \frac{1}{\eta}2K\tilde{Q}\right] \\
\kappa_2 &= \bar{W}\tilde{Q}\left[-\frac{1}{6}\rho_\theta^3 + \frac{1}{\eta}\left[\nu - W\right]\frac{1}{6} + \frac{1}{\eta}\frac{4}{3}K\tilde{Q}\right] \\
\kappa_3 &= K\tilde{Q}^2\rho_\theta^3\sigma_\theta^2 \\
\kappa_4 &= \bar{m}\left[\rho_\theta^2 - \frac{1}{\eta}\left[\nu - W\right] - \frac{1}{\eta}2K\tilde{Q}\right] \\
\kappa_5 &= \bar{W}\left[\rho_\theta^3 - \frac{1}{\eta}\left[\nu - W\right] - \frac{1}{\eta}4K\tilde{Q}\right] \\
\kappa_6 &= 12K\rho_\theta^3\sigma_\theta^2 \\
\kappa_7 &= -8K\tilde{Q}\rho_\theta^3\sigma_\theta^2 \\
\kappa_8 &= \bar{W}\left[-\rho_\theta^3 + \frac{1}{\eta}\left[\nu - W\right] + \frac{1}{\eta}2K\tilde{Q}\right] \\
\kappa_9 &= 6K\tilde{Q}\rho_\theta^3\sigma_\theta^2 \\
\kappa_{10} &= -12K\rho_\theta^3\sigma_\theta^2
\end{aligned}
$$

Substituting (64) and (65) in (66) we get:

$$
g_0 + g_1 v + g_2 v^2 + g_3 v^3 + g_4 v^4 = 0 \tag{68}
$$

where

$$
g_0 = \lambda_5\alpha_1 + \lambda_6\alpha_1^2 + \lambda_9\left(\beta_1 + \beta_5\alpha_1\right)
$$

$$
\begin{aligned}
g_1 &= \lambda_1 + \lambda_7 \alpha_1 + \lambda_9 (\beta_2 + \beta_6 \alpha_1) \\
&\quad + \lambda_{10} (\beta_1 + \beta_5 \alpha_1) \\
g_2 &= \lambda_2 + \lambda_5 \alpha_2 + 2\lambda_6 \alpha_1 \alpha_2 + \lambda_8 \alpha_1 \\
&\quad + \lambda_9 (\beta_3 + \beta_5 \alpha_2) + \lambda_{10} (\beta_2 + \beta_6 \alpha_1) \\
g_3 &= \lambda_3 + \lambda_7 \alpha_2 + \lambda_9 (\beta_4 + \beta_6 \alpha_2) + \lambda_{10} (\beta_3 + \beta_5 \alpha_2) \\
g_4 &= \lambda_4 + \lambda_6 \alpha_2^2 + \lambda_8 \alpha_2 + \lambda_{10} (\beta_4 + \beta_6 \alpha_2)
\end{aligned}
$$

Substituting (64) and (65) in (67) we get:

$$
h_0 + h_1 v + h_2 v^2 + h_3 v^3 + h_4 v^4 + h_5 v^5 = 0 \tag{69}
$$

where:

$$
\begin{aligned}
h_0 &= \kappa_4 \alpha_1 + \kappa_8 (\beta_1 + \beta_5 \alpha_1) + +\kappa_{10} \alpha_1 (\beta_1 + \beta_5 \alpha_1) \\
h_1 &= \kappa_5 \alpha_1 + \kappa_6 \alpha_1^2 + \kappa_8 (\beta_2 + \beta_6 \alpha_1) + \kappa_{10} \alpha_1 (\beta_2 + \beta_6 \alpha_1) \\
h_2 &= \kappa_1 + \kappa_4 \alpha_2 + \kappa_8 (\beta_3 + \beta_5 \alpha_2) + \kappa_9 (\beta_1 + \beta_5 \alpha_1) \\
&\quad + \kappa_{10} \alpha_1 (\beta_3 + \beta_5 \alpha_2) + \kappa_{10} \alpha_2 (\beta_1 + \beta_5 \alpha_1) \\
h_3 &= \kappa_2 + \kappa_5 \alpha_2 + 2\kappa_6 \alpha_1 \alpha_2 + \kappa_7 \alpha_1 + \kappa_8 (\beta_4 + \beta_6 \alpha_2) \\
&\quad + \kappa_9 (\beta_2 + \beta_6 \alpha_1) + \kappa_{10} \alpha_1 (\beta_4 + \beta_6 \alpha_2) \\
&\quad + \kappa_{10} \alpha_2 (\beta_2 + \beta_6 \alpha_1) \\
h_4 &= \kappa_9 (\beta_3 + \beta_5 \alpha_2) + \kappa_{10} \alpha_2 (\beta_3 + \beta_5 \alpha_2) \\
h_5 &= \kappa_3 + \kappa_6 \alpha_2^2 v + \kappa_7 \alpha_2 + \kappa_9 (\beta_4 + \beta_6 \alpha_2) \\
&\quad + \kappa_{10} \alpha_2 (\beta_4 + \beta_6 \alpha_2)
\end{aligned}
$$

II.D Solution with switching equilibria

We now consider equilibria that allow for the possibility of switching between high risk and low risk states. Define state 1 as the low risk state and let p_1 be the probability that next period we will be in the low risk state when this period we are in the low risk state. Similarly, let p_2 be the probability that next period we are in the high risk state if this period we are in the high risk state. The equity prices in the low and high risk states are state-specific functions of θ_t of the same form as (53):

$$
Q_t^{\text{low risk}} = \tilde{Q}_1 \exp\left[-v_1 \theta_t - V_1 \theta_t^2 - Z_1 \theta_t^3\right]
$$

$$Q_t^{\text{high risk}} = \tilde{Q}_2 \exp\left[-v_2\theta_t - V_2\theta_t^2 - Z_2\theta_t^3\right]$$

Using the same approximations as above, we write that in the low risk state:

$$\begin{aligned} Q_{t+1}^{\text{low risk}} &= a_{1,low} + a_{2,low}\theta_t + a_{3,low}\theta_t^2 + a_{4,low}\theta_t^3 \\ &\quad + \left[a_{5,low} + a_{6,low}\theta_t + a_{7,low}\theta_t^2\right]\epsilon_{t+1}^{\theta} \end{aligned} \tag{70}$$

where

$$\begin{aligned} a_{1,low} &= \tilde{Q}_1\left(1 + \omega_1\sigma_\theta^2\right) \\ a_{2,low} &= \tilde{Q}_1\left[-v_1 + 3\eta_1\sigma_\theta^2\right]\rho_\theta \\ a_{3,low} &= \tilde{Q}_1\omega_1\rho_\theta^2 \\ a_{4,low} &= \tilde{Q}_1\eta_1\rho_\theta^3 \\ a_{5,low} &= -\tilde{Q}_1 v_1 \\ a_{6,low} &= 2\tilde{Q}_1\omega_1\rho_\theta \\ a_{7,low} &= 3\tilde{Q}_1\eta_1\rho_\theta^2 \end{aligned}$$

and $\omega_1 = -V_1 + \frac{1}{2}v_1^2$, $\eta_1 = -\frac{1}{6}v_1^3 + v_1 V_1 - Z_1$.

Similarly in the high risk state:

$$\begin{aligned} Q_{t+1}^{\text{high risk}} &= a_{1,high} + a_{2,high}\theta_t + a_{3,high}\theta_t^2 + a_{4,high}\theta_t^3 \\ &\quad + \left[a_{5,high} + a_{6,high}\theta_t + a_{7,high}\theta_t^2\right]\epsilon_{t+1}^{\theta} \end{aligned} \tag{71}$$

where

$$\begin{aligned} a_{1,high} &= \tilde{Q}_2\left(1 + \omega_2\sigma_\theta^2\right) \\ a_{2,high} &= \tilde{Q}_2\left[-v_2 + 3\eta_2\sigma_\theta^2\right]\rho_\theta \\ a_{3,high} &= \tilde{Q}_2\omega_2\rho_\theta^2 \\ a_{4,high} &= \tilde{Q}_2\eta_2\rho_\theta^3 \\ a_{5,high} &= -\tilde{Q}_2 v_2 \\ a_{6,high} &= 2\tilde{Q}_2\omega_2\rho_\theta \\ a_{7,high} &= 3\tilde{Q}_2\eta_2\rho_\theta^2 \end{aligned}$$

and $\omega_2 = -V_2 + \frac{1}{2}v_2^2$, $\eta_2 = -\frac{1}{6}v_2^3 + v_2 V_2 - Z_2$.

The expected asset price, conditional on the future state, is then:

$$\begin{aligned} E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) &= a_{1,low} + a_{2,low}\theta_t + a_{3,low}\theta_t^2 + a_{4,low}\theta_t^3 \\ E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) &= a_{1,high} + a_{2,high}\theta_t + a_{3,high}\theta_t^2 + a_{4,high}\theta_t^3 \end{aligned}$$

If the economy is in the low risk state at time t, the expectation of Q_{t+1} is:

$$\begin{aligned} E_t\left(Q_{t+1}|t \text{ } is \text{ } low\right) &= p_1 E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) + (1-p_1) E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) \\ &= d_{1,low} + d_{2,low}\theta_t + d_{3,low}\theta_t^2 + d_{4,low}\theta_t^3 \end{aligned} \tag{72}$$

where

$$\begin{aligned} d_{1,low} &= p_1 a_{1,low} + (1-p_1) a_{1,high} \\ d_{2,low} &= p_1 a_{2,low} + (1-p_1) a_{2,high} \\ d_{3,low} &= p_1 a_{3,low} + (1-p_1) a_{3,high} \\ d_{4,low} &= p_1 a_{4,low} + (1-p_1) a_{4,high} \end{aligned}$$

Similarly, if the economy is in the high risk state at time t, the expectation of Q_{t+1} is:

$$\begin{aligned} E_t\left(Q_{t+1}|t \text{ } is \text{ } high\right) &= (1-p_2) E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } low\right) + p_2 E_t\left(Q_{t+1}|t+1 \text{ } is \text{ } high\right) \\ &= d_{1,high} + d_{2,high}\theta_t + d_{3,high}\theta_t^2 + d_{4,high}\theta_t^3 \end{aligned} \tag{73}$$

where

$$\begin{aligned} d_{1,high} &= (1-p_2) a_{1,low} + p_2 a_{1,high} \\ d_{2,high} &= (1-p_2) a_{2,low} + p_2 a_{2,high} \\ d_{3,high} &= (1-p_2) a_{3,low} + p_2 a_{3,high} \\ d_{4,high} &= (1-p_2) a_{4,low} + p_2 a_{4,high} \end{aligned}$$

Next, we compute the variance of the equity price in period $t+1$ from the point of view of period t. The variance of Q_{t+1} is

$$var\left(Q_{t+1}\right) = E_t Q_{t+1}^2 - \left(E_t Q_{t+1}\right)^2$$

In a state $i = 1, 2$ we have:

$$Q_{i,t+1} = \left[a_{1,i} + a_{2,i}\theta_t + a_{3,i}\theta_t^2 + a_{4,i}\theta_t^3\right] + \left[a_{5,i} + a_{6,i}\theta_t + a_{7,i}\theta_t^2\right]\epsilon_{t+1}^{\theta} \tag{74}$$

It follows that in a specific $t+1$ state (dropping terms in θ_t^4 and above):

$$E_t\left(Q_{t+1}^2 | t+1\ is\ i\right) = b_{1,i} + b_{2,i}\theta_t + b_{3,i}\theta_t^2 + b_{4,i}\theta_t^3$$

where for $i = high$, low:

$$\begin{aligned} b_{1,i} &= a_{1,i}^2 + a_{5,i}^2\sigma_\theta^2 \\ b_{2,i} &= 2a_{1,i}a_{2,i} + 2a_{5,i}a_{6,i}\sigma_\theta^2 \\ b_{3,i} &= a_{2,i}^2 + 2a_{1,i}a_{3,i} + \left[a_{6,i}^2 + 2a_{5,i}a_{7,i}\right]\sigma_\theta^2 \\ b_{4,i} &= 2a_{1,i}a_{4,i} + 2a_{2,i}a_{3,i} + 2a_{6,i}a_{7,i}\sigma_\theta^2 \end{aligned}$$

Consider that the economy is in the low risk state at time t. We drop the terms where θ_t to as a fourth or higher power. The expectation of Q_{t+1}^2 is then:

$$\begin{aligned} E_t\left(Q_{t+1}^2 | t\ is\ low\right) &= p_1 E_{t+1}\left(Q_{t+1}^2 | t+1\ is\ low\right) + (1-p_1)\, E_{t+1}\left(Q_{t+1}^2 | t+1\ is\ high\right) \\ &= c_{1,low} + c_{2,low}\theta_t + c_{3,low}\theta_t^2 + c_{4,low}\theta_t^3 \end{aligned}$$

where

$$\begin{aligned} c_{1,low} &= p_1 b_{1,low} + (1-p_1)\, b_{1,high} \\ c_{2,low} &= p_1 b_{2,low} + (1-p_1)\, b_{2,high} \\ c_{3,low} &= p_1 b_{3,low} + (1-p_1)\, b_{3,high} \\ c_{4,low} &= p_1 b_{4,low} + (1-p_1)\, b_{4,high} \end{aligned}$$

Using (72) we write:

$$\begin{aligned} \left(E_t\left(Q_{t+1} | t\ is\ low\right)\right)^2 &= d_{1,low}^2 + 2d_{1,low}d_{2,low}\theta_t + \left[d_{2,low}^2 + 2d_{1,low}d_{3,low}\right]\theta_t^2 \\ &\quad + 2\left[d_{1,low}d_{4,low} + d_{2,low}d_{3,low}\right]\theta_t^3 \end{aligned}$$

The variance of the asset price in period $t+1$ from the point of view of period t, when the economy is in the low risk state in period t, is thus:

$$\begin{aligned} var\left(Q_{t+1} | t\ is\ low\right) &= \left(c_{1,low} - d_{1,low}^2\right) + \left(c_{2,low} - 2d_{1,low}d_{2,low}\right)\theta_t \\ &\quad + \left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right)\theta_t^2 \\ &\quad + \left(c_{4,low} - 2d_{1,low}d_{4,low} - 2d_{2,low}d_{3,low}\right)\theta_t^3 \end{aligned} \tag{75}$$

Similarly, the variance of the asset price in period $t+1$ from the point of view of period t, when the economy is in the high risk state in period t, is:

$$\begin{aligned} var\left(Q_{t+1}|t\ is\ high\right) &= \left(c_{1,high} - d_{1,high}^2\right) + \left(c_{2,high} - 2d_{1,high}d_{2,high}\right)\theta_t \\ &\quad + \left(c_{3,high} - d_{2,high}^2 - 2d_{1,high}d_{3,high}\right)\theta_t^2 \\ &\quad + \left(c_{4,high} - 2d_{1,high}d_{4,high} - 2d_{2,high}d_{3,high}\right)\theta_t^3 \end{aligned} \tag{76}$$

where:

$$\begin{aligned} c_{1,high} &= (1-p_2)\,b_{1,low} + p_2 b_{1,high} \\ c_{2,high} &= (1-p_2)\,b_{2,low} + p_2 b_{2,high} \\ c_{3,high} &= (1-p_2)\,b_{3,low} + p_2 b_{3,high} \\ c_{4,high} &= (1-p_2)\,b_{4,low} + p_2 b_{4,high} \end{aligned}$$

We now compute the product of the interest rate and the equity price in period $t+1$ from the point of view of period t. If the economy is in the low risk state at time t, we use (58) to get:

$$\left(Q_t R_{t+1}|t\ is\ low\right) = e_{1,low} + e_{2,low}\theta_t + e_{3,low}\theta_t^2 + e_{4,low}\theta_t^3 \tag{77}$$

where:

$$\begin{aligned} e_{1,low} &= \frac{\tilde{Q}_1}{\eta}\left(\nu - W + K\tilde{Q}_1\right) \\ e_{2,low} &= -\frac{\tilde{Q}_1}{\eta}\left(\nu - W + 2K\tilde{Q}_1\right)v_1 \\ e_{3,low} &= \frac{\tilde{Q}_1}{\eta}\left[(\nu - W)\,\omega_1 + 2K\tilde{Q}_1\left(v_1^2 - V_1\right)\right] \\ e_{4,low} &= \frac{\tilde{Q}_1}{\eta}\left[(\nu - W)\,\eta_1 + K\tilde{Q}_1\left(-\frac{4}{3}v_1^3 + 4v_1V_1 - 2Z_1\right)\right] \end{aligned}$$

Similarly, if the economy is in the high risk state at time t, we have:

$$\left(Q_t R_{t+1}|t\ is\ high\right) = e_{1,high} + e_{2,high}\theta_t + e_{3,high}\theta_t^2 + e_{4,high}\theta_t^3 \tag{78}$$

where:

$$e_{1,high} = \frac{\tilde{Q}_2}{\eta}\left(\nu - W + K\tilde{Q}_2\right)$$

$$
\begin{aligned}
e_{2,high} &= -\frac{\tilde{Q}_2}{\eta}\left(\nu - W + 2K\tilde{Q}_2\right)v_2 \\
e_{3,high} &= \frac{\tilde{Q}_2}{\eta}\left[(\nu - W)\,\omega_2 + 2K\tilde{Q}_2\left(v_2^2 - V_2\right)\right] \\
e_{4,high} &= \frac{\tilde{Q}_2}{\eta}\left[(\nu - W)\,\eta_2 + K\tilde{Q}_2\left(-\frac{4}{3}v_2^3 + 4v_2V_2 - 2Z_2\right)\right]
\end{aligned}
$$

We can now compute the equity market clearing condition. Consider that the economy is in the low risk state at time t. Using (72), (75) and (77), (52) is written as:

$$
X_{t=low}\left(\bar{W} - \bar{m}\theta_t\right) = Y_{t=low} \tag{79}
$$

where

$$
\begin{aligned}
X_{t=low} &= \bar{A} + d_{1,low} - e_{1,low} + \left(d_{2,low} - e_{2,low}\right)\theta_t \\
&\quad + \left(d_{3,low} - e_{3,low}\right)\theta_t^2 + \left(d_{4,low} - e_{4,low}\right)\theta_t^3
\end{aligned}
$$

and

$$
\begin{aligned}
Y_{t=low} &= K\bar{A}^2\sigma_a^2 + K\left(c_{1,low} - d_{1,low}^2\right) + K\left(c_{2,low} - 2d_{1,low}d_{2,low}\right)\theta_t \\
&\quad +K\left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right)\theta_t^2 \\
&\quad +K\left(c_{4,low} - 2d_{1,low}d_{4,low} - 2d_{2,low}d_{3,low}\right)\theta_t^3
\end{aligned}
$$

We focus on the terms in (79) that are constant, proportional to θ_t, proportional to θ_t^2, and proportional to θ_t^3. The constant terms in (79) are:

$$
\begin{aligned}
&\bar{W}\left(\bar{A} + d_{1,low} - e_{1,low}\right) \\
&= K\bar{A}^2\sigma_a^2 + K\left(c_{1,low} - d_{1,low}^2\right)
\end{aligned} \tag{80}
$$

The linear terms in (79) are:

$$
\begin{aligned}
&\bar{W}\left(d_{2,low} - e_{2,low}\right) - \bar{m}\left(\bar{A} + d_{1,low} - e_{1,low}\right) \\
&= K\left(c_{2,low} - 2d_{1,low}d_{2,low}\right)
\end{aligned} \tag{81}
$$

The quadratic terms in (79) are:

$$
\begin{aligned}
&\bar{W}\left(d_{3,low} - e_{3,low}\right) - \bar{m}\left(d_{2,low} - e_{2,low}\right) \\
&= K\left(c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low}\right)
\end{aligned} \tag{82}
$$

The cubic terms in (79) are:

$$\begin{aligned}& \bar{W}\left(d_{4,low}-e_{4,low}\right)-\bar{m}\left(d_{3,low}-e_{3,low}\right) \\ =\;& K\left(c_{4,low}-2d_{1,low}d_{4,low}-2d_{2,low}d_{3,low}\right)\end{aligned} \tag{83}$$

If the economy is in the high risk state at time t, we use (73), (76) and (78), to write (52) as:

$$X_{t=high}\left(\bar{W}-\bar{m}\theta_t\right)=Y_{t=high} \tag{84}$$

where

$$\begin{aligned}X_{t=high} &= \bar{A}+d_{1,high}-e_{1,high}+\left(d_{2,high}-e_{2,high}\right)\theta_t \\ &\quad +\left(d_{3,high}-e_{3,high}\right)\theta_t^2+\left(d_{4,high}-e_{4,high}\right)\theta_t^3\end{aligned}$$

and

$$\begin{aligned}Y_{t=high} &= K\bar{A}^2\sigma_a^2+K\left(c_{1,high}-d_{1,high}^2\right)+K\left(c_{2,high}-2d_{1,high}d_{2,high}\right)\theta_t \\ &\quad +K\left(c_{3,high}-d_{2,high}^2-2d_{1,high}d_{3,high}\right)\theta_t^2 \\ &\quad +K\left(c_{4,high}-2d_{1,high}d_{4,high}-2d_{2,high}d_{3,high}\right)\theta_t^3\end{aligned}$$

We focus on the terms in (84) that are constant, proportional to θ_t, proportional to θ_t^2 and proportional to θ_t^3. The constant terms in (84) are:

$$\begin{aligned}& \bar{W}\left(\bar{A}+d_{1,high}-e_{1,high}\right) \\ =\;& K\bar{A}^2\sigma_a^2+K\left(c_{1,high}-d_{1,high}^2\right)\end{aligned} \tag{85}$$

The linear terms in (84) are:

$$\begin{aligned}& \bar{W}\left(d_{2,high}-e_{2,high}\right)-\bar{m}\left(\bar{A}+d_{1,high}-e_{1,high}\right) \\ =\;& K\left(c_{2,high}-2d_{1,high}d_{2,high}\right)\end{aligned} \tag{86}$$

The quadratic terms in (84) are:

$$\begin{aligned}& \bar{W}\left(d_{3,high}-e_{3,high}\right)-\bar{m}\left(d_{2,high}-e_{2,high}\right) \\ =\;& K\left(c_{3,high}-d_{2,high}^2-2d_{1,high}d_{3,high}\right)\end{aligned} \tag{87}$$

The cubic terms in (84) are:

$$\begin{aligned}& \bar{W}\left(d_{4,high}-e_{4,high}\right)-\bar{m}\left(d_{3,high}-e_{3,high}\right) \\ =\;& K\left(c_{4,high}-2d_{1,high}d_{4,high}-2d_{2,high}d_{3,high}\right)\end{aligned} \tag{88}$$

The system of eight equations (80)-(83) and (85)-(88) can be solved numerically for the eight unknowns $\tilde{Q}_1$, v_1, V_1, Z_1, $\tilde{Q}_2$, v_2, V_2, Z_2.

II.E Computation of the Volatility of Risk

Equity price risk is the standard deviation of Q_{t+1}/Q_t. Using the expression (75) we write the standard deviation of Q_{t+1}/Q_t in the low risk state (at time t) as:

$$risk_t\,(t\ is\ low) = \frac{\exp\left[v_1\theta_t + V_1\theta_t^2 + Z_1\theta_t^3\right]}{\tilde{Q}_1}\sqrt{f_{1,low} + f_{2,low}\theta_t + f_{3,low}\theta_t^2 + f_{4,low}\theta_t^3} \tag{89}$$

where:

$$\begin{aligned}
f_{1,low} &= c_{1,low} - d_{1,low}^2 \\
f_{2,low} &= c_{2,low} - 2d_{1,low}d_{2,low} \\
f_{3,low} &= c_{3,low} - d_{2,low}^2 - 2d_{1,low}d_{3,low} \\
f_{4,low} &= c_{4,low} - 2d_{1,low}d_{4,low} - 2d_{2,low}d_{3,low}
\end{aligned}$$

We now take a cubic approximation of (89). Start with the exponential term:

$$\exp\left[v_1\theta_t + V_1\theta_t^2 + Z_1\theta_t^3\right] = 1 + r_{1,low}\theta_t + r_{2,low}\theta_t^2 + r_{3,low}\theta_t^3$$

where:

$$\begin{aligned}
r_{1,low} &= v_1 \\
r_{2,low} &= \frac{1}{2}v_1^2 + V_1 \\
r_{3,low} &= \frac{1}{6}v_1^3 + v_1V_1 + Z_1
\end{aligned}$$

A cubic expansion of the square root terms gives:

$$\sqrt{f_{1,low} + f_{2,low}\theta_t + f_{3,low}\theta_t^2 + f_{4,low}\theta_t^3} = \left(f_{1,low}\right)^{0.5} + r_{4,low}\theta_t + r_{5,low}\theta_t^2 + r_{6,low}\theta_t^3$$

where:

$$\begin{aligned}
r_{4,low} &= \frac{1}{2}\left(f_{1,low}\right)^{-0.5} f_{2,low} \\
r_{5,low} &= -\frac{1}{8}\left(f_{1,low}\right)^{-1.5}\left(f_{2,low}\right)^2 + \frac{1}{2}\left(f_{1,low}\right)^{-0.5} f_{3,low} \\
r_{6,low} &= \frac{1}{16}\left(f_{1,low}\right)^{-2.5}\left(f_{2,low}\right)^3 - \frac{1}{4}\left(f_{1,low}\right)^{-1.5} f_{2,low}f_{3,low} + \frac{1}{2}\left(f_{1,low}\right)^{-0.5} f_{4,low}
\end{aligned}$$

(89) then becomes, omitting terms in θ_t^4 or above:

$$risk_t\,(t\ is\ low) = g_{1,low} + g_{2,low}\theta_t + g_{3,low}\theta_t^2 + g_{4,low}\theta_t^3$$

where:

$$\begin{aligned}
g_{1,low} &= \frac{1}{\tilde{Q}_1}\left(f_{1,low}\right)^{0.5} \\
g_{2,low} &= \frac{1}{\tilde{Q}_1}\left[r_{4,low} + r_{1,low}\left(f_{1,low}\right)^{0.5}\right] \\
g_{3,low} &= \frac{1}{\tilde{Q}_1}\left[r_{5,low} + r_{1,low}r_{4,low} + r_{2,low}\left(f_{1,low}\right)^{0.5}\right] \\
g_{4,low} &= \frac{1}{\tilde{Q}_1}\left[r_{6,low} + r_{1,low}r_{5,low} + r_{2,low}r_{4,low} + r_{3,low}\left(f_{1,low}\right)^{0.5}\right]
\end{aligned}$$

Conditional on being in future low states, the volatility of risk is the standard deviation of:

$$\begin{aligned}
& risk_{t+1}\left(t+1 \ is \ low\right) \\
= \ & g_{1,low} + g_{2,low}\left[\rho_\theta\theta_t + \epsilon^\theta_{t+1}\right] + g_{3,low}\left[\rho_\theta\theta_t + \epsilon^\theta_{t+1}\right]^2 + g_{4,low}\left[\rho_\theta\theta_t + \epsilon^\theta_{t+1}\right]^3 \\
= \ & g_{1,low} + \left(g_{3,low} + 3g_{4,low}\rho_\theta\theta_t\right)\sigma^2_\theta \\
& + g_{2,low}\rho_\theta\theta_t + g_{3,low}\left(\rho_\theta\theta_t\right)^2 + g_{4,low}\left(\rho_\theta\theta_t\right)^3 \\
& + \left[g_{2,low} + 2g_{3,low}\rho_\theta\theta_t + 3g_{4,low}\left(\rho_\theta\theta_t\right)^2\right]\epsilon^\theta_{t+1}
\end{aligned}$$

From the point of view of period t the standard deviation of of risk is then:

$$SD_t\left(risk_{t+1}\left(t+1 \ is \ low\right)\right) = \left(g_{2,low} + 2g_{3,low}\rho_\theta\theta_t + 3g_{4,low}\left(\rho_\theta\theta_t\right)^2\right)\sigma_\theta \tag{90}$$

Results are analogous if we are in the high risk state at time t. The standard deviation of Q_{t+1}/Q_t in the high risk state as

$$risk_t\left(t \ is \ high\right) = \frac{\exp\left[v_2\theta_t + V_2\theta^2_t + Z_2\theta^3_t\right]}{\tilde{Q}_2}\sqrt{f_{1,high} + f_{2,high}\theta_t + f_{3,high}\theta^2_t + f_{4,high}\theta^3_t}$$

where:

$$\begin{aligned}
f_{1,high} &= c_{1,high} - d^2_{1,high} \\
f_{2,high} &= c_{2,high} - 2d_{1,high}d_{2,high} \\
f_{3,high} &= c_{3,high} - d^2_{2,high} - 2d_{1,high}d_{3,high} \\
f_{4,high} &= c_{4,high} - 2d_{1,high}d_{4,high} - 2d_{2,high}d_{3,high}
\end{aligned}$$

Taking a cubic approximation of this, the standard deviation of Q_{t+1}/Q_t in the high risk state becomes

$$risk_t\left(t \ is \ high\right) = g_{1,high} + g_{2,high}\theta_t + g_{3,high}\theta^2_t + g_{4,high}\theta^3_t$$

where:

$$
\begin{aligned}
g_{1,high} &= \frac{1}{\tilde{Q}_2}\left(f_{1,high}\right)^{0.5} \\
g_{2,high} &= \frac{1}{\tilde{Q}_2}\left[r_{4,high} + r_{1,high}\left(f_{1,high}\right)^{0.5}\right] \\
g_{3,high} &= \frac{1}{\tilde{Q}_2}\left[r_{5,high} + r_{1,high}r_{4,high} + r_{2,high}\left(f_{1,high}\right)^{0.5}\right] \\
g_{4,high} &= \frac{1}{\tilde{Q}_2}\left[r_{6,high} + r_{1,high}r_{5,high} + r_{2,high}r_{4,high} + r_{3,high}\left(f_{1,high}\right)^{0.5}\right]
\end{aligned}
$$

and:

$$
\begin{aligned}
r_{1,high} &= v_2 \\
r_{2,high} &= \frac{1}{2}v_2^2 + V_2 \\
r_{3,high} &= \frac{1}{6}v_2^3 + v_2V_2 + Z_2 \\
r_{4,high} &= \frac{1}{2}\left(f_{1,high}\right)^{-0.5} f_{2,high} \\
r_{5,high} &= -\frac{1}{8}\left(f_{1,high}\right)^{-1.5}\left(f_{2,high}\right)^2 + \frac{1}{2}\left(f_{1,high}\right)^{-0.5} f_{3,high} \\
r_{6,high} &= \frac{1}{16}\left(f_{1,high}\right)^{-2.5}\left(f_{2,high}\right)^3 - \frac{1}{4}\left(f_{1,high}\right)^{-1.5} f_{2,high}f_{3,high} + \frac{1}{2}\left(f_{1,high}\right)^{-0.5} f_{4,high}
\end{aligned}
$$

The volatility of risk is then

$$
SD_t\left(risk_{t+1}\left(t+1 \text{ } is \text{ } high\right)\right) = \left(g_{2,high} + 2g_{3,high}\rho_\theta\theta_t + 3g_{4,high}\left(\rho_\theta\theta_t\right)^2\right)\sigma_\theta \tag{91}
$$

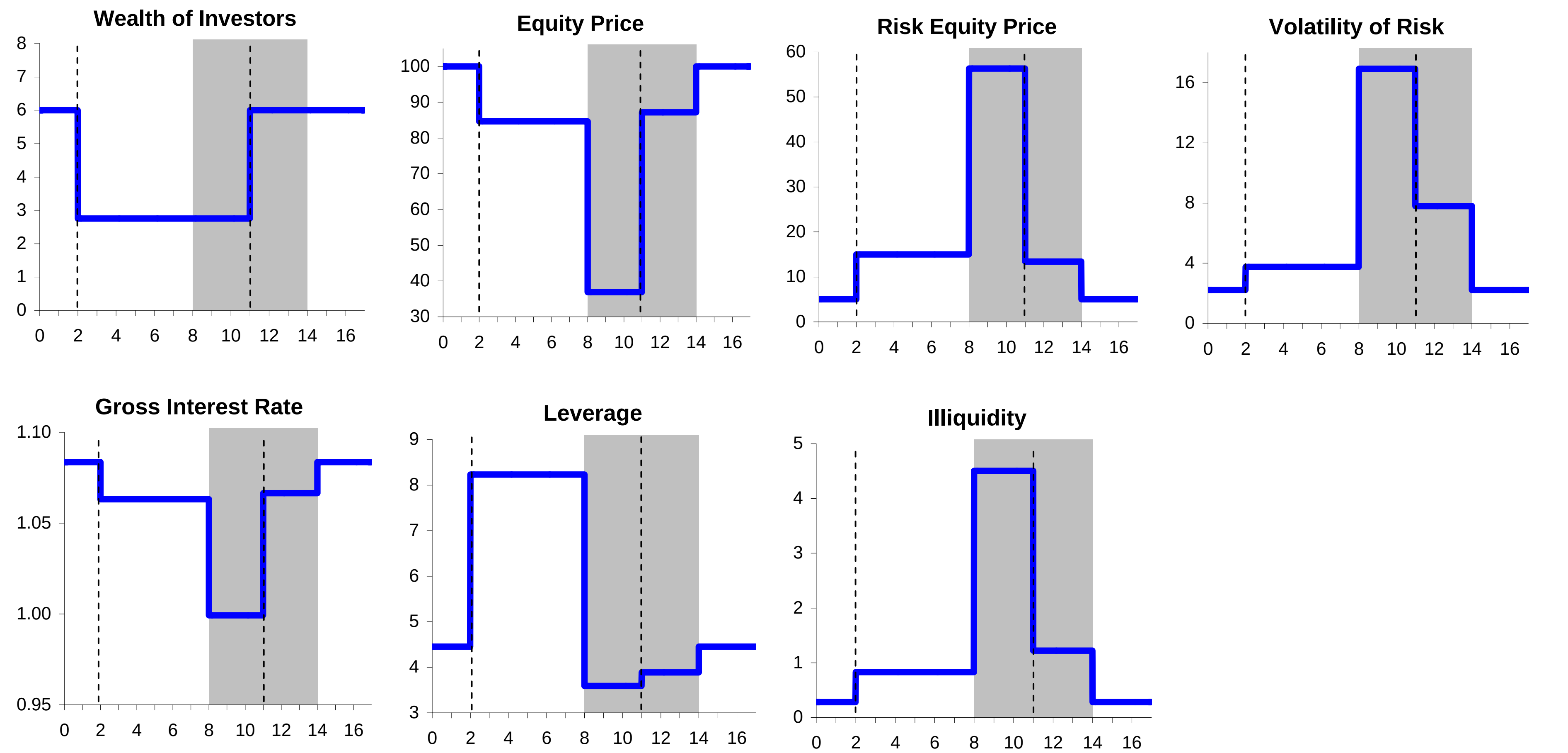
III Sensitivity Analysis

This section reports the results from sensitivity analysis discussed in section 6.3 of the paper. Figure 1 repeats the benchmark simulation reported in Figure 8 of the paper. The other 19 Figures consider different types of sensitivity analysis. In Figures 2 to 18 we consider the sensitivity to various model parameters. The parameter that is changed is listed in the title of the Figure. All other parameters remain the same as in the benchmark parameterization listed at the bottom of Figure 1. Figures 2 and 3 respectively halve and double $\bar{A}$. Figures 4 and 5 respectively halve and double the parameters η and $\nu - W$. Their ratio, which is the constant term in the interest rate equation, is therefore held constant. Figures 6 and 7 change $\nu - W$ to respectively 180 and 200. Figures 8 and 9 respectively halve and double σ_a. Figures 10 and 11 change ρ_θ to respectively 0.6 and 0.8. Figures 12 and 13 respectively halve and double γ. Figures 14 and 15 respectively halve and double m. Figures 16 and 17 respectively halve and double K.

Figure 18 reports results when only investors are hit by wealth shocks. This is then an aggregate wealth shock. There is no redistribution between households and investors. This corresponds to setting $n = 0$ (and $\delta = 0$) in the generalized version of the model in section I of this Appendix. Figure 19 reports results when a fraction of the wealth is held in the form of trees. This corresponds to setting $\delta = 0.1$ (and $m = n$) in the generalized version of the model in section I. Finally, Figure 20 reports results when we use a cubic approximation of the equity market clearing condition, as discussed in section II of this Appendix.

Figure 1. Benchmark simulation (figure 8 in the paper; δ = 0 and m = n)

shaded area = high risk equilibrium; vertical lines = endowment shock



The economy starts in the low risk equilibrium. At time 2 the endowment of investors falls from 6 to 2.8. The economy stays in the low risk equilibrium until time 8, at which point is shifts to the high risk equilibrium. At time 11 endowments shift back towards the initial allocation. The economy remains in the high risk equilibrium until time 14, at which points it shifts back to the low risk equilibrium.

$\overline{A} = 0.15; \upsilon - W = 190; \eta = 200; \sigma_a = 0.1; \sigma_\theta = 0.1; \rho_\theta = 0.7; \gamma = 1; \overline{W}_I = 6; m = 2; K = 20; p_1 = 0.95; p_2 = 0.7$

Figure 2. Model simulation: $\overline{A} = 0.075$

shaded area = high risk equilibrium; vertical lines = endowment shock

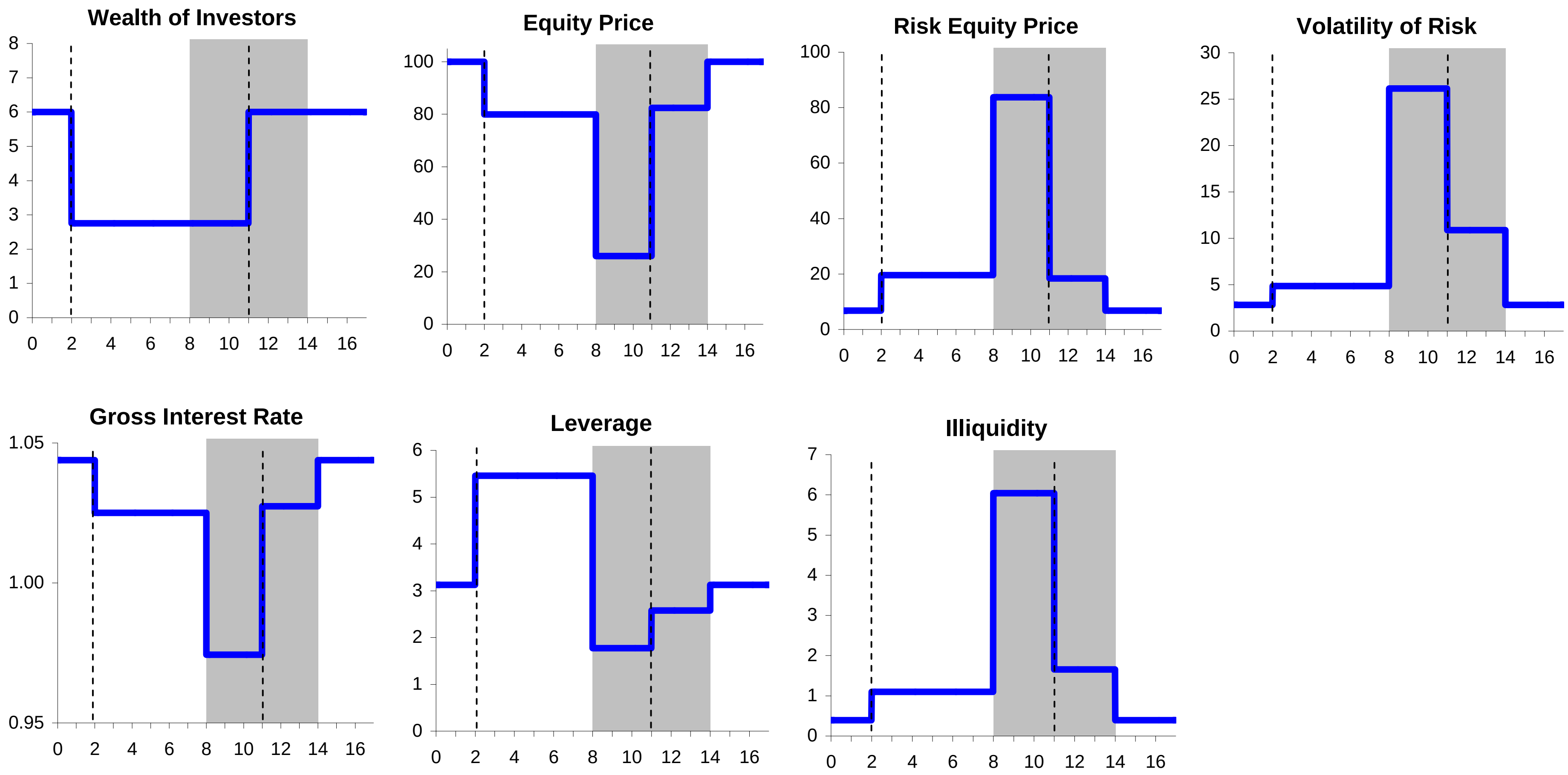


Figure 3. Model simulation: $\overline{A} = 3$

shaded area = high risk equilibrium; vertical lines = endowment shock

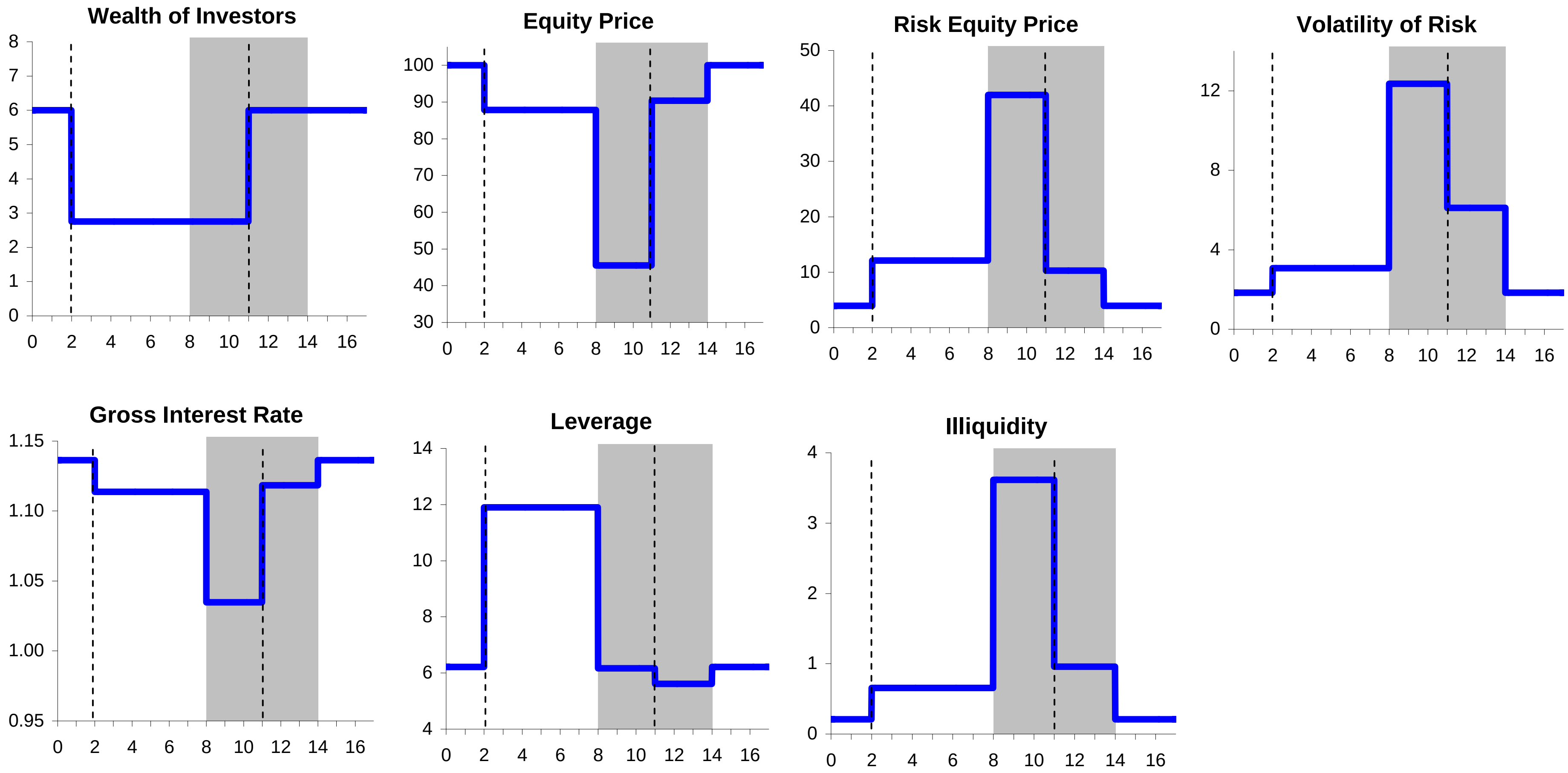


Figure 4. Model simulation: $\upsilon - W = 95; \eta = 100$

shaded area = high risk equilibrium; vertical lines = endowment shock

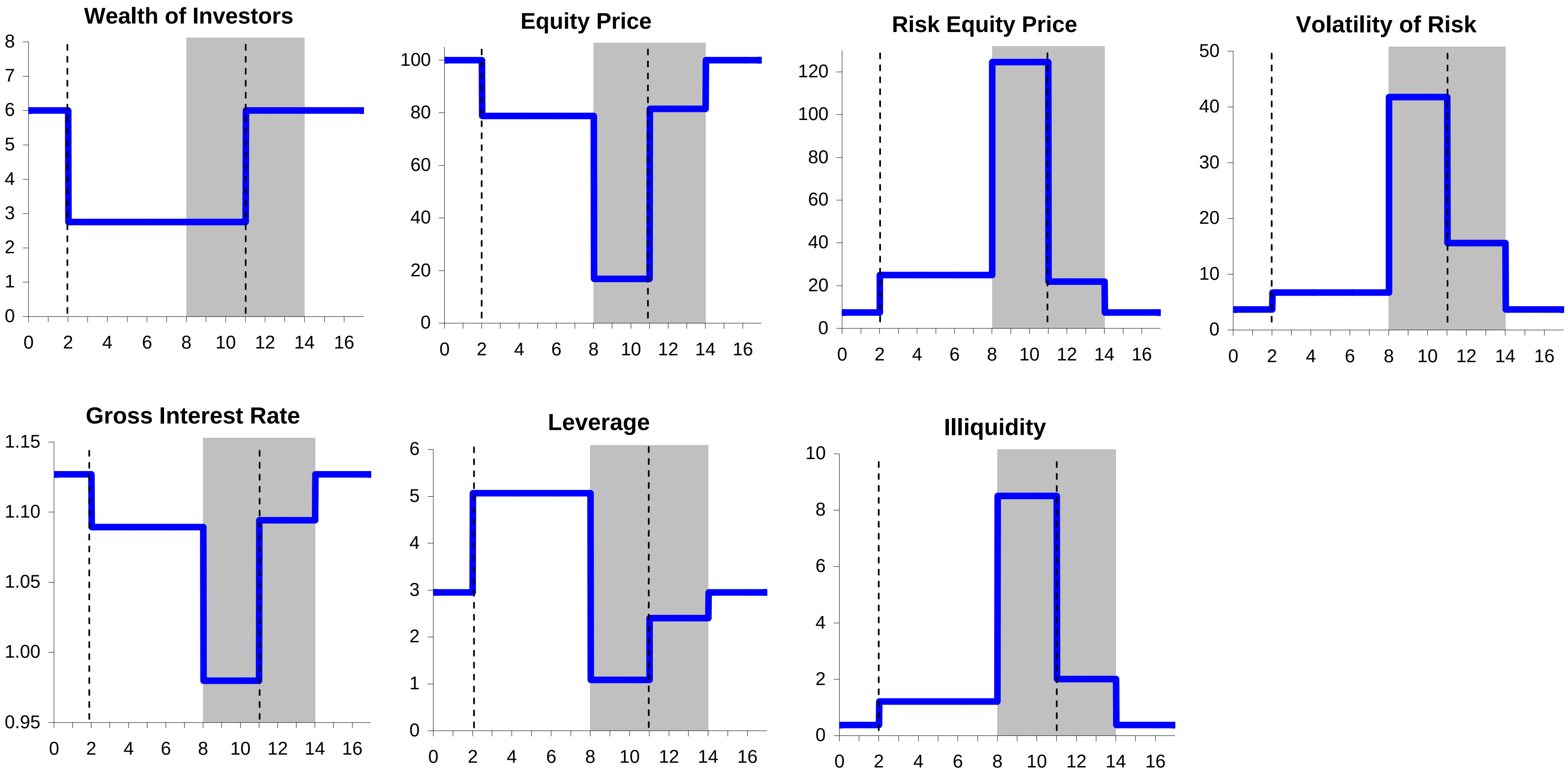


Figure 5. Model simulation: $\upsilon - W = 380\,;\eta = 400$

shaded area = high risk equilibrium; vertical lines = endowment shock

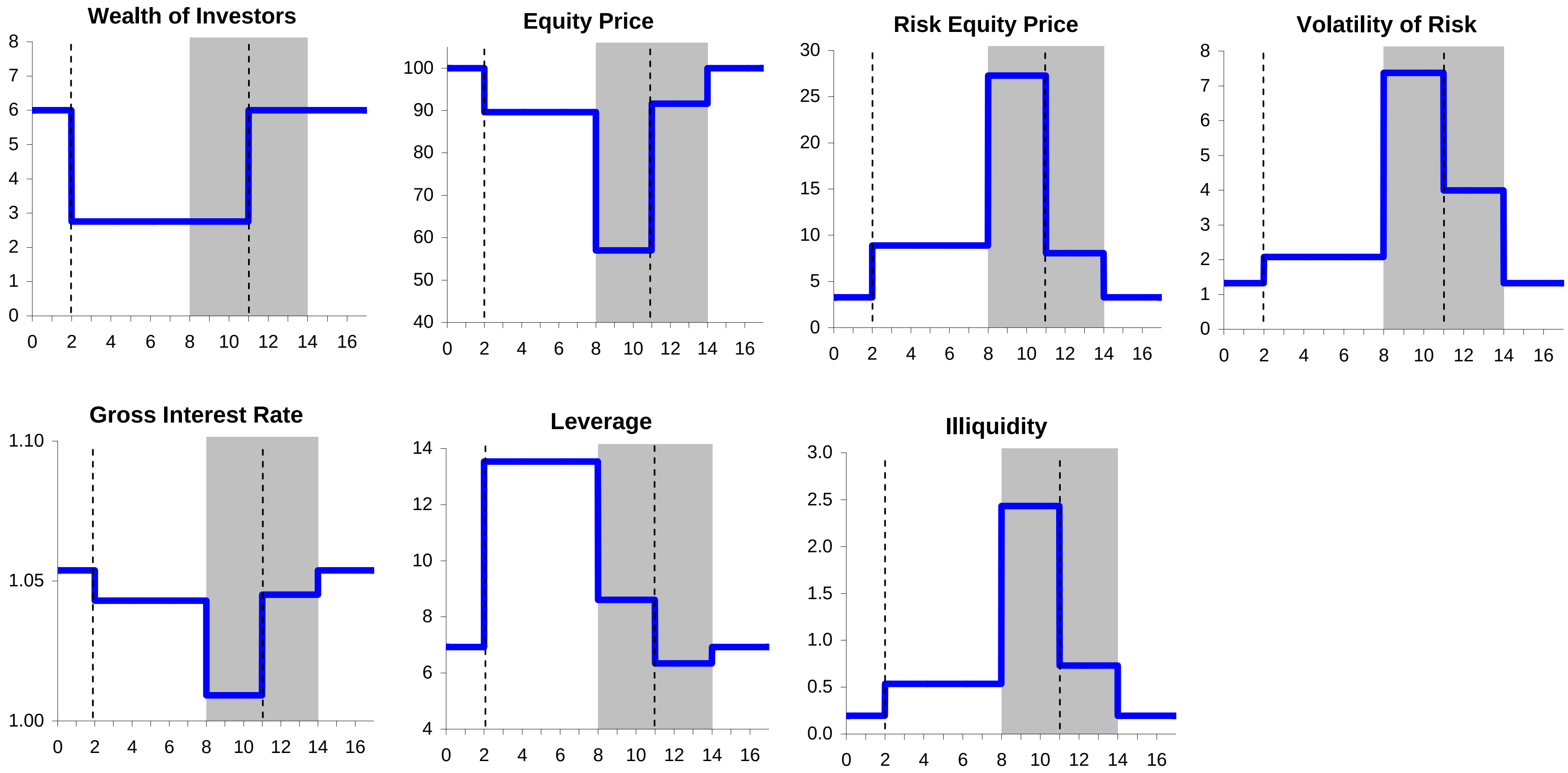


Figure 6. Model simulation: $\upsilon - W = 180$

shaded area = high risk equilibrium; vertical lines = endowment shock

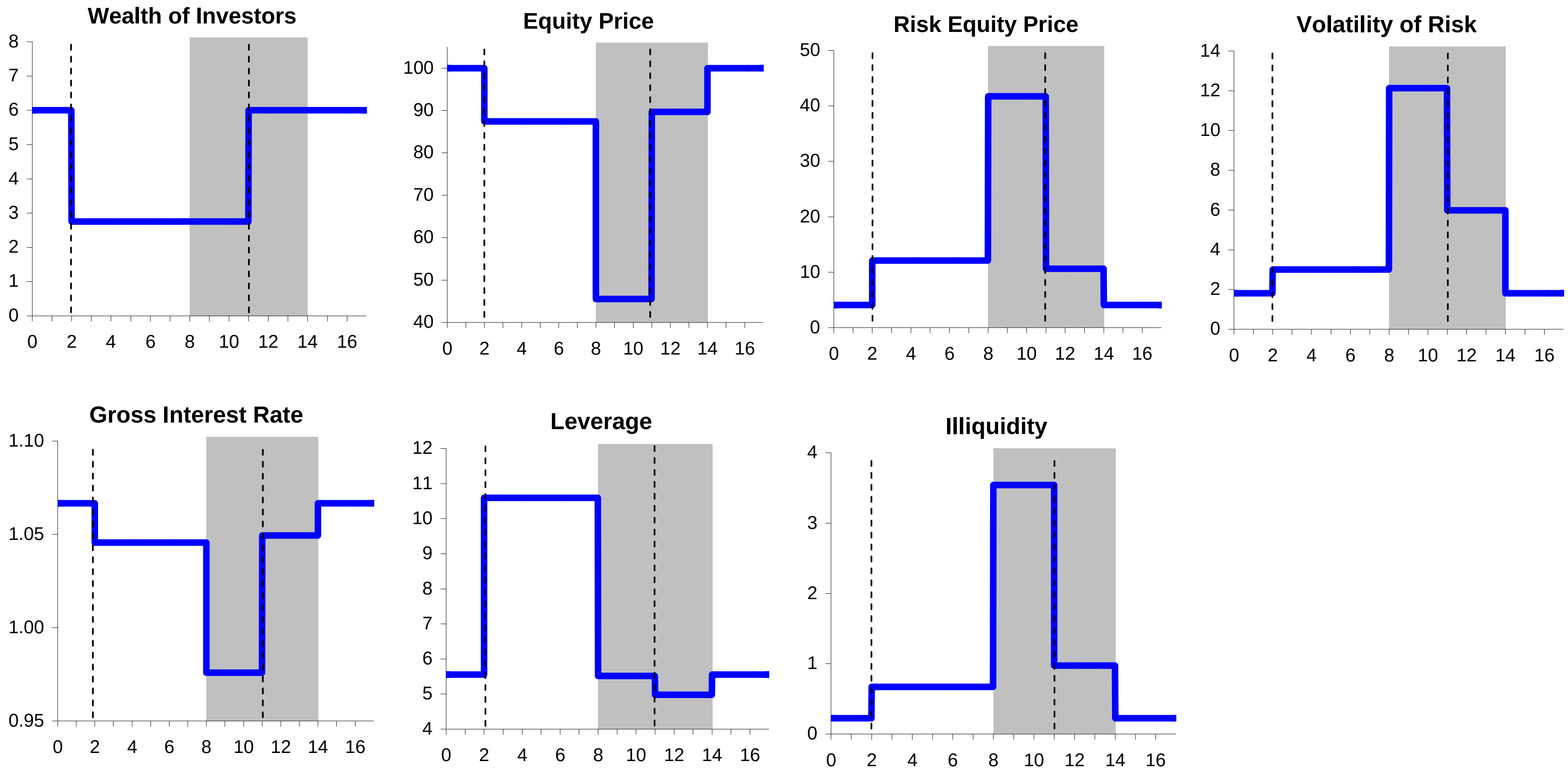


Figure 7. Model simulation: $\upsilon - W = 200$

shaded area = high risk equilibrium; vertical lines = endowment shock

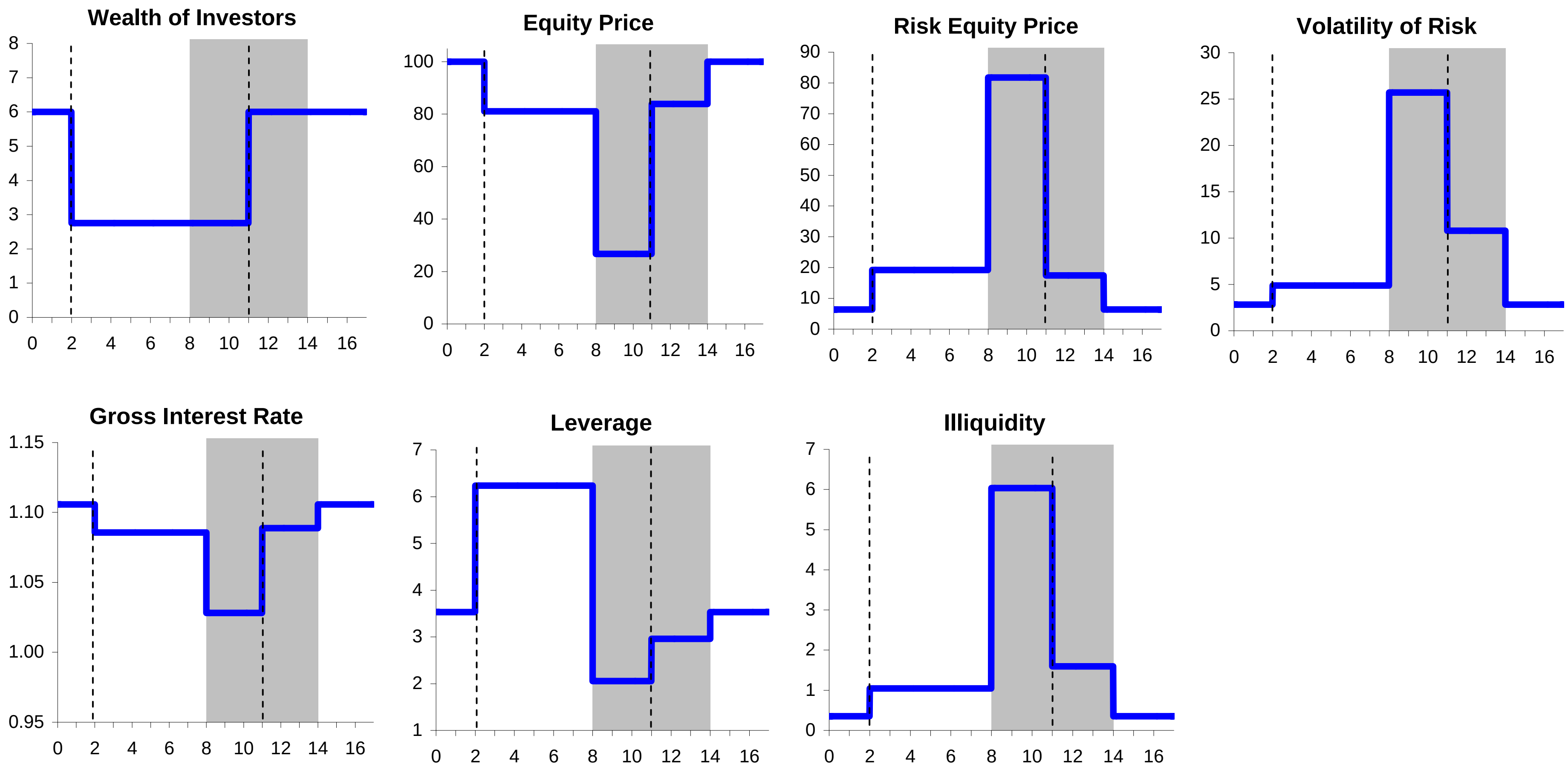


Figure 8. Model simulation: $\sigma_a = 0.05$

shaded area = high risk equilibrium; vertical lines = endowment shock

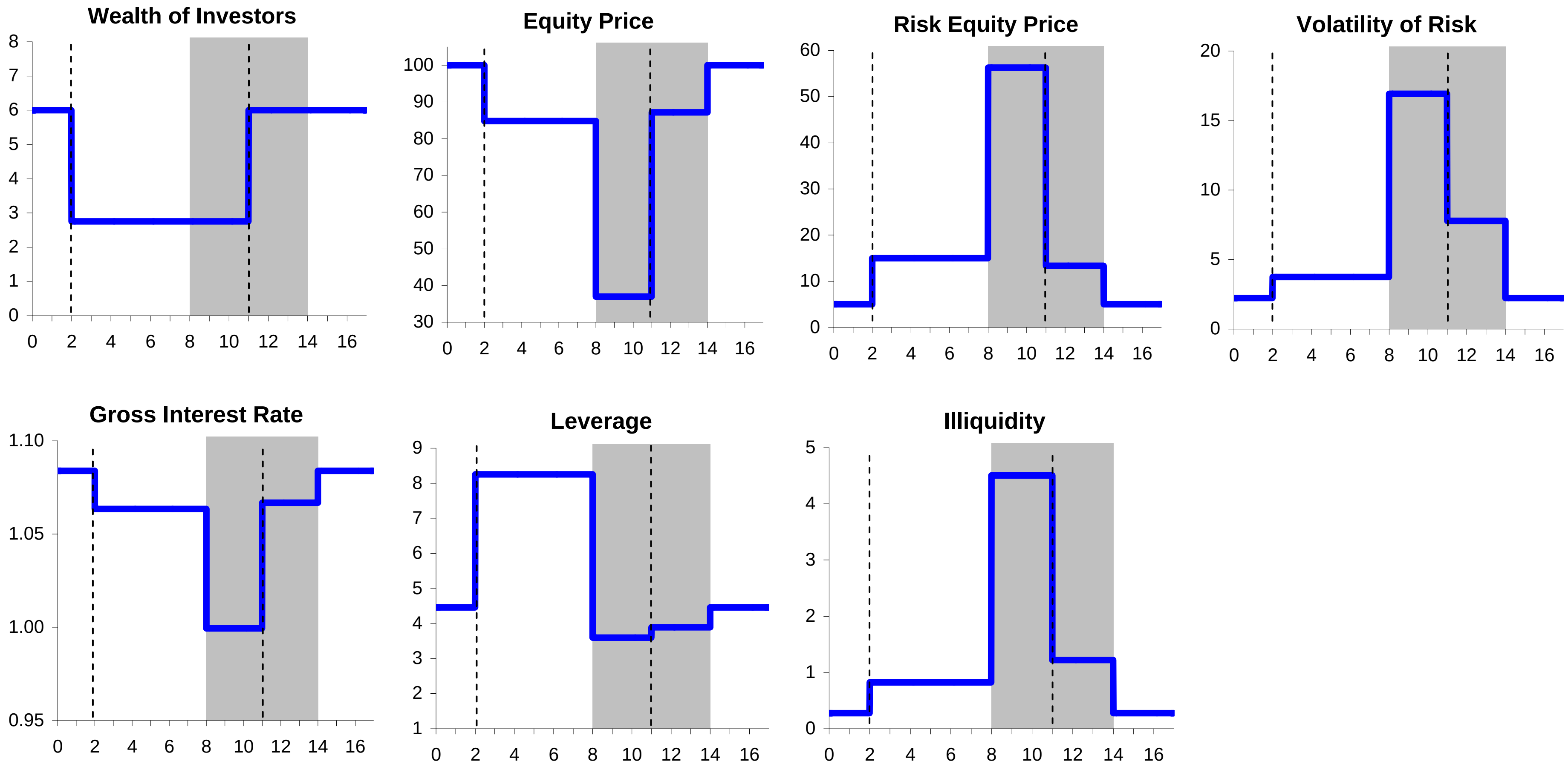


Figure 9. Model simulation: $\sigma_a = 0.2$

shaded area = high risk equilibrium; vertical lines = endowment shock

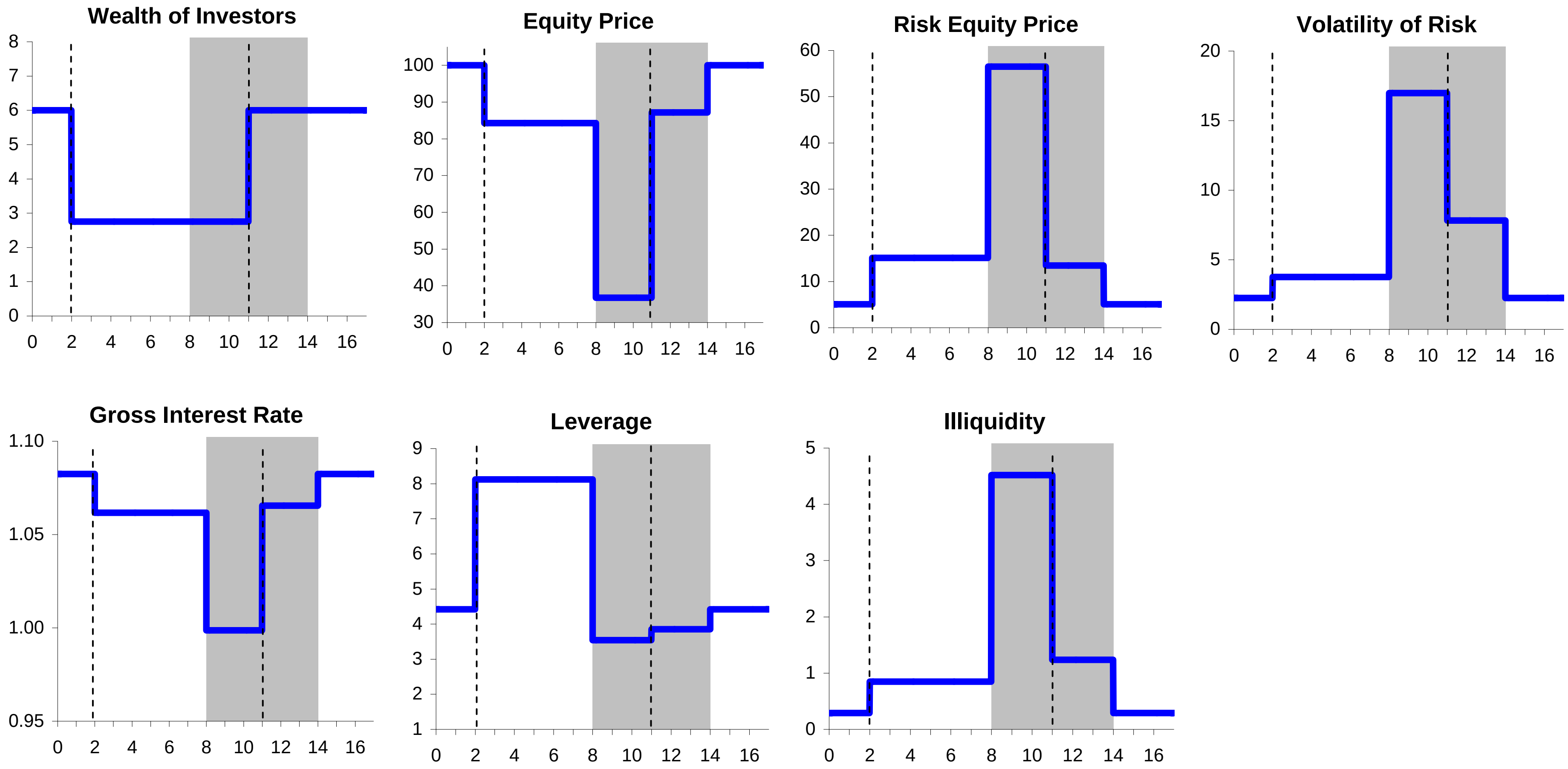


Figure 10. Model simulation: $\rho_\theta = 0.6$

shaded area = high risk equilibrium; vertical lines = endowment shock

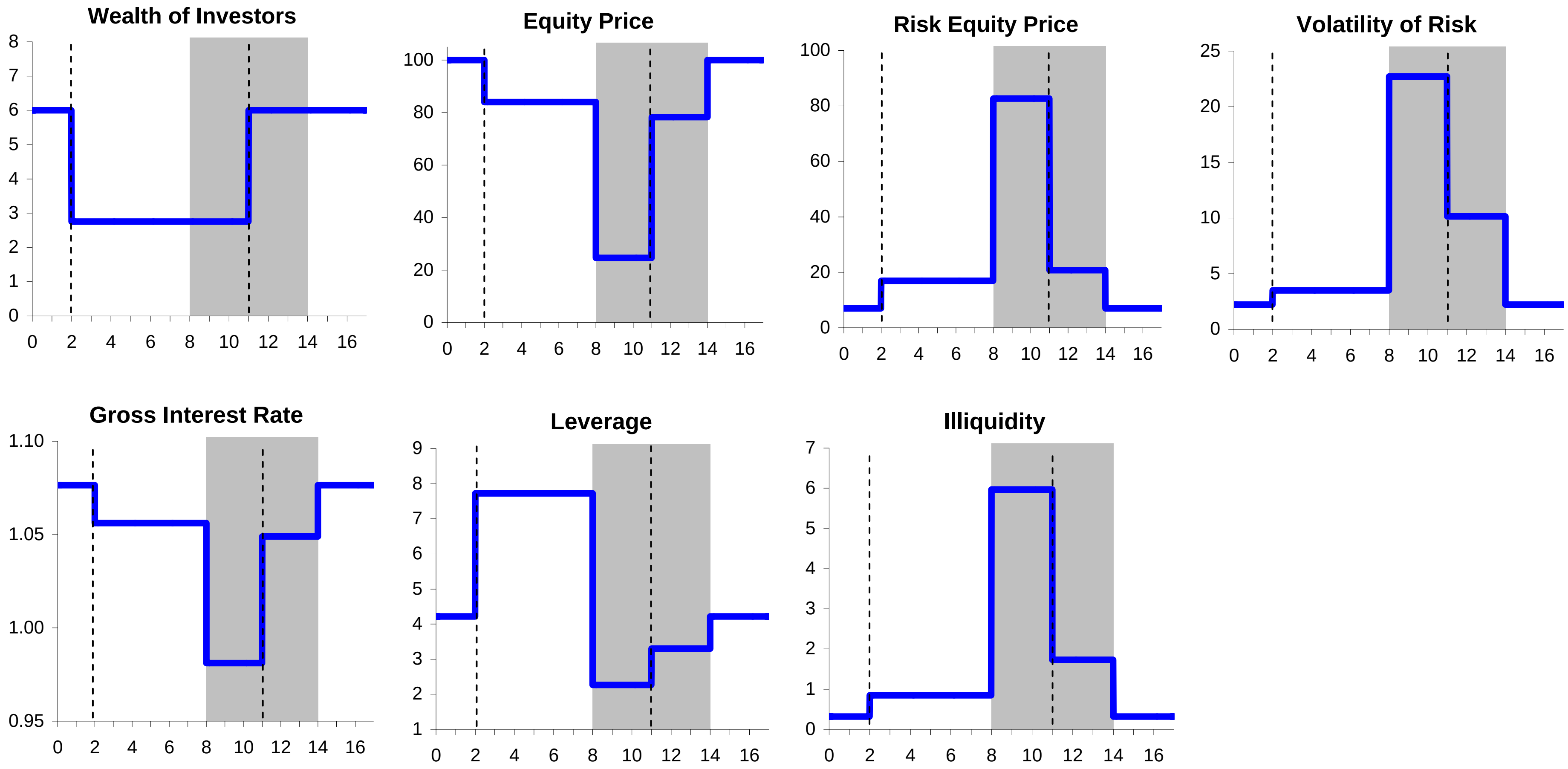


Figure 11. Model simulation: $\rho_\theta = 0.8$

shaded area = high risk equilibrium; vertical lines = endowment shock

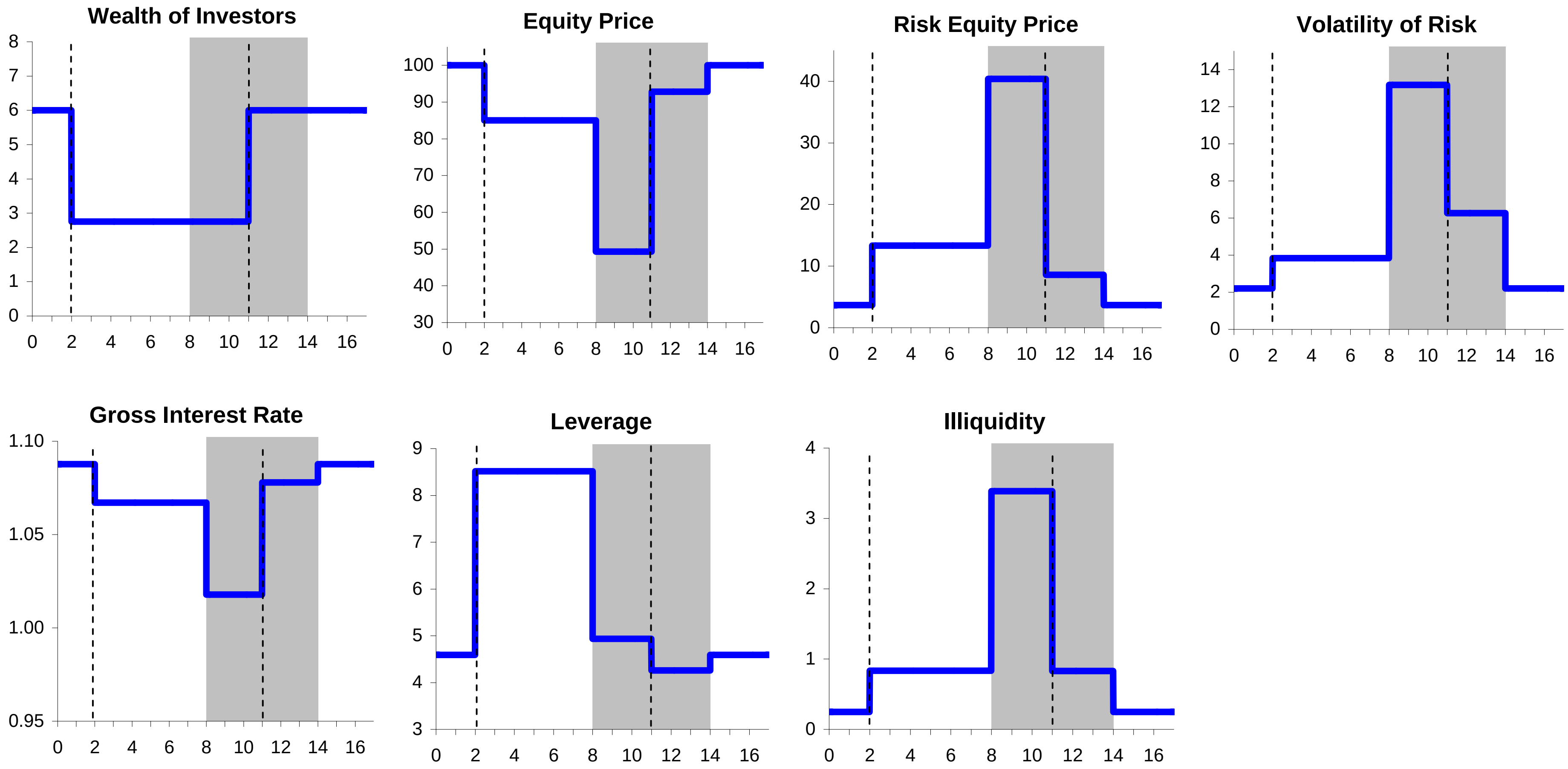


Figure 12. Model simulation: $\gamma = 0.5$

shaded area = high risk equilibrium; vertical lines = endowment shock

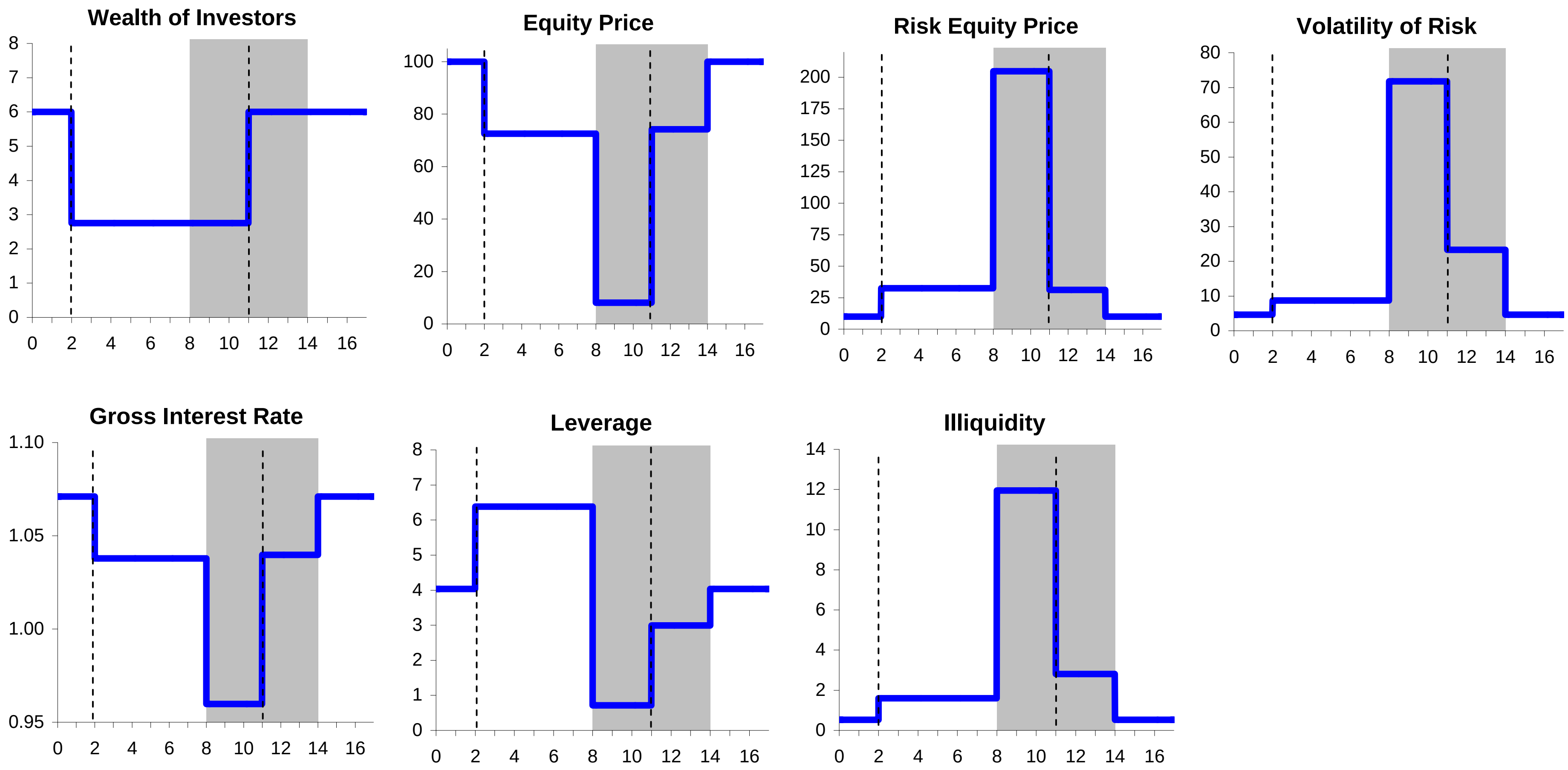


Figure 13. Model simulation: $\gamma = 2$

shaded area = high risk equilibrium; vertical lines = endowment shock

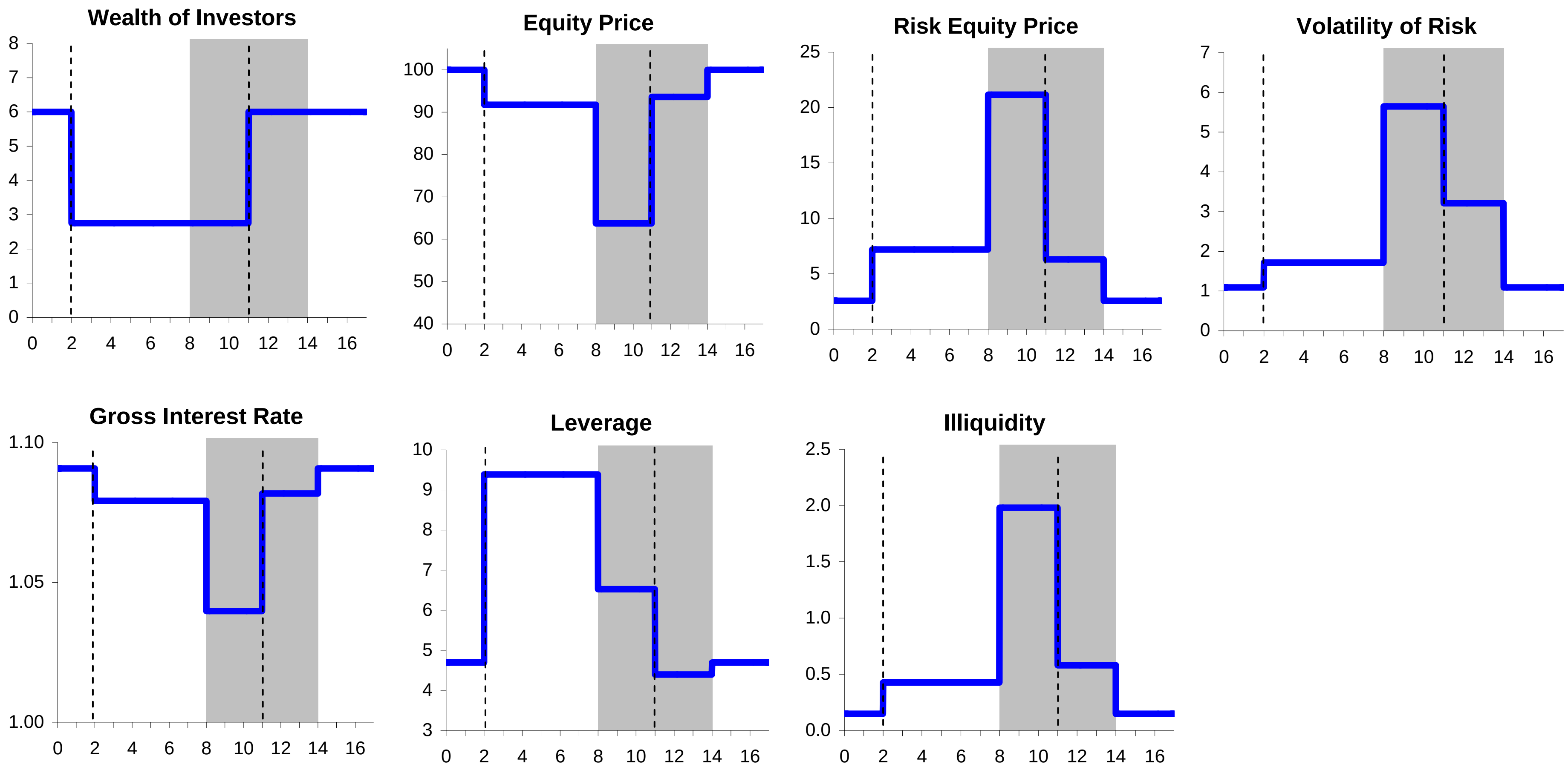


Figure 14. Model simulation: $m = 1$

shaded area = high risk equilibrium; vertical lines = endowment shock

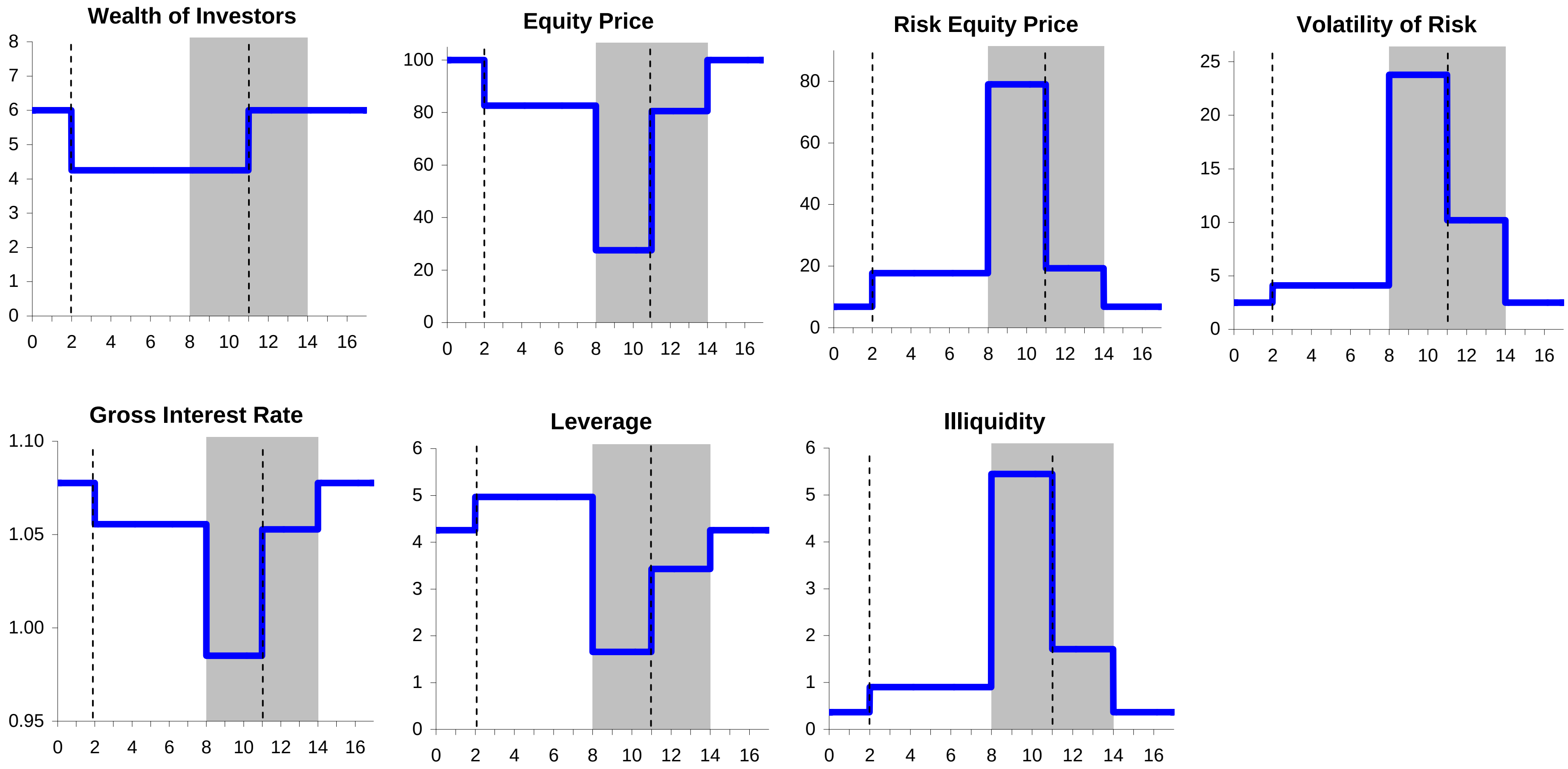


Figure 15. Model simulation: $m = 4$

shaded area = high risk equilibrium; vertical lines = endowment shock

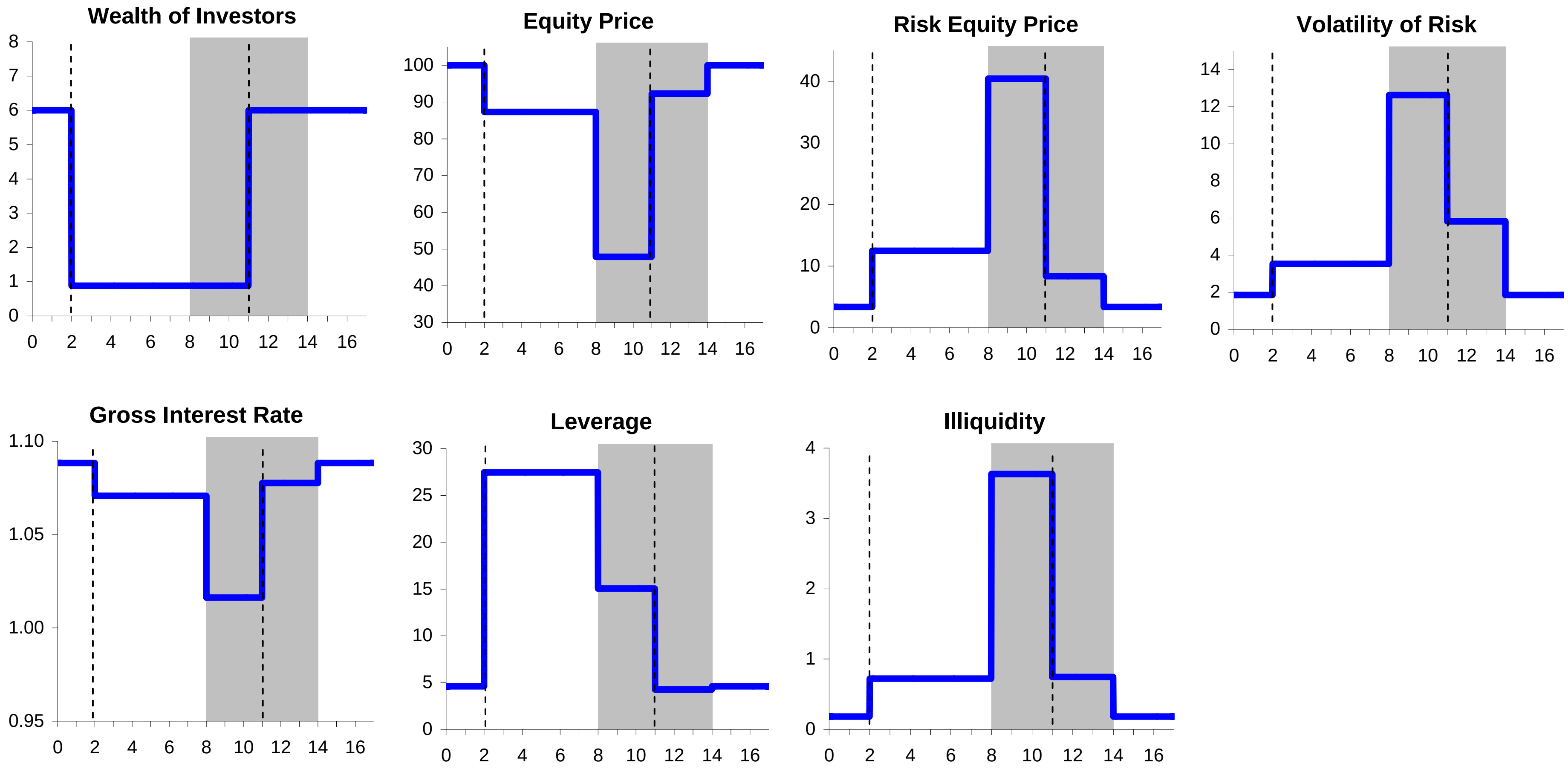


Figure 16. Model simulation: $K = 10$

shaded area = high risk equilibrium; vertical lines = endowment shock

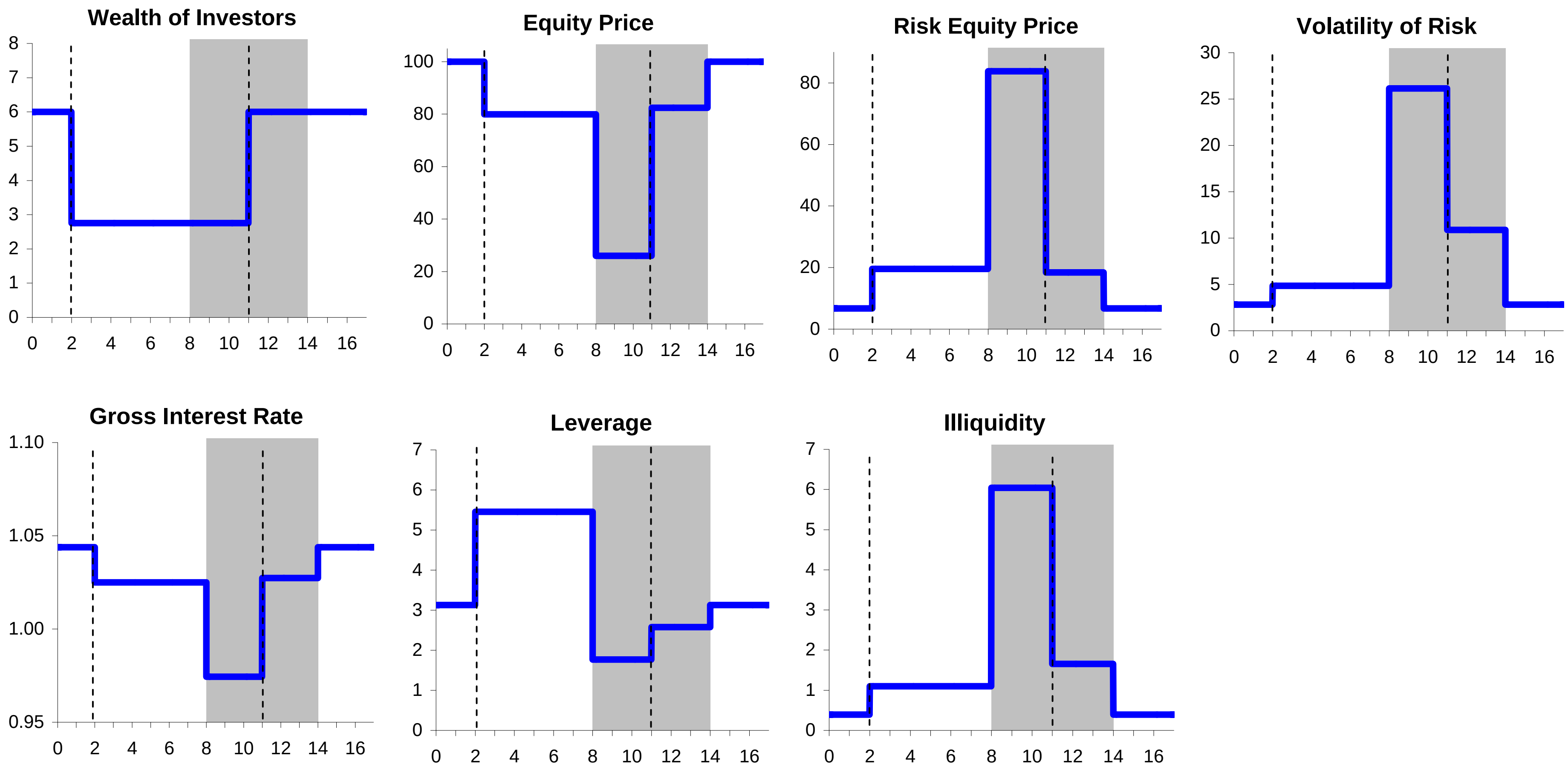


Figure 17. Model simulation: $K = 40$

shaded area = high risk equilibrium; vertical lines = endowment shock

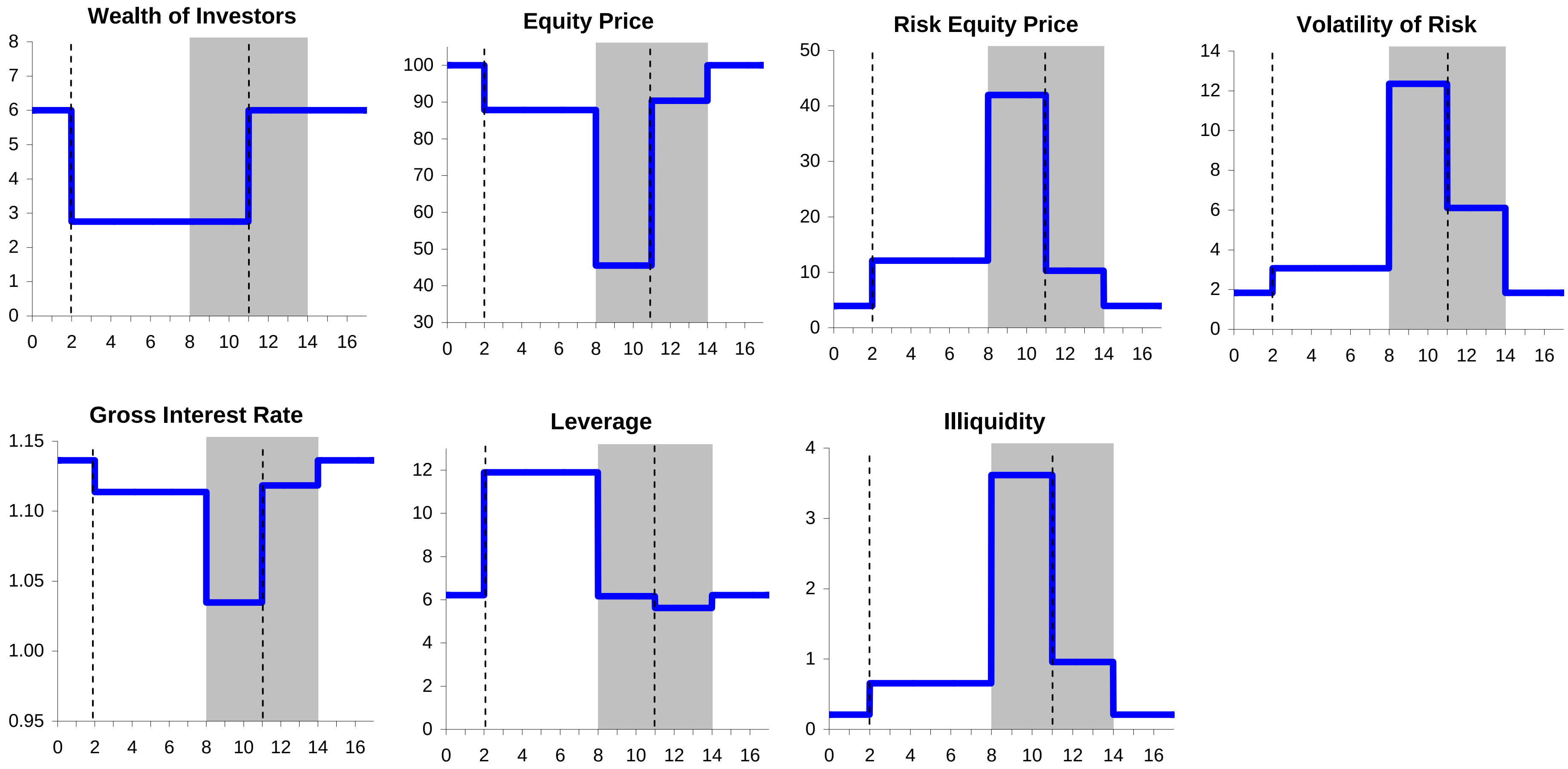


Figure 18. Model simulation: Endowment shocks for investors only (δ = 0 and n = 0)

shaded area = high risk equilibrium; vertical lines = endowment shock

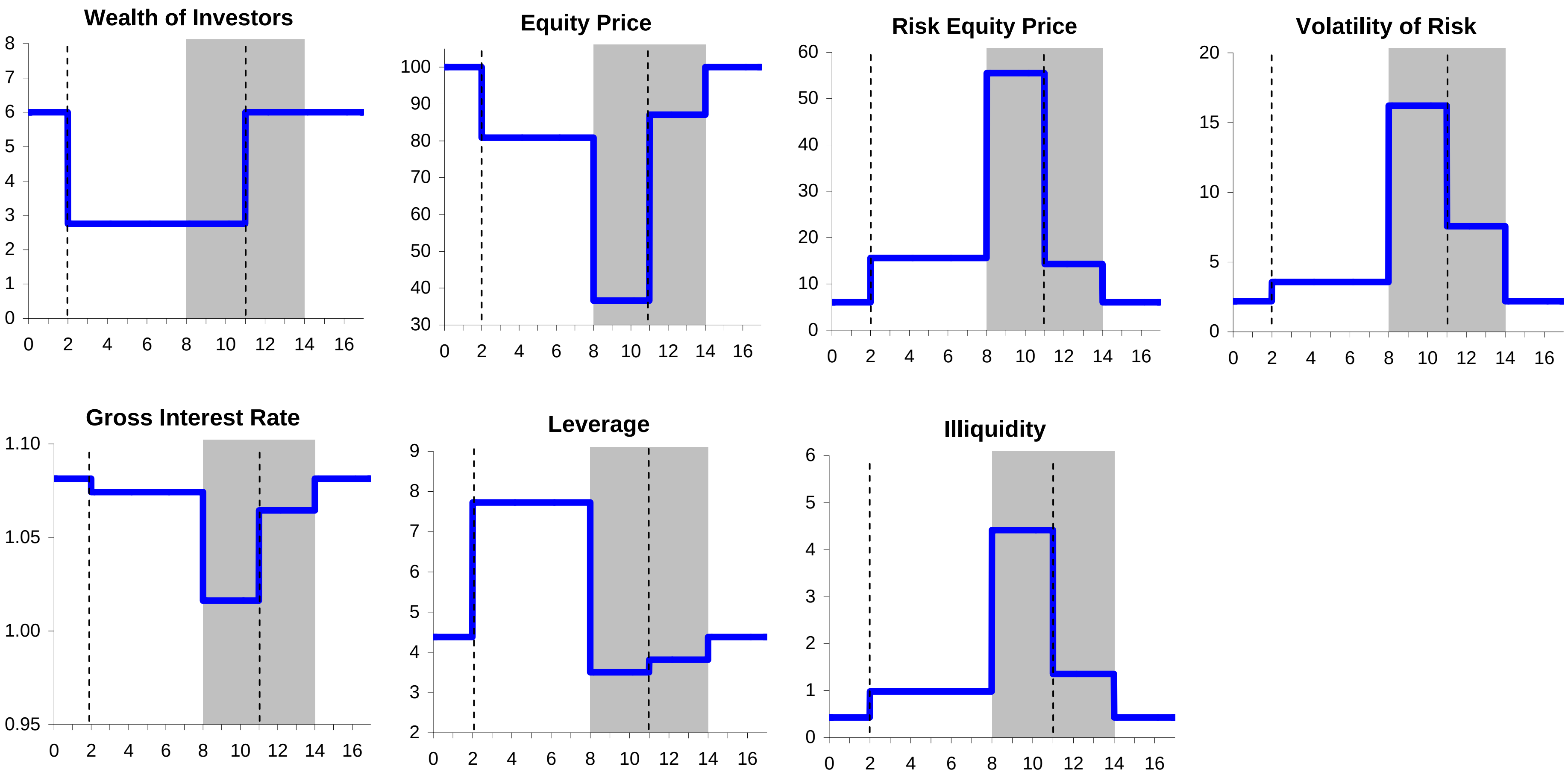


Figure 19. Model simulation: Endowment in goods and trees (δ = 0.1 and $n = m$)

shaded area = high risk equilibrium; vertical lines = endowment shock

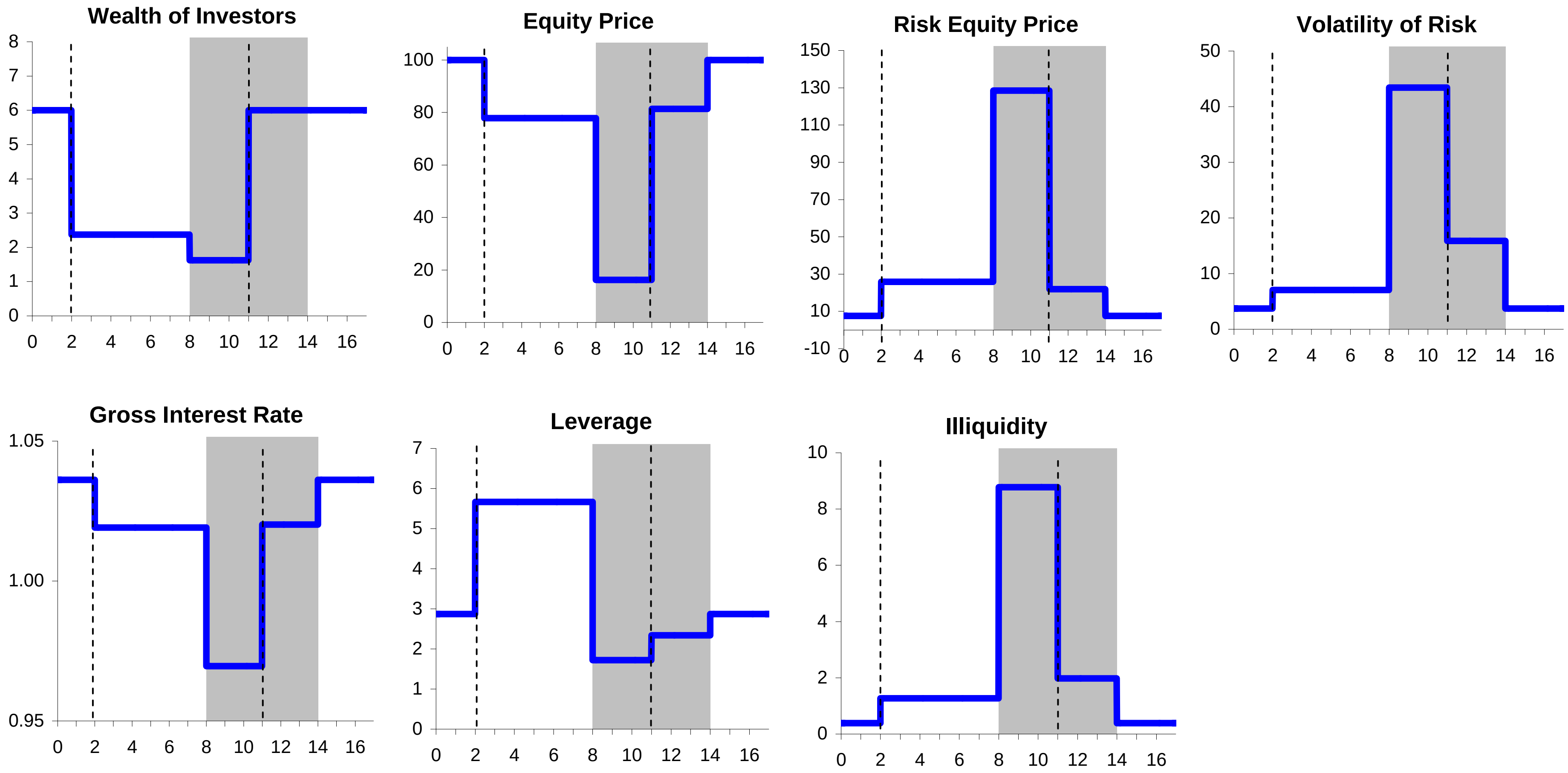


Figure 20. Model simulation: Cubic approximation

shaded area = high risk equilibrium; vertical lines = endowment shock