Technical Appendix: General Relationships Among Local Labor Supply Elasticities*

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This appendix serves two purposes. First, it examines what happens if the assumption of scale symmetry in consumption holds only approximately. The empirical literature often finds non-zero η^X or long-run elasticities, but since these estimates are generally close to zero, it is important to show what happens when our restriction holds approximately. Second, it uses this more general assumption to derive the expressions among the local labor elasticities discussed in brief in "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?" (2008).

At an interior solution to the household's problem, it is convenient to use the Frisch dual problem to study relationships among local labor supply elasticities. Defining

$$\mu = \frac{1}{\lambda},$$

let

$$\Phi(\mu, W_1, W_2) = \max_{C,N_1,N_2} \mu U(C, N_1, N_2) + W_1 N_1 + W_2 N_2 - C.$$

(The single and single earner cases can be seen as special cases of this dual earner case in which the share of labor income for one household member is zero.) By the envelope theorem,

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$$\frac{\partial \Phi}{\partial \mu} = U(\mu, W_1, W_2)$$

$$\frac{\partial \Phi}{\partial W_i} = N_i(\mu, W_1, W_2).$$

Define "net expenditure" X by

$$X = C - W_1 N_1 - W_2 N_2.$$

Then

$$C(\mu, W_1, W_2) = \mu \frac{\partial \Phi}{\partial \mu} + W_1 \frac{\partial \Phi}{\partial W_1} + W_2 \frac{\partial \Phi}{\partial W_2} - \Phi$$

and

$$X(\mu, W_1, W_2) = \mu \frac{\partial \Phi}{\partial \mu} - \Phi.$$

We will begin by expressing elasticities in terms of the labor income ratios

$$h_i = \frac{W_i N_i}{C}$$

and the standardized second derivatives of Φ defined by

$$\phi_{\mu\mu} = \frac{\mu^2}{C} \frac{\partial^2 \Phi}{\partial^2 \mu}$$

$$\phi_{\mu i} = \phi_{i\mu} = \frac{\mu W_i}{C} \frac{\partial^2 \Phi}{\partial \mu \partial W_i}$$

$$\phi_{ij} = \frac{W_i W_j}{C} \frac{\partial^2 \Phi}{\partial W_i \partial W_j}.$$

With X as one alternative out of X, λ, C, U A general notation for the wage elasticities we are interested in is

$$\eta_{ij}^X = \left.\frac{\partial \ln N_i}{\partial \ln W_j}\right|_{X=\text{constant},\ W_k=\text{constant for } k\neq j},$$

$$\eta_i^X = \eta_{i1}^X + \eta_{i2}^X = \left.\frac{\partial \ln N_i}{\partial \ln W}\right|_{X=\text{constant},\ W_2/W_1=\text{constant}}.$$

$$\eta^X = \frac{h_1\eta_1^X + h_2\eta_2^X}{h_1 + h_2}.$$

Thus, η_i^X is an elasticity with respect to a proportional increase in both wages, while η^X is a labor income weighted average of the individual η_i^X elasticities.

These definitions and the fact that $\mu =$ constant is the same thing as $\lambda =$ constant allow one to lay out the following:

$$\frac{\partial \ln N_i(\mu, W_1, W_2)}{\partial \ln \mu} = \frac{\phi_{\mu i}}{h_i} \tag{1}$$

$$\frac{\partial \ln N_i(\mu, W_1, W_2)}{\partial \ln W_j} = \eta_{ij}^\lambda = \frac{\phi_{ij}}{h_i} \tag{2}$$

$$\eta_i^\lambda = \frac{\phi_{i1} + \phi_{i2}}{h_i} \tag{3}$$

$$\eta^\lambda = \frac{\phi_{11} + 2\phi_{12} + \phi_{22}}{h_1 + h_2} \tag{4}$$

$$\frac{\partial \ln C(\mu, W_1, W_2)}{\partial \ln \mu} = \phi_{\mu\mu} + \phi_{\mu 1} + \phi_{\mu 2} \tag{5}$$

$$\frac{\partial \ln C(\mu, W_1, W_2)}{\partial \ln W_i} = \phi_{i\mu} + \phi_{i1} + \phi_{i2} \tag{6}$$

$$\frac{1}{C}\frac{\partial X}{\partial \ln \mu} = \phi_{\mu\mu} \tag{7}$$

$$\frac{1}{C}\frac{\partial X}{\partial \ln W_i} = \phi_{\mu i} - h_i. \tag{8}$$

$$\frac{\mu}{C}\frac{\partial U}{\partial \ln \mu} = \phi_{\mu\mu} \tag{9}$$

$$\frac{\mu}{C}\frac{\partial U}{\partial \ln W_i} = \phi_{\mu i}. \tag{10}$$

The absolute values of the local marginal propensities to earn are given by the fraction of extra net expenditure devoted to reduced work hours when μ varies, holding W_1 and W_2 constant:

$$\ell_i = \frac{-W_i \frac{\partial N_i}{\partial \ln \mu}}{\frac{\partial X}{\partial \ln \mu}} = \frac{-h_i \frac{\partial \ln N_i}{\partial \ln \mu}}{\frac{1}{C}\frac{\partial X}{\partial \ln \mu}} = -\frac{\phi_{\mu i}}{\phi_{\mu\mu}}. \tag{11}$$

The marginal propensity to consume out of an increase in net expenditure X is

$$1 - \ell_1 - \ell_2 = \frac{\phi_{\mu\mu} + \phi_{\mu 1} + \phi_{\mu 2}}{\phi_{\mu\mu}} = \frac{\frac{\partial \ln C}{\partial \mu}}{\frac{1}{C}\frac{\partial X}{\partial \ln \mu}} = \left.\frac{\partial C}{\partial X}\right|_{W_1, W_2 = \text{constant}} \tag{12}$$

Given the nature of our evidence, which is first and foremost about income effects, it is reasonable to think of the marginal propensities to earn ℓ_1 and ℓ_2 as the most robustly identified of all the local elasticities if the functional form is loosened up. Therefore, we focus on deriving equations that determine other quantities in terms of ℓ_1 and ℓ_2, among other fundamentals. In particular, hereafter we will routinely write $-\ell_i\phi_{\mu\mu}$ in place of $\phi_{\mu i}$:

$$\phi_{\mu i} = -\ell_i \phi_{\mu\mu} \tag{13}$$

Given h_1 and h_2, knowing ℓ_1 and ℓ_2 determine two of the six dimensions of the standardized second derivatives ϕ. We need four more restrictions to pin down the other four dimensions. The degree of departure from scale symmetry in consumption, or alternatively the value of the overall uncompensated labor supply elasticity η^X will provide one more restriction. Two more restrictions will come from imposing the degree of additive nonseparability between consumption and each of the two types of labor. The last restriction will come from imposing either the value of ϕ_{12} or the closely related elasticity of substitution between N_1 and N_2. But in the leading case the elasticity of substitution between N_1 and N_2 does not affect the elasticities η_i with respect to proportional increases in both wages.

A convenient way to measure the degree of nonseparability between consumption and the two type of labor by α_1 and α_2 in the definition

$$d\ln C = sd\ln\mu + \alpha_1 h_1 d\ln N_1 + \alpha_2 h_2 d\ln N_2. \tag{14}$$

Literally, the parameter s is the labor-constant elasticity of intertemporal substitution for consumption. Ultimately we will use α_1, α_2 and the degree of departure from scale symmetry in consumption to eliminate s since in our context where the interest rate is constant and always equal to ρ it cannot be functioning as the elasticity of intertemporal substitution for consumption. To relate α_i to the standardized second derivatives ϕ, substitute

$$d\ln N_i = \frac{1}{h_i}[-\ell_i\phi_{\mu\mu}d\ln\mu + \phi_{i1}d\ln W_1 + \phi_{i2}d\ln W_2] \tag{15}$$

into (14):

$$\begin{aligned} d\ln C &= [s-(\alpha_1\ell_1+\alpha_2\ell_2)\phi_{\mu\mu}]d\ln\mu + [\alpha_1\phi_{11}+\alpha_2\phi_{12}]d\ln W_1 \\ &\quad +[\alpha_1\phi_{12}+\alpha_2\phi_{22}]d\ln W_2 \end{aligned} \tag{16}$$

Comparing (16) to (5) and (6), it is clear after using (13) and rearranging that

$$[1-\ell_1(1-\alpha_1)-\ell_2(1-\alpha_2)]\phi_{\mu\mu} = s \tag{17}$$

$$\phi_{\mu\mu}\begin{bmatrix}\ell_1\\ \ell_2\end{bmatrix} = \begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{12} & \phi_{22}\end{bmatrix}\begin{bmatrix}1-\alpha_1\\ 1-\alpha_2\end{bmatrix} \tag{18}$$

There is a close relationship between the degree of nonseparability between consumption and labor indicated by α_1 and α_2 and how closely the utility function comes to scale symmetry in consumption. Define

$$\theta_i = \left.\frac{\partial\ln W_i}{\partial\ln C}\right|_{N_1,N_2=\text{constant}}.$$

Scale symmetry in consumption implies $\theta_1 = \theta_2 = 1$. More generally, weak separability between consumption and an aggregate of the two types of labor implies $\theta_1 = \theta_2 = \theta$, since weak separability means that a change in C holding N_1 and N_2 constant should not change the slope of the indifference curve between N_1 and N_2, which is W_1/W_2. From equation (15), one can see that $d\ln N_1 = d\ln N_2 = 0$ requires

$$\phi_{\mu\mu}\begin{bmatrix}\ell_1\\ \ell_2\end{bmatrix} d\ln\mu = \begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{12} & \phi_{22}\end{bmatrix}\begin{bmatrix}d\ln W_1\\ d\ln W_2\end{bmatrix} \tag{19}$$

As long as

$$\begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{12} & \phi_{22}\end{bmatrix}$$

is nonsingular (equivalent to the reasonable assumption of a nonzero Frisch labor supply elasticity for any linear combination of N_1 and N_2), (18) and (19) together imply that

$$\left.\frac{\partial \ln W_i}{\partial \ln \mu}\right|_{N_1,N_2=\text{constant}} = 1-\alpha_i \tag{20}$$

Combining (20) with the definition in (14) that

$$\left.\frac{\partial C}{\partial \ln \mu}\right|_{N_1,N_2=\text{constant}} = s, \tag{21}$$

one can solve for θ_i:

$$\theta_i = \left.\frac{\partial \ln W_i}{\partial \ln C}\right|_{N_1,N_2=\text{constant}} = \frac{1-\alpha_i}{s} \tag{22}$$

One consequence of equation (22) is that weak separability between consumption and a labor aggregate implies not only $\theta_1 = \theta_2 = \theta$, but also $\alpha_1 = \alpha_2 = \alpha$. Another consequence is that s can be eliminated by substituting

$$s = \frac{1-\alpha_i}{\theta_i}. \tag{23}$$

Also, given (17),

$$\phi_{\mu\mu} = \frac{1-\alpha_i}{\theta_i[1-\ell_1(1-\alpha_1)-\ell_2(1-\alpha_2)]}. \tag{24}$$

The assumption of weak separability between consumption and a labor aggregate (or equivalently between consumption and a leisure aggregate) is attractive. We will focus on that case from here on. With weak separability between consumption and a labor aggregate, equation (24) becomes

$$\phi_{\mu\mu} = \frac{1-\alpha}{\theta[1-(1-\alpha)(\ell_1+\ell_2)]} \tag{25}$$

Also, substituting $\alpha_1 = \alpha_2 = \alpha$ into (18),

$$\phi_{i1} + \phi_{i2} = \frac{\ell_i \phi_{\mu\mu}}{1-\alpha} = \frac{\ell_i}{\theta[1-(1-\alpha)(\ell_1+\ell_2)]} \tag{26}$$

One obvious consequence is that

$$\frac{\phi_{11}+\phi_{12}}{\phi_{12}+\phi_{22}} = \frac{\ell_1}{\ell_2} \tag{27}$$

Also, by (3),

$$\eta_i^\lambda = \frac{\ell_i \phi_{\mu\mu}}{h_i(1-\alpha)} = \frac{\ell_i}{\theta h_i[1-(1-\alpha)(\ell_1+\ell_2)]}, \tag{28}$$

and by (4),

$$\eta^\lambda = \frac{(\ell_1+\ell_2)\phi_{\mu\mu}}{(h_1+h_2)(1-\alpha)} = \frac{\ell_1+\ell_2}{\theta(h_1+h_2)[1-(1-\alpha)(\ell_1+\ell_2)]}. \tag{29}$$

It is useful to relate η_i^λ to η^λ by the following implication of (28) and (29):

$$\eta_i^\lambda = \frac{\frac{\ell_i}{\ell_1+\ell_2}}{\frac{h_i}{h_1+h_2}} \eta^\lambda. \tag{30}$$

Both η_i^λ and η^λ are inversely proportional to θ. Therefore, a modest departure from scale symmetry in consumption leads to only a modest modification in the implied value of η_i^λ and η^λ as a function of ℓ_1, ℓ_2 and α. For example, if $\theta = 1.1$, so that consumption growing 2 percent per year with no trend in labor would imply W_i/C up 20 percent (or 22 percent after compounding) over the course of a century, then the implied value of η_i^λ would be $\frac{10}{11}$ as large as if strict scale symmetry in consumption held.

By (2) and (3), in terms of the unknown value of ϕ_{12},

$$\eta_{ii}^\lambda = \eta_i^\lambda - \frac{\phi_{12}}{h_i}$$

while

$$\eta_{12}^{\lambda} = \frac{\phi_{12}}{h_1}$$

$$\eta_{21}^{\lambda} = \frac{\phi_{12}}{h_2}.$$

These are quite useful formulas if N_1 and N_2 are Frisch separable, so that $\phi_{12} = 0$, as we assume in our primary functional form. If not, to get further, let's relate ϕ_{12} to the Frisch elasticity of substitution between N_1 and N_2.

Define the Frisch elasticity of substitution between N_1 and N_2 by

$$\sigma_{12}^{\lambda} = \left.\frac{\partial \ln(N_1/N_2)}{\partial \ln(W_1/W_2)}\right|_{\lambda=\text{constant},\ U=\text{constant}}. \tag{31}$$

From (9), (10) and (13),

$$\frac{\mu}{C} dU = \phi_{\mu\mu}[d\ln\mu - \ell_1 d\ln W_1 - \ell_2 d\ln W_2] = 0 \tag{32}$$

If $d\ln mu = -d\ln\lambda = 0$, this implies

$$\ell_1 d\ln W_1 + \ell_2 d\ln W_2 = 0 \tag{33}$$

or

$$d\ln W_1 = \frac{\ell_2}{\ell_1 + \ell_2} d\ln(W_1/W_2)$$

$$d\ln W_2 = \frac{-\ell_1}{\ell_1 + \ell_2} d\ln(W_1/W_2).$$

Thus (remembering that $d\ln\mu = 0$),

$$d\ln N_1 = \frac{\ell_2\phi_{11} - \ell_1\phi_{12}}{h_1(\ell_1 + \ell_2)} d\ln(W_1/W_2) \tag{34}$$

$$d\ln N_2 = \frac{\ell_2\phi_{12} - \ell_1\phi_{22}}{h_2(\ell_1 + \ell_2)} d\ln(W_1/W_2). \tag{35}$$

Combining (34) and (35),

$$h_1 d\ln N_1 + h_2 d\ln N_2 = \frac{\ell_2(\phi_{11} + \phi_{12}) - \ell_1(\phi_{12} + \phi_{22})}{\ell_1 + \ell_2}$$

Weak separability between consumption and a labor aggregate implies that

$$h_1 d\ln N_1 + h_2 d\ln N_2 = 0.$$

That is, N_1 and N_2 change in such a say as to stay on the same indifference curve between N_1 and N_2. Because the labor aggregate remains unchanged, consumption C must also remain unchanged to keep λ fixed.

Subtracting (35) from (34) and dividing through by $d\ln(W_1/W_2)$ yields after simplification using (28),

$$\sigma_{12}^{\lambda} = \frac{\ell_2\eta_1^{\lambda} + \ell_1\eta_2^{\lambda}}{\ell_1+\ell_2} - \frac{(h_1+h_2)\phi_{12}}{h_1h_2}. \tag{36}$$

Thus, ϕ_{12} is given by

$$\phi_{12} = \frac{h_1h_2}{h_1+h_2}\left\{\frac{\ell_2\eta_1^{\lambda} + \ell_1\eta_2^{\lambda}}{\ell_1+\ell_2} - \sigma_{12}^{\lambda}\right\}. \tag{37}$$

Thus, ϕ_{12} differs from zero when the elasticity of substitution between N_1 and N_2 differs from a weighted average of the Frisch labor supply elasticities of N_1 and N_2. The lower the elasticity of substitution between N_1 and N_2, the more Frisch complementarity there is between N_1 and N_2.

Let us examine uncompensated labor supply elasticity η^X next, since the size of η^X is an alternative way of measuring the degree of departure from strict scale symmetry in consumption. Equations (7) and (8), together with (13) imply that

$$\frac{dX}{C} = \phi_{\mu\mu} d\ln\mu - [\ell_1\phi_{\mu\mu} + h_1] d\ln W_1 - [\ell_2\phi_{\mu\mu} + h_2] d\ln W_2.$$

Therefore, $dX = 0$ implies

$$d\ln\mu = \left(\frac{h_1}{\phi_{\mu\mu}} + \ell_1\right) d\ln W_1 + \left(\frac{h_2}{\phi_{\mu\mu}} + \ell_2\right) d\ln W_2$$

and by (15),

$$\eta_{ij}^{X} = \frac{1}{h_i}\left[\phi_{ij} - \ell_i\phi_{\mu\mu}\left(\frac{h_j}{\phi_{\mu\mu}} + \ell_j\right)\right] = \eta_{ij}^{\lambda} - \frac{\ell_i\ell_j\phi_{\mu\mu}}{h_i} - \frac{\ell_i h_j}{h_i}. \tag{38}$$

Adding up,

$$\eta_i^X = \eta_i^\lambda - \frac{\ell_i}{h_i}\left[(\ell_1+\ell_2)\phi_{\mu\mu} + \frac{\ell_i(h_1+h_2)}{h_i}\right], \tag{39}$$

and averaging with labor income weights,

$$\eta^X = \eta^\lambda - \frac{(\ell_1+\ell_2)^2}{h_1+h_2}\phi_{\mu\mu} - (\ell_1+\ell_2). \tag{40}$$

Note that using (30), we obtain a similar relationship:

$$\eta_i^X = \frac{\frac{\ell_i}{\ell_1+\ell_2}}{\frac{h_i}{h_1+h_2}}\eta^X. \tag{41}$$

The similarity to (30) is a consequence of the structure imposed by weak separability between consumption and a labor aggregate.

Adding $\ell_1+\ell_2$ to both sides of (40) and substituting in the expression for η^λ in (29),

$$\eta^X + \ell_1 + \ell_2 = \phi_{\mu\mu}\left(\frac{(\ell_1+\ell_2)[1-(1-\alpha)(\ell_1+\ell_2)]}{(h_1+h_2)(1-\alpha)}\right) \tag{42}$$

Equations (42) and (25) imply

$$\begin{aligned}\phi_{\mu\mu} &= (\eta^X+\ell_1+\ell_2)\frac{(h_1+h_2)(1-\alpha)}{(\ell_1+\ell_2)[1-(1-\alpha)(\ell_1+\ell_2)]} \\ &= \frac{1-\alpha}{\theta[1-(1-\alpha)(\ell_1+\ell_2)]}.\end{aligned} \tag{43}$$

Thus,

$$\theta = \frac{\ell_1+\ell_2}{(h_1+h_2)(\eta^X+\ell_1+\ell_2)} \tag{44}$$

and

$$\eta^X = (\ell_1+\ell_2)\left[\frac{1}{\theta(h_1+h_2)} - 1\right] \tag{45}$$

Since η^X is more easily observed than θ, it is good to have an expressions for η^λ in terms of η^X instead of θ. Substituting in from (44), (28) and (29) become

$$\eta_i^\lambda = \frac{\ell_i(h_1+h_2)(\eta^X+\ell_1+\ell_2}{(\ell_1+\ell_2)h_i[1-(1-\alpha)(\ell_1+\ell_2)]}, \tag{46}$$

$$\eta^\lambda = \frac{\eta^X+\ell_1+\ell_2}{[1-(1-\alpha)(\ell_1+\ell_2)]}. \tag{47}$$

This implies that as long as $\ell_1+\ell_2$ is substantial compared to the size of η^X, the difference between η^X and zero does not change the overall picture of the size of the elasticity η^λ.

In addition to using η^X to gauge the size of θ, it is possible to use either cross-elasticity η_{12}^X or η_{21}^X to gauge ϕ_{12}. Substitute in from (43) for $\phi_{\mu\mu}$ into (38) and rearrange to get

$$\begin{aligned}\phi_{12} &= h_1\eta_{12}^X + h_2\ell_1 + \ell_1\ell_2(\eta^X+\ell_1+\ell_2)\frac{(h_1+h_2)(1-\alpha)}{(\ell_1+\ell_2)[1-\alpha(\ell_1+\ell_2)]} \\ &= h_2\eta_{21}^X + h_1\ell_2 + \ell_1\ell_2(\eta^X+\ell_1+\ell_2)\frac{(h_1+h_2)(1-\alpha)}{(\ell_1+\ell_2)[1-\alpha(\ell_1+\ell_2)]}.\end{aligned} \tag{48}$$

The two versions of the formula reflect the Slutsky symmetry condition.

To complete the set of elasticities, formulas for η^C and η^U are in order. By (5), (6) and (13),

$$d\ln C = (1-\ell_1-\ell_2)\phi_{\mu\mu}d\ln\mu + [\phi_{11}+\phi_{12}-\ell_1\phi_{\mu\mu}]d\ln W_1 + [\phi_{12}+\phi_{22}-\ell_2\phi_{\mu\mu}]d\ln W_2. \tag{49}$$

Thus, $d\ln C = 0$ implies

$$d\ln\mu = \frac{1}{[1-\ell_1-\ell_2]\phi_{\mu\mu}}\left[(\ell_1\phi_{\mu\mu}-\phi_{11}-\phi_{12})d\ln W_1 + (\ell_2\phi_{\mu\mu}-\phi_{12}-\phi_{22})d\ln W_1\right].$$

Then

$$\begin{aligned}\eta_{ij}^C &= \frac{1}{h_i}\left\{\phi_{ij} - \frac{\ell_i\phi_{\mu\mu}}{(1-\ell_1-\ell_2)\phi_{\mu\mu}}\left[\ell_j\phi_{\mu\mu}-\phi_{j1}-\phi_{j2}\right]\right\} \\ &= \eta_{ij}^\lambda + \frac{\ell_i}{h_i(1-\ell_1-\ell_2)}\left\{h_j\eta_j^\lambda - \ell_j\phi_{\mu\mu}\right\}.\end{aligned} \tag{50}$$

Adding over j, and using (29), (30) and (43),

$$\begin{aligned}\eta_i^C &= \frac{\ell_i\phi_{\mu\mu}[1-(1-\alpha)(\ell_1+\ell_2)]}{h_i(1-\ell_1-\ell_2)(1-\alpha)} \\ &= \frac{\ell_i}{\theta h_i(1-\ell_1-\ell_2)} \\ &= \frac{\ell_i(h_1+h_2)(\eta^X+\ell_1+\ell_2)}{h_i(\ell_1+\ell_2)(1-\ell_1-\ell_2)} \end{aligned} \tag{51}$$

Averaging with labor income weights,

$$\eta^C = \frac{\ell_1+\ell_2}{\theta(h_1+h_2)(1-\ell_1-\ell_2)} = \frac{\eta^X+\ell_1+\ell_2}{1-\ell_1-\ell_2} \tag{52}$$

Note that

$$\eta_i^C = \frac{\frac{\ell_i}{\ell_1+\ell_2}}{\frac{h_i}{h_1+h_2}}\eta^C. \tag{53}$$

Again, this is a reflection of the assumption of weak separability between consumption and a labor aggregate.

To find η^U, use (32) in the form

$$d\ln\mu = \ell_1 d\ln W_1 - \ell_2 d\ln W_2 \tag{54}$$

Then

$$\eta_{ij}^U = \frac{\phi_{ij}-\ell_i\phi_{\mu\mu}}{h_i} = \eta_{ij}^\lambda - \frac{\ell_i\ell_j\phi_{\mu\mu}}{h_i}. \tag{55}$$

Adding up over j and using (43), (29) and (30)

$$\begin{aligned}\eta_i^U &= \frac{\ell_i[1-(1-\alpha)(\ell_1+\ell_2)]\phi_{\mu\mu}}{h_i(1-\alpha)} \\ &= \frac{\ell_i}{\theta h_i} \\ &= \frac{\ell_i(h_1+h_2)(\eta^X+\ell_1+\ell_2)}{h_i(\ell_1+\ell_2)} \end{aligned} \tag{56}$$

Finally, averaging over i with labor income weights,

$$\eta^U = \frac{\ell_1 + \ell_2}{\theta(h_1 + h_2)} = \eta^X + \ell_1 + \ell_2 \tag{57}$$

Not surprisingly,

$$\eta_i^U = \frac{\frac{\ell_i}{\ell_1+\ell_2}}{\frac{h_i}{h_1+h_2}} \eta^U. \tag{58}$$

The foregoing equations show the most important relationships. The one remaining task is show how to find the other elasticities from η^λ, which is what we literally do after finding η^λ from the parameteric model. Inverting equation (29) yields

$$\ell_1 + \ell_2 = \frac{\theta(h_1 + h_2)\eta^\lambda}{1 + \theta(1-\alpha)(h_1 + h_2)\eta^\lambda}. \tag{59}$$

Substituting from (59) into (52) and (57) yields

$$\eta^C = \frac{\eta^\lambda}{1 - \theta\alpha(h_1 + h_2)\eta^\lambda} \tag{60}$$

$$\eta^U = \frac{\eta^\lambda}{1 + \theta(1-\alpha)(h_1 + h_2)\eta^\lambda} \tag{61}$$

Using (57) again to find η^X from $\eta^X = \eta^U - \ell_1 - \ell_2$, one finds that

$$\eta^X = \frac{[1 - \theta(h_1 + h_2)]\eta^\lambda}{1 + \theta(1-\alpha)(h_1 + h_2)\eta^\lambda} \tag{62}$$

Equation (30) implies

$$\frac{\frac{\ell_i}{\ell_1+\ell_2}}{\frac{h_i}{h_1+h_2}} = \frac{\eta_i^\lambda}{\eta^\lambda}. \tag{63}$$

Together with (53), (58) and (41), (63) implies that one can find the individual elasticities η_i^C, η_i^U and η_i^X by multiplying the corresponding household average elasticities by $\frac{\eta_i^\lambda}{\eta^\lambda}$. The individual local MPE ℓ_i can be found as

$$\ell_i = \frac{h_i\eta_i^\lambda}{(h_1 + h_2)\eta^\lambda}(\ell_1 + \ell_2) = \frac{\theta h_i \eta_i^\lambda}{1 + \theta(1-\alpha)(h_1 + h_2)\eta^\lambda} \tag{64}$$

Finally, in the main text, we discuss the individual own-wage uncompensated elasticity in a dual earner setting: η_{ii}^X. Equations (38), (43), (59) and (64) imply

$$\eta_{ii}^X = \eta_{ii}^\lambda - \frac{\theta h_i \eta_i^\lambda [1 + \theta(1-\alpha) h_i \eta_i^\lambda]}{1 + \theta(1-\alpha)(h_1 + h_2)\eta^\lambda}. \qquad (65)$$

In translating these formulas into those in the main text, set $\theta = 1$ to impose scale symmetry in consumption and $\eta_{ii}^\lambda = \eta_i^\lambda$ to impose Frisch independence of N_1 and N_2. Also, remember that similarly to the other overall household elasticities designated by η,

$$\eta^\lambda = \frac{h_1 \eta_1^\lambda + h_2 \eta_2^\lambda}{h_1 + h_2}$$

and that

$$h_i = \frac{W_i N_i}{C}.$$