

# Fiscal Cost to Exit Quantitative Easing: The Case of Japan

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## Abstract

This paper simulates the cash flows and balance sheets of the Bank of Japan (BoJ) before and after the end of Quantitative and Qualitative Monetary Easing (QQE) under various scenarios. The simulations show that the BoJ will record significant accounting losses after the end of QQE, because the yields on Japanese government bonds (JGBs) acquired during QQE will be lower than the interest rate on excess reserves due to the normalization of the short-term interest rate after the end of QQE. These losses are fiscal costs for the consolidated Japanese government, as they correspond to increased interest expenses to the public. The extent of the BoJ's accounting losses depends crucially on the duration of QQE and the interest-rate elasticity of banknote demand.

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# 1 Introduction

The Bank of Japan (BoJ) commenced its “Quantitative and Qualitative Monetary Easing” (QQE) policy in April 2013. The purpose of this policy was to achieve a two percent annual CPI inflation rate at the earliest possible time, with a time horizon of about two years. Through this policy, the BoJ has more than doubled the monetary base and the holdings of Japanese government bonds (JGBs).<sup>1</sup> The BoJ has also doubled the average remaining maturity of newly purchased JGBs. As a result, the size of the BoJ’s balance sheet relative to GDP has become far larger than those in the US and the euro area, as shown in Figure 1. Figure 2 summarizes the composition of the BoJ’s assets between 1998 and 2015.

QQE remains ongoing. In April 2015, two years after the onset of QQE, the BoJ announced that it expected to achieve the two percent annual inflation target in the first half of fiscal year 2016; thus it could not achieve the inflation target as initially planned. In April 2016, the BoJ revised the expected time of the achievement of the inflation target to fiscal year 2017.

While QQE continues, the BoJ has not made any announcement on a QQE exit strategy yet. For example, at the Standing Committee on Audit in the Upper House of the National Diet of Japan in June 2015, BoJ Governor Haruhiko Kuroda said that it was too early to discuss an exit strategy as the appropriate strategy would depend on economic and financial market conditions at the time of exit.<sup>2</sup>

Nevertheless, it is important to discuss the BoJ’s exit strategy before exit because of a possibility that the BoJ will report significant accounting losses after the end of QQE. If the BoJ raises the policy interest rate after the achievement of the inflation target, it may then incur a capital loss associated with the decline in the market price of its long-term bond holdings, as suggested by Goodfriend (2000) and simulated by Fujiki, Okina, and Shiratsuka

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<sup>1</sup>The monetary base increased from 138 trillion yen at the end of 2012 to 380 trillion yen as of June 2016. Over the same period, its JGB holdings increased from 89 trillion yen to 373 trillion yen.

<sup>2</sup>There is a dissenting BOJ Policy Board member who is expressing his view on the possible effects of QQE on the BOJ’s profit and balance sheet in the course of exiting QQE in the future (See Kiuchi 2015).

(2001). While the BoJ has adopted an amortized cost method for its accounting since 2004, so that a mark-to-market capital loss will not appear on its balance sheet if it holds long-term assets until maturity, it will still need to maintain central-bank current account balances held by banks (i.e., reserve balances) at a high level for an extended period of time in such a case. In this case, the BoJ will need to pay interest on excess reserves to control the policy rate. It is possible that the interest expenses on excess reserves exceed the interest revenues from asset holdings, causing the BoJ to run accounting losses. The BoJ's payments to the Japanese government will then be zero, or even negative if it receives government subsidies to cover any part of the losses.

In theory, the central bank is part of the consolidated government, so its accounting loss per se should not matter as long as the government and the central bank arrange a loss-sharing rule in advance. Such a rule, however, is not envisioned by the law governing the Bank of Japan, the Bank of Japan Act, currently. Also, Ueda (2003), a then Policy Board member at the BoJ, pointed out that if the government could be committed to a loss-sharing rule with the central bank without a significant problem, then central-bank independence would not be an issue in the first place. He raised a concern that if the central bank runs negative equity, then the government may intervene in the central bank's monetary policy through capital injection. Indeed, the current level of fiscal debt in Japan is very high.<sup>3</sup> Some argue that there is a concern that fiscal policy will dominate monetary policy to maintain a low interest rate on government fiscal debt (Ikeo 2013; Okina 2015).

In the US, several existing studies examine the Federal Reserve's balance sheet, which was expanded due to the Large-Scale Asset Purchases programs operating between 2008 and 2014. For example, Carpenter et al. (2015) and Greenlaw et al. (2013) simulate the Federal Reserve's net profit and net assets after the end of the quantitative easing (QE). Also, other studies consider the effect of the Federal Reserve's net assets on price stability. For example, Del Negro and Sims (2014) argue that the central bank cannot guarantee price

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<sup>3</sup>The ratio of fiscal debt to GDP in Japan is 232% in 2016, according to the estimate by the OECD.

stability if the government is not committed to sharing losses with the central bank, because the central bank may tolerate a high inflation rate to earn seigniorage. They show that this possibility leads to multiple equilibria because of self-fulfilling public expectations. Likewise, Reis (2015) classifies three types of central bank insolvency (negative current profit; violation of the rule for the distribution of current profit; and intertemporal insolvency). Reis argues that the inflation rate will depend on the type of insolvency applied to the central bank, as the central bank will adjust seigniorage to achieve the given condition for its solvency. Regarding a loss-sharing rule between the central bank and the government, Goodfriend (2014) has proposed that the Federal Reserve should retain its profit before the exit from QE to build a loss-absorbing capital buffer for post-exit losses.

In this paper, we simulate the transition of the BoJ's balance sheet to analyze how QQE will account for accounting losses to the BoJ in the course of the exit from QQE. Our analysis largely draws on Carpenter et al.'s (2015) work in the US and also Iwata et al. (2014), which applied Carpenter et al.'s (2014) analysis to Japan in June 2014. Iwata et al. (2014) shows that the BoJ will report accounting losses after the end of QQE, even without the expansion of the BOJ's asset purchase program in October 2014. The contribution of our analysis to the literature is to incorporate subsequent policy changes by the BoJ and also to simulate the transition of the BoJ's balance sheet under various scenarios to identify key determinants of its losses after the end of QQE. We find that a longer duration of QQE and a higher interest-rate elasticity of banknote demand will increase the BoJ's accounting losses significantly. Thus, the BoJ should allow for an increased cost when extending the duration of QQE, and should not take the current banknote demand, about 19% of nominal GDP in June 2016, for granted. Furthermore, we find that if the BoJ does not cease, but instead tapers its asset purchase program in the exit from QQE, its accounting losses will decrease as it will earn profit through the term spread. This effect is significantly stronger if there is no safety channel, i.e., the effect of the bond supply on the long-term interest rate.

The remainder of the paper is organized as follows. Section 2 models the BoJ’s balance sheet. Section 3 sets up the simulation, including the description of data and the calibration of parameters. We report the results of the benchmark simulation in Section 4 and conduct a sensitivity analysis in Section 5. Section 6 concludes the paper.

## 2 Model of the Bank of Japan’s Accounts

We consider a simple model of the BoJ’s balance sheet, consisting of the following items:

JGBs	Reserve balances held by financial institutions
Other assets	Reserve balances held by the government
	Banknotes
	Other liabilities
	Net assets

Here, reserve balances are current account balances at the BoJ. Other assets and liabilities comprise various items, including Treasury bills (T-bills), commercial paper and bonds, exchange-traded funds (ETFs), loans and bills discounted, foreign currency assets, and repos.

To focus on the effect of large-scale purchases of JGBs on the BoJ’s balance sheet, we simplify the balance sheet by defining the difference between other assets and liabilities as “net short-term assets”. This approach ignores the presence of long-term assets other than JGBs, such as ETFs, real estate investment funds (REITs), and stocks, which are included in “Other assets” on the BoJ’s balance sheet. As of March 31, 2016, the total book value of these items is 9.3 trillion yen while the BoJ’s JGB holdings sum to 269 trillion yen. If a significant capital gain arises for these assets, then the BoJ’s cumulative accounting profit over time may increase up to a few trillion yen in total. As shown below, this figure is small compared to the simulated change in the BoJ’s cumulative accounting profit. Thus, our simplifying assumption does not affect the main results of the model described below.

In addition, in the model, we ignore the reserve balances held by the government. For most of the 2000s, the government’s reserve balances are between one and three percent

of those held by financial institutions at the end of each fiscal year. Thus, ignoring these balances also does not significantly affect the results of the model.<sup>4</sup>

Accordingly, we consider the following simplified BoJ's balance sheet:

JGBs ( $B_t$ )	Reserve balances held by financial institutions ( $D_t$ )
Net short-term assets ( $S_t$ )	Banknotes ( $C_t$ )
	Net assets ( $E_t$ )

The notations of variables are in parentheses. The subscript  $t$  denotes the fiscal year. Each variable represents the value at the beginning of the fiscal year.

## 2.1 Demand for banknotes and required reserves

The BoJ supplies required reserves and banknotes passively in response to the demands for bank deposits and cash from the public. We assume banknote demand depends on nominal GDP and the nominal interest rates:

$$C_t = \mu_t GDP_t, \tag{1}$$

where  $GDP_t$  denotes annual nominal GDP, and  $\mu_t$  is the banknotes-to-GDP ratio. We also assume a regime shift in  $\mu_t$  to capture the interest-rate elasticity of banknote demand:

$$\mu_t = \begin{cases} \bar{\mu} & \text{if } t \leq \tau, \\ \underline{\mu} & \text{if } t > \tau, \end{cases} \tag{2}$$

where  $\tau$  denotes the fiscal year when the BoJ achieves the inflation target. As assumed below, the BoJ raises the policy rate after this year. This simple specification of the interest rate elasticity is sufficient for our simulations, as our scenario analysis does not consider detailed business-cycle fluctuations in the nominal GDP growth rate and the nominal interest rate after  $t$  hits  $\tau$ .

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<sup>4</sup>We confirm this result by running simulations under the assumption that the government's reserve balance is one percent of those held by financial institutions.

Reserve balances held by financial institutions are split into required reserves ( $D_{REQ,t}$ ) and excess reserves ( $D_{EX,t}$ ):

$$D_{REQ,t} + D_{EX,t} = D_t. \quad (3)$$

The demand for required reserves is proportional to bank-deposit demand by the public:

$$D_{REQ,t} = \gamma \eta_t GDP_t, \quad (4)$$

where  $\gamma$  is the average reserve requirement ratio and  $\eta_t$  is the bank deposits-to-GDP ratio.

For a similar reason to (2), there is a regime shift in  $\eta_t$  when  $t$  hits  $\tau$ :

$$\eta_t = \begin{cases} \bar{\eta} & \text{if } t \leq \tau, \\ \underline{\eta} & \text{if } t > \tau. \end{cases} \quad (5)$$

## 2.2 Supply of excess reserves

The BoJ's flow of funds constraint determines the supply of excess reserves, given the BoJ's asset purchase policy on JGBs and net short-term assets. The BoJ's flow of funds constraint is written as

$$D_{t+1} - D_t = V_t - R_t - P_t + S_{t+1} - S_t - i_{ST,t}S_t + X_t + J_t + F_t + T_t - (C_{t+1} - C_t), \quad (6)$$

where  $V_t$  is the purchase value of newly purchased JGBs,  $R_t$  is the coupon payments on JGBs,  $P_t$  is the redemption of JGBs held by the BoJ at maturity,  $i_{ST,t}$  is the short-term nominal interest rate,  $X_t$  is the interest expense on reserves,  $J_t$  is the payment of after-tax profit to the government,  $F_t$  is the general and administrative expenses and costs, such as personnel expenses, and  $T_t$  is the corporate and residential taxes incurred by the BoJ. Note that on the right-hand side of (6), the BoJ's expenditures have a positive sign while the revenues have a negative sign. This is because the increment in reserve balances on the left-hand side of (6) is the BoJ's cash flow, but with the opposite sign. The new issue of banknotes,  $C_{t+1} - C_t$ , appears as the last term on the right-hand side of (6) with a minus sign, as banks obtain new banknotes by withdrawing their reserve balances.

In February 2016, the Bank of Japan introduced a three-tier system for interest payment on reserves. In the first tier, each bank receives 0.1% effective annual interest on the difference between the average reserve balance held by the bank in 2015, which amounts to around 220 trillion yen in total, and required reserves for the bank in each month. This difference is called Basic Reserves. On the second tier, called Macro Add-On Reserves, each bank receives 0% interest. This tier includes required reserves. The third tier, called Policy-Rate Reserves, is the remaining reserve balance. For this tier, each bank receives the policy interest rate set by the central bank, which is -0.1% as of June 2016. The size of Macro Add-on Reserves is determined so that the size of Policy-Rate Reserves carrying a negative interest rate is minimized while the interest rate on Policy-Rate Reserves still anchors the short-term interest rate in the financial market. The total amount of Policy-Rate Reserves is set to around 10 trillion yen as of June 2016.

Reflecting the three-tier system, the interest expense on reserves,  $X_t$ , is determined by the following rule:

$$X_t = \begin{cases} 0.001 \times (220 \text{ trillion yen} - D_{REQ,t}) + i_{CB,t} \times (10 \text{ trillion yen}) & \text{if } t \leq \tau, \\ i_{CB,t} D_{EX,t} & \text{if } t > \tau, \end{cases} \quad (7)$$

where  $i_{CB,t}$  is the central-bank interest rate on Policy-Rate Reserves before the end of QQE and on excess reserves after the end of QQE. We assume that the short-term interest rate is anchored by this interest rate:

$$i_{CB,t} = i_{ST,t}. \quad (8)$$

We simply assume that the general and administrative expenses and costs grow at the same rate as GDP:

$$F_t = \phi GDP_t. \quad (9)$$



## 2.3 Accounting profit and net assets

The BoJ transfers a fraction of its accounting profit to the government. The BoJ's pretax accounting profit,  $\pi_t$ , is defined as

$$\pi_t = R_t + A_t + i_{ST,t}S_t - X_t - F_t, \quad (10)$$

where  $A_t$  is the amortization of the discount on JGBs under the amortized cost method. Accordingly, the book value of JGBs,  $B_t$ , is updated through this method:<sup>5</sup>

$$B_{t+1} - B_t = V_t - P_t + A_t. \quad (11)$$

For simplicity, we assume that the corporate and residential taxes incurred by the BoJ are proportional to positive pretax profit:

$$T_t = \kappa \max\{0, \pi_t\}. \quad (12)$$

Under the current rule, the BoJ pays out a fraction of its after-tax annual accounting profit to the government the following year, and retains the remainder as part of its net assets. Thus, the payment of profit to the government,  $J_t$ , is determined by the following rule:

$$J_t = \max\{0, \theta(1 - \kappa)\pi_{t-1}\},$$

where  $\theta$  is the proportion of after-tax accounting profit to be transferred to the government.<sup>6</sup> Here, we assume that the BoJ does not make any transfer if it incurs an accounting loss for the year.

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<sup>5</sup>The BoJ has adopted this method since 2004, under which the difference between the face value and purchase value of JGBs held to maturity is evenly split over maturity to compute the amortization of the discount for each year. The amortized discount is then gradually recognized as part of interest revenues each year and added to the book value of JGBs, so that the book value equals the face value at maturity.

<sup>6</sup>Precisely speaking, the BoJ's remittance of profit to the government is accounted for as part of ordinary expenses when calculating the BoJ's pretax accounting profit. At the same time, the remittance is set to equal a certain percent of the BoJ's after-tax accounting profit. For simplicity, we ignore this practice to avoid the iteration, because the resulting adjustment is small.

The BoJ's net assets are simply the difference between the book value of its assets and its liabilities:

$$E_t = B_t + S_t - D_t - C_t. \quad (13)$$

Overall, (3)-(13) determine the transition of the BoJ's balance sheet and accounting profit, given the characteristics of the JGBs held by the BoJ ( $R_t$  and  $P_t$ ), the BoJ's asset purchase policy ( $V_t$  and  $S_{t+1} - S_t$ ), macroeconomic variables ( $i_{ST,t}$  and  $GDP_t$ ), and the timing of the achievement of the inflation target, which ends QQE ( $\tau$ ).

### 3 Simulation setup

Hereafter, we simulate the model by inserting data, including the current BoJ's asset purchase policy, under simple macroeconomic scenarios. This exercise aims to identify important factors that determine the cost to exit QQE. The starting month of the simulations is June 2016.

#### 3.1 Timing of the end of QQE

Currently, the BoJ is committed to achieving the inflation target in fiscal year 2017. Accordingly, we set  $\tau$  to 2017 for a benchmark. This assumption implies that the BoJ achieves the inflation target at the end of fiscal year 2017 (or March 2018) in the model, given the annual frequency of the model.

#### 3.2 BoJ's asset purchase policy before the end of QQE

In October 2014, the BoJ announced an expansion of QQE, which aims to increase the reserve balances held by financial institutions by 80 trillion yen per year until it achieves the inflation target. Thus, we assume

$$D_{t+1} - D_t = 80 \text{ trillion yen} \quad \text{if } t \leq \tau. \quad (14)$$

Table 1: Benchmark assumption on the BoJ’s monthly purchases of JGBs before the achievement of the inflation target

Maturity (years)	20	10	5	2	10 (Inflation-indexed)	15 (Floating-rate)
Purchase value (trillion yen)	2.25	2.7	2.4	2.4	0.05	0.05

To achieve this goal, the BoJ is committed to purchasing JGBs worth between 8 and 12 trillion yen each month. It sets a range for the monthly purchase value of JGBs for each maturity (see Appendix A). Using the median value of each range, we assume the BoJ’s monthly purchases of JGBs for  $t \leq \tau$  as shown in Table 1. The figures in the table imply

$$V_t = 118.2 \text{ trillion yen} \quad \text{if } t \leq \tau. \quad (15)$$

For simplicity, we assume that the terms of JGB purchases for each maturity, such as the price and the coupon rate, are the same as in the primary market at the end of May 2016.<sup>7</sup> Under this assumption, the average maturity of newly purchased JGBs is around nine years, which is consistent with the BoJ’s policy as of June 2016 that sets the average remaining maturity of newly purchased JGBs to between seven and twelve years.<sup>8</sup>

Given the JGB purchase policy set by (15), the BoJ adjusts its net short-term assets to increase the reserve balances held by financial institutions by 80 trillion yen per year. Thus,

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<sup>7</sup>There has been no new issuance of floating-rate JGBs since 2008. For this category, we assume the BoJ purchases these bonds at the same terms as those most recently issued in the secondary market at the end of May 2016. For simplicity, the remaining maturity for this category of bonds is set to seven years for any year of purchase.

<sup>8</sup>In this regard, an alternative assumption is to follow Carpenter et al. (2015) and use an estimated yield curve to calculate the price of discounted bonds for each maturity. We do not choose this option as we obtain a negative yield for short maturities if we insert the long-term rate before the introduction of a negative policy interest rate in February 2016 into the estimated yield curve. This observation implies that there is nonlinearity in the effect of the long-term interest rate on the yield curve.

(6) implies that

$$S_{t+1} - S_t = -V_t + R_t + P_t + i_{ST,t}S_t - X_t - J_t - F_t - T_t + C_{t+1} - C_t + D_{t+1} - D_t \text{ if } t \leq \tau \quad (16)$$

where  $D_{t+1} - D_t$  and  $V_t$  are set by (14) and (15), respectively.

### 3.3 BoJ's asset purchase policy after the end of QQE

We assume that after the end of QQE, the BoJ holds JGBs until maturity and does not make any new purchases:

$$V_t = 0 \text{ if } t > \tau. \quad (17)$$

Following the BoJ's policy in regular time, we also assume that the BoJ purchases T-bills, part of net short-term assets in the model, using the revenues from any redeemed JGBs for one year.<sup>9</sup> After that, the BoJ receives reserve balances from the government for the repayment of T-bills. Thus, reserve balances at the end of fiscal year,  $D_{t+1}$ , decline through (6) as JGBs held by the BoJ gradually mature.

In addition, the BoJ's balance of net short-term assets at the end of QQE is higher than the value of redeemed JGBs in the previous year in the simulations. We assume that the BoJ maintains this excess balance of net short-term assets after the end of QQE, because otherwise the balance of net short-term assets becomes unrealistically close to zero as JGBs held by the BoJ mature over time.

If reserve balances become insufficient for required reserves, then the BoJ supplies the necessary amount of reserve balances by adjusting its net short-term assets. Overall, the

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<sup>9</sup>This policy aims to mitigate the imbalance between demand and supply in the JGB market by rolling over redeemed JGBs for a short period of time.

BoJ's adjustment of net short-term assets is specified as

$$S_{t+1} - S_t = \begin{cases} P_t - P_{t-1} & \text{if } t > \tau \text{ and } D_{t+1} \geq D_{REQ,t+1}, \\ -V_t + R_t + P_t + i_{ST,t}S_t - X_t - J_t & \\ -F_t - T_t + C_{t+1} - C_t + D_{REQ,t+1} - D_t & \text{if } t > \tau \text{ and } D_{t+1} < D_{REQ,t+1} \\ & \text{under } S_{t+1} - S_t = P_t - P_{t-1}, \end{cases} \quad (18)$$

where the right-hand side of the second line is derived by substituting  $D_{t+1} = D_{REQ,t+1}$  into (6).

### 3.4 Data on BoJ's JGB holdings and balance sheet

The coupon payments ( $R_t$ ) and redemption ( $P_t$ ) of JGBs depend on the terms of past asset purchases by the BoJ. The BoJ publishes the face value of its JGBs for each issuance date. We combine these data with the coupon rate and the maturity for each issuance date published by the Ministry of Finance. The BoJ, however, does not publish the book value of its JGBs for each issuance date, but only the aggregate book value for each maturity. To compute the book value for each issuance date, we divide the aggregate book value for each maturity proportionally to the face value of each issuance date with that maturity.

For the initial value of the balance sheet items, we use the Bank of Japan Accounts as of the end of May 2016.

### 3.5 Benchmark scenario on the path of the inflation rate, the short-term interest rate, and the nominal GDP growth rate

For a benchmark, we assume that the annual inflation rate remains at 0% until the BoJ hits the inflation target (i.e.,  $t \leq \tau$ ) and stays at the targeted rate, 2%, after the achievement of the target (i.e.,  $t > \tau$ ). The short-term interest rate,  $i_{ST,t}$ , is set to -0.1% for  $t \leq \tau$ , which is the same as the current interest rate on Policy-Rate Balances. Then it is assumed to rise to 1.25% at  $t = T + 1$  and 2.75% for  $t \geq T + 2$ . The long-run short-term interest rate, 2.75%, is based on the observation that the spread between the geometric averages of

the overnight call rate and the CPI inflation rate over 1990–2012 was 0.8%. Here we use sample averages during the Lost Decades before the beginning of QQE in 2013. We assume that the BoJ takes two years to raise the short-term interest rate to its long-run level after the end of QQE, as the policy interest rate is usually adjusted gradually in practice. The nominal GDP growth rate is set to the inflation rate plus 0.3%, which is the spread between the geometric averages of the two rates over 1990–2012.

Even though this simple scenario ignores feedback from the BoJ’s balance sheet to aggregate economic activity, it allows us to run simulations under a transparent assumption. In this paper, we choose the benefit of considering a simple scenario based on past data, given the difficulty in precisely forecasting the long-term trend of GDP and other macroeconomic variables. Also, we do not use survey forecasts or financial market data, such as the yield curve, to set the time path of the interest rate, because these data reflect the average expectations over different contingencies, including the case in which the BoJ cannot achieve the inflation target within the announced deadline. In the benchmark scenario, we use the inflation rate as the anchor for the interest rate and nominal GDP by fixing the spread between each of the two variables and the inflation rate. Thus, the interest rate and nominal GDP move parallel to the inflation rate as the inflation target is achieved in this scenario.

### **3.6 Calibration of the remaining parameters**

For banknote demand before the achievement of the inflation target, we set  $\bar{\mu} = 0.178$ , which is the banknotes-to-GDP ratio in 2014. Here, we use the latest year for which the Annual Report on National Accounts is available, because if we use a sample average in some post-war period, the difference between the last data point and the sample average must be absorbed by an artificial jump in the BoJ’s net assets. For the period after the achievement of the inflation target, we set  $\underline{\mu} = 0.0797$ . This value is the sample average of the banknotes-to-GDP ratio over the 1990s, which is the most recent period before quantitative easing in Japan in the 2000s.

We set a similar assumption for the demand for required reserves. The bank deposits-to-GDP ratios before and after the achievement of the inflation target (i.e.,  $\bar{\eta}$  and  $\underline{\eta}$ ) are set to 2.08 and 1.53, respectively. These values are derived from the same sample periods as those used to set  $\bar{\mu}$  and  $\underline{\mu}$ . We set  $\gamma = 0.00593$ , which is the average reserve requirement ratio between 1992 and 2014, as there was a regulatory change in reserve requirement ratios in October 1991.<sup>10</sup> The ratio between the general and administrative expenses and costs and GDP,  $\phi$ , is set to 0.000395, which makes  $F_t$  equal 193 billion yen in 2014, as observed in the data. The corporate income and residential tax rate on the BoJ's pretax profit,  $\kappa$ , is set to 0.275, which is the average rate over the fiscal years between 2012 and 2014. We use only the data during these three years, as the corporate income tax rate changed in 2012. Finally, the proportion of the BoJ's after-tax accounting profit to be transferred to the government,  $\theta$ , is set to 0.95, following the Bank of Japan Act.<sup>11</sup>

## 4 Benchmark results

In this section, we summarize the result of the benchmark simulation. Figure 3 depicts the transition of the BoJ's balance sheet items. As shown in the figure, reserve balances gradually decline as JGBs held by the BoJ mature over time. It takes around 30 years until the excess reserves due to QQE disappear.

Figure 4 displays the major components of the BoJ's accounting profit. As shown, the BoJ records accounting losses between 2017 and 2037. Accordingly, the payment of profit to the government becomes zero for this period. The maximum annual loss amounts to 7.54 trillion yen in 2019.

These losses arise because of maturity transformation on the BoJ's balance sheet. As

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<sup>10</sup>The reserve requirement ratio differs across different categories of bank deposits. To set the value of  $\gamma$ , we compute the ratio between the sum of required reserves and the balance of bank deposits subject to reserve requirements for each fiscal year, and then take the sample average over 1992–2014.

<sup>11</sup>Under this act, the five percent of after-tax accounting profit must be added to the BoJ's legal reserve each year.

the low yields on JGBs acquired during QQE are predetermined, the interest expenses on excess reserves surpass the interest revenues from JGBs as the BoJ raises the interest rate on excess reserves to increase the short-term interest rate after QQE. These losses continue until reserve balances sufficiently decline with the gradual redemption of JGBs.

The fiscal cost of QQE for the consolidated government appears in the BoJ's accounting loss. Note that if there were no QQE, the public would hold JGBs purchased by the BoJ, instead of the massive amount of reserve balances supplied to their banks. In this case, the consolidated government would pay interest on JGBs to the public, rather than that on excess reserves. Thus, the difference between interest on JGBs held by the BoJ and that on excess reserves paid by the BoJ to banks corresponds to increased interest expenses made by the consolidated government to the public. A caveat is that this loss is the gross fiscal cost under a given path of the short-term interest rate. To measure the opportunity cost of QQE, we need to know the precise path of the macroeconomic variables in the instance of no QQE.

Figure 5 illustrates the transition of the BoJ's net assets. As shown in the figure, the accumulation of the accounting losses over 20 years results in an extended period of negative net assets for the BoJ. In the benchmark simulation, the BoJ recovers positive net assets in 2125 by gradually offsetting the accumulated loss with normal profit from the interest margin between net short-term assets and banknotes. Does the BoJ remain intertemporally solvent in the benchmark scenario? Following Del Negro and Sims (2014) and Reis (2015), we compute the BoJ's net worth, which takes into account the present discounted value of future pretax profit.<sup>12</sup> We use the short-term interest rate for the discount rate, as the BoJ's assets consist of net short-term assets in the long run. We find that the time path of the BoJ's net worth is always positive, about 183 trillion yen in 2018, and steadily increases afterwards as nominal GDP grows, as shown in Figure 6. Thus, the present discounted value

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<sup>12</sup>The BoJ's tax payments to the government are equivalent to the dividend payouts by a private company to its shareholders. Thus, we include pretax profit in the computation of the BoJ's net worth.



of future profit is sufficiently large to absorb the cumulative accounting losses after the end of QQE.

## 5 Sensitivity analysis

In this section, we discuss the sensitivity of the simulation result to alternative scenarios concerning the BoJ's behavior and the macroeconomic environment.

### 5.1 Longer duration of QQE

Figure 7 shows the BoJ's balance sheet when QQE lasts until fiscal year 2019 (i.e.,  $\tau = 2019$ ). In this case, the BoJ's JGB holdings reach 550 trillion yen at their peak to achieve the large amount of annual reserve supply targeted under QQE. As a result, the BoJ's accounting losses after the end of QQE increase because of the larger amount of maturity transformation on the BoJ's balance sheet, as indicated by Figure 8. Thus, the fiscal cost after the exit from QQE should be taken into account when the BoJ compares the cost and benefit of extending the duration of QQE.

### 5.2 Interest-rate elasticity of banknote demand

In the benchmark scenario, we assume that the banknotes-to-GDP ratio falls to its 1990s level as the short-term interest rate is normalized after the achievement of the inflation target. This assumption is important for the BoJ's accounting profit after the end of QQE, because the BoJ's profit in normal times stems from purchasing interest-bearing assets to supply banknotes to the public via banks. The supply of reserve balances is small relative to banknotes in normal times.

We consider three alternative scenarios for the interest-rate elasticity of banknote demand. In the first scenario, the banknotes-to-GDP ratio remains the same before and after the end of QQE (i.e.,  $\bar{\mu} = \underline{\mu} = 0.178$ ). Thus, in this scenario, banknote demand does not

respond to an increase in the short-term interest rate. Iwata et al. (2014) set this assumption for their simulation of the BoJ's balance sheet.

The second scenario uses the statistical estimation of the real banknote demand function. We consider two specifications:

$$\ln(RC_t) = a_0 + a_1 \ln(RGDP_t) + a_2(CALL_t) + \varepsilon_{1t}, \quad (19)$$

$$\ln(RC_t) = b_0 + b_1 \ln(RGDP_t) + b_2 \ln(CALL_t) + \varepsilon_{2t}, \quad (20)$$

where  $RC_t$  is real banknote demand,  $RGDP_t$  is real GDP, and  $CALL_t$  is the nominal overnight call rate. Using annual data between 1959 and 2013 for Japan, we find that the second specification includes cointegration with a structural break in 2004. Following Hayashi (2000), we estimate the cointegration coefficients for 1959–2003 using dynamic ordinary least squares (DOLS). The estimation results are  $b_1 = 1.029$  ( $s.e. = 0.43$ ) and  $b_2 = -0.133$  ( $s.e. = 0.010$ ).<sup>13</sup> As DOLS does not identify the constant term, we set  $b_0 = -3.31$  so that the fitted value of (20) coincides with the data in 2003. In the second scenario, we assume that the structural break in the real banknote demand function occurs because of the effective zero-interest rate policy adopted by the BoJ. Thus, the real banknote demand after the end of QQE is determined by (20) with the DOLS estimates of the coefficients for 1959–2003.

The third scenario uses an estimate of the cash stash held as part of household savings. The BoJ issues banknotes in denominations of one, two, five and 10 thousand yen, of which the one and 10 thousand yen notes are most commonly used. Otani and Suzuki (2008) assume that people only use 10 thousand yen notes when they hoard cash for savings, and that the transaction demands for one and 10 thousand yen notes are proportional to each other. Under these assumptions, Otani and Suzuki estimate the cash stash using the excess balance of 10 thousand yen notes above the transaction demand for these notes, which is assumed to grow at the same rate as the outstanding balance of one thousand yen notes.<sup>14</sup>

<sup>13</sup>Static ordinary least squares (SOLS) estimates are similar to the DOLS estimates (see Appendix B for details).

<sup>14</sup>Kohno and Shiraishi (2013) use Otani and Suzuki's estimates of cash stash in their simulation of the

They set 1995 as the base year in which they assume there is no cash stash in Japan.<sup>15</sup>

Using Otani and Suzuki's method, we estimate the cash stash to be about 35 trillion yen (see Figure 9). For the third scenario, we assume that only the cash stash disappears as the BoJ raises the short-term interest rate, while the transaction demand for banknotes remains unaffected. This assumption implies that the banknotes-to-GDP ratio drops to 0.104 at the time of the end of QQE (i.e.,  $\underline{\mu} = 0.104$ ).<sup>16</sup>

Figure 10 shows the BoJ's net assets under the benchmark and the three alternative scenarios. The figure demonstrates that the interest-rate elasticity of banknote demand is a crucial determinant of the BoJ's accounting losses and the duration in which the BoJ records negative net assets.

### 5.3 Building a loss-absorbing capital buffer before the end of QQE

Goodfriend (2014) has recommended the Federal Reserve to retain earnings to build a loss-absorbing capital buffer before the end of QE. Also, on November 13, 2015, the Bank of Japan formally requested the Ministry of Finance to allow the Bank of Japan to provide a loss allowance for possible future losses due to QQE. The Bank of Japan reported such a loss allowance of 450 billion yen in its annual report for fiscal year 2015.

Given these recent developments, Figure 11 shows the change in the BoJ's net assets when the BoJ retains all the after-tax profit to increase its net assets from the first month of the simulation, June 2016, until the end of QQE. The figure indicates that the accumulation of retained earnings before the end of QQE does not absorb much the cumulative losses after the end of QQE, because the BoJ is assumed to achieve the inflation target relatively soon in the simulation. Also, the term spread before the end of QQE reflects uncertainty on the

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BoJ's balance sheet as we do in this paper.

<sup>15</sup>The so-called Jyu-sen problem occurred in Japan in 1995. Jyu-sen is the abbreviation of a nonbank mortgage loan lender. Japanese banks made real estate loans through Jyu-sens in the late 1980s, and the losses to Jyu-sens came to the surface in 1995. Thus, Otani and Suzuki (2008) measure the cash stash associated with the Japanese public's concerns with the financial health of Japanese banks.

<sup>16</sup>This ratio is computed in such a way that if the banknotes-to-GDP ratio drops to this ratio in 2014, only the estimated cash stash for this year, 36.04 trillion yen, disappears from banknote demand.

achievement of the inflation target. Because the time path shown in the simulation is one of possible events ex-ante, it is natural to observe that the term spread before the end of QQE is not high enough to reflect the expected gains from reserve holdings after an increase in the short-term interest rate at the end of QQE.

## 5.4 Tapering after the exit from QQE

In the benchmark simulation, we assume that the BoJ rolls over its JGB holdings for only one year by purchasing T-bills with the revenues from redeemed JGBs. For an alternative scenario, we consider the case in which the BoJ decreases its new purchases of JGBs gradually over a certain period of years after the end of QQE. In this case, the amount of newly purchased JGBs each year is set to decline linearly so that it becomes zero after the adjustment period. The average maturity of newly purchased JGBs after the end of QQE is assumed to be 10 years.<sup>17</sup>

In this case, we need to set a time path for the long-term nominal interest rate to compute the yield on JGBs purchased after the end of QQE. Following Krishnamurthy and Vissing-Jorgensen (2011), we assume the existence of a safety channel such that

$$i_{LT,t} = i_{ST,t} + \min \left\{ 0.01, \quad 0.003 - 0.0031 \min \left\{ 0, \ln \left( \frac{B_t}{GDP_t} \right) - \ln(0.041) \right\} \right\}, \quad (21)$$

where  $i_{LT,t}$  denotes the long-term nominal interest rate. On the right-hand side, 0.01 is the long-run term spread, 0.003 is the term spread under QQE before the introduction of a negative policy interest rate in February 2016, -0.0031 is the supply elasticity of the long-term interest rate implied by Krishnamurthy and Vissing-Jorgensen's (2011) estimate of the safety channel in the US, and 0.041 is the ratio of the book value of JGBs held by the BoJ to nominal GDP in December 2014. The value of the long-run term spread equals the spread

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<sup>17</sup>More precisely, we assume that the outstanding balance of newly purchased JGBs follows a simple process such that

$$NB_{t+1} = V_t + 0.9NB_t,$$

where  $NB_t$  is the outstanding principal of JGBs purchased after the end of QQE at the beginning of fiscal year  $t$  and  $V_t$  is the amount of newly purchased JGBs in fiscal year  $t$ .

between the geometric averages of the 10-year JGB yield and the overnight call rate over 1990–2012, which is the Lost Decades before the onset of QQE. The value of the term spread under QQE is based on the fact that the 10-year JGB yield fluctuated around 0.3–0.5% after the expansion of JGB purchases in December 2014 and until the introduction of a negative policy interest rate in February 2016, while the overnight call rate was set to 0.1% by the BoJ through the interest rate on excess reserves during that period.<sup>18</sup> For the supply elasticity of the long-term interest rate, we use Krishnamurthy and Vissing-Jorgensen’s US estimate, as we do not know of comparable analysis using Japanese data.<sup>19</sup>

Under (21), the long-term interest rate gradually increases to the long-run level after the BoJ’s JGB holdings-to-GDP ratio becomes smaller than the level before December 2014. We set this assumption because the long-term interest rate did not show a downward trend in 2015, despite continuing large-scale JGB purchases each month (see Figure 12). For simplicity, we set no coupon on JGBs purchased after the end of QQE, and also assume that the average yield on these JGBs equals the long-term nominal interest rate,  $i_{LT,t}$ .

Figure 13 shows the BoJ’s balance sheet when the adjustment period for the BoJ’s JGB purchases after the end of QQE is set at 10 years. Because of the continuation of new purchases of JGBs, the BoJ’s JGB holdings peak in around 2020.

Figure 14 compares the BoJ’s net assets between the benchmark and alternative scenarios with adjustment periods of 10 and 20 years. As shown, tapering reduces the BoJ’s accounting losses and the size of negative net assets. This is because the BoJ can profit from the term spread after the end of QQE, as shown in Figure 15.

The safety channel plays an important role in determining the effect of tapering. Figure 16

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<sup>18</sup>We do not use the average term spread under the negative interest rate, because the sample period is too short.

<sup>19</sup>Strictly speaking, Krishnamurthy and Vissing-Jorgensen (2011) compute the 10-year equivalent supply of Treasury bonds by using the remaining maturity of each issue of Treasury bonds as a weight. However, they also report that the simple sum of the market values of Treasury bonds is highly correlated with the 10-year equivalent supply of Treasury bonds. Given no public data on the market value of JGBs held by the BoJ, we use the book value of JGBs in (21).

compares the BoJ's net assets with and without the safety channel, given that the adjustment period for the BoJ's JGB purchases is set at 10 years. In the case of no safety channel, the term spread immediately jumps to the long-run level (i.e.,  $i_{LT,t} = i_{ST,t} + 0.01$ ) after the end of QQE. The figure demonstrates that if the term spread returns to its pre-QQE level immediately after the end of QQE, then the BoJ will earn significant profit from the term spread through continuing purchases of JGBs. As a result, the BoJ will narrowly avoid negative net assets. Thus, tapering saves more fiscal cost for the consolidated government as the safety channel is less significant. A caveat is that the term spread is assumed to be always positive after the end of QQE in the simulation. A longer duration of tapering increases the risk of losses to the consolidated government due to negative term spreads, which can occur from time to time over the business cycle.

## 6 Conclusions

We have simulated the BoJ's cash flows and balance sheet before and after the end of QQE under various scenarios. The simulations show that the BoJ will record significant accounting losses after the end of QQE. These losses are fiscal costs for the consolidated government, as they correspond to increased interest expenses to the public due to the replacement of JGBs with reserve balances under QQE. We have also found that a longer duration of QQE and a higher interest-rate elasticity of banknote demand increase the BoJ's accounting losses significantly. Thus, the BoJ should take into account the fiscal cost when deciding the duration of the large-scale purchase of JGBs under QQE, and also should not take the current banknote demand for granted.

The cumulative accounting losses make the BoJ's net assets negative over 100 years in the simulations. While this is a ballpark estimate given simple macroeconomic scenarios to run simulations under transparent assumptions, it still demonstrates that it is a realistic possibility that the BoJ will report negative net assets for a sustained period of time after

the end of QQE. This result indicates that it is necessary to clarify the legal status of the BoJ when the BoJ reports negative net assets. Even though the BoJ's net assets per se do not matter for the consolidated government, lack of a transparent accounting rule that handles negative net assets of the BoJ may impose a financial constraint on the BoJ or bias the BoJ's monetary policy decisions, as argued by Del Negro and Sims (2014). A possible solution is that the BoJ and the Ministry of Finance set a loss-sharing rule in advance of the end of QQE. Alternatively, the diet can pass such a law that the Bank of Japan can run with negative accounting net assets. Such a rule, however, is not currently envisioned in the Bank of Japan Act. The discussion of the accounting rule for the BoJ with negative net assets must be started and completed before the end of QQE to avoid repercussions to the BoJ's monetary policy after the end of QQE.

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# Appendices

## A BoJ's JGB purchase policy under QQE

In October 2014, the BoJ announced an expansion of QQE, which aimed to increase the reserve balances held by financial institutions by 80 trillion yen per year. In December 2014, it set a new JGB purchase policy such that it purchases JGBs worth between eight and twelve trillion yen in purchase value per month. The average remaining maturity of the purchased JGBs under this policy is set to be between seven and 10 years. Every month, the BoJ announces the approximate range of its monthly purchase value for each type of JGBs. The policy announced for June 2016 is as shown in Table 2.

Table 2: Monthly purchase value of JGBs under QQE

Bond type	Remaining maturity	Approximate range of the purchase value of JGBs per month (billion yen)
Coupon-bearing bonds	< 1 year	100-300
	1-5 years	3600-6000
	5-10 years	1800-3600
	> 10 years	1500-3000
Floating-rate bonds		50
Inflation-indexed bonds		100 (bimonthly)

The monthly purchase value for each maturity of JGBs in the model, i.e., the figure in Table 1, equals the median value of the following range in Table 2: two- and five-year bonds fall into the second row; ten-year bonds fall into the third row; and twenty-year bonds fall into the fourth row. For two- and five-year bonds, we split into half the median value of the range shown in the second row of Table 2. The figures in Table 1 imply that the average maturity of newly purchased JGBs before the exit from QQE is 9.1 years. This figure satisfies the BoJ's actual policy on the average remaining maturity of purchased JGBs, which is set between seven and twelve years as of June 2016.

## B Estimation of the real banknote demand function

In the literature, Nakashima and Saito (2012) and Miyao (2002, 2005) analyze the interest rate elasticity of M1 (the sum of banknotes and demand deposits) in Japan by specifying real M1 as the dependent variable in (19) and (20), respectively. In addition, while Miyao (2002, 2005) set the income elasticity of M1,  $b_1$ , to zero, Fujiki and Watanabe (2004) use cross-sectional data across Japanese prefectures to set the value of  $b_1$ . In our analysis, we follow this literature to estimate the real banknotes demand in Japan.

For the estimation of (19) and (20), we use the following data for 1959–2013:

- $C_t$ : the annual average balance of banknotes for each fiscal year.
- $NGDP_t$ : GDP deflator for each fiscal year; 1993SNA data for 1993–2013 extended with the annual growth rates in 1968SNA data for 1959–1992.
- $Call_t$ : the annual average overnight call rate; the collateralized rate until 1984 and the uncollateralized rate from 1985.

In the sample period, 2013 is the most recent year in which data on GDP in the Annual Report on National Accounts is available, and 1959 is the first year in which the call rate data exist. We define the real balance of banknotes and the real GDP by

$$RC_t = \frac{C_t}{PGDP_t}, \quad (22)$$

$$RGDP_t = \frac{NGDP_t}{PGDP_t}, \quad (23)$$

respectively.

We first conduct an augmented Dickey–Fuller (ADF) test and find that we cannot reject the null hypothesis of a unit root with a time trend, except for  $CALL_t$  (see Table 3). For  $CALL_t$ , we cannot reject the null hypothesis of a unit root based on Elliott, Rothenberg and Stock (1996). (The test statistics is  $-1.762$ , and the 10% level critical value is  $-2.748$ .)

Table 3: ADF test on the components of the real banknote demand function

	ADF statistics	P-value	Lag	Time trend
$\ln(RC_t)$	-2.873	0.1713	1	Exist
$\ln(RGDP_t)$	-2.526	0.3151	1	Exist
$CALL_t$	-4.391	0.0023	3	Exist
$\ln(CALL_t)$	-2.448	0.3543	1	Exist

Notes: The sample period is the fiscal years between 1959 and 2013. We use the `dfuller` command in Stata 13.

Table 4: Engel–Granger test on cointegration

	Test statistics	Critical value		
		1%	5%	10%
Eq. (19)	-1.992	-4.572	-3.903	-3.569
Eq. (20)	-1.452	-4.572	-3.903	-3.569

Notes: The sample period is the fiscal years between 1959 and 2013. We use the `egranger` command in Stata 13.

Next, we conduct the Engel–Granger test and find that we cannot reject the null hypothesis of no cointegration among the variables appearing in each of (19) and (20) (see Table 4).

Given this result, we test a structural break in cointegration using Gregory and Hansen’s (1996) test. In this test, the null hypothesis is no cointegration, and the alternative hypothesis is cointegration with a structural break. While we cannot reject the null hypothesis for (19), we find that there is a structural break in 2004 according to the ADF and  $Z_t$  statistics (see Table 5).

Given a cointegration relationship for (20) over the period 1959–2003, we estimate the cointegration coefficients using SOLS and DOLS, following Hayashi (2000) (see Table 6). The estimation result for DOLS does not change significantly, even if we set the number of lags to one year.

Table 5: Gregory and Hansen's (1996) test for a structural break in cointegration

	Test statistics for each function		Critical value		
	Eq. (19)	Eq. (20)	1%	5%	10%
ADF	-4.02 (2003)	-5.59 (2004)	-5.44	-4.92	-4.69
$Z_t$	-4.42 (2003)	-5.64 (2004)	-5.44	-4.92	-4.69
$Z_\alpha$	-29.08 (1999)	-40.92 (2004)	-57.01	-46.98	-42.49

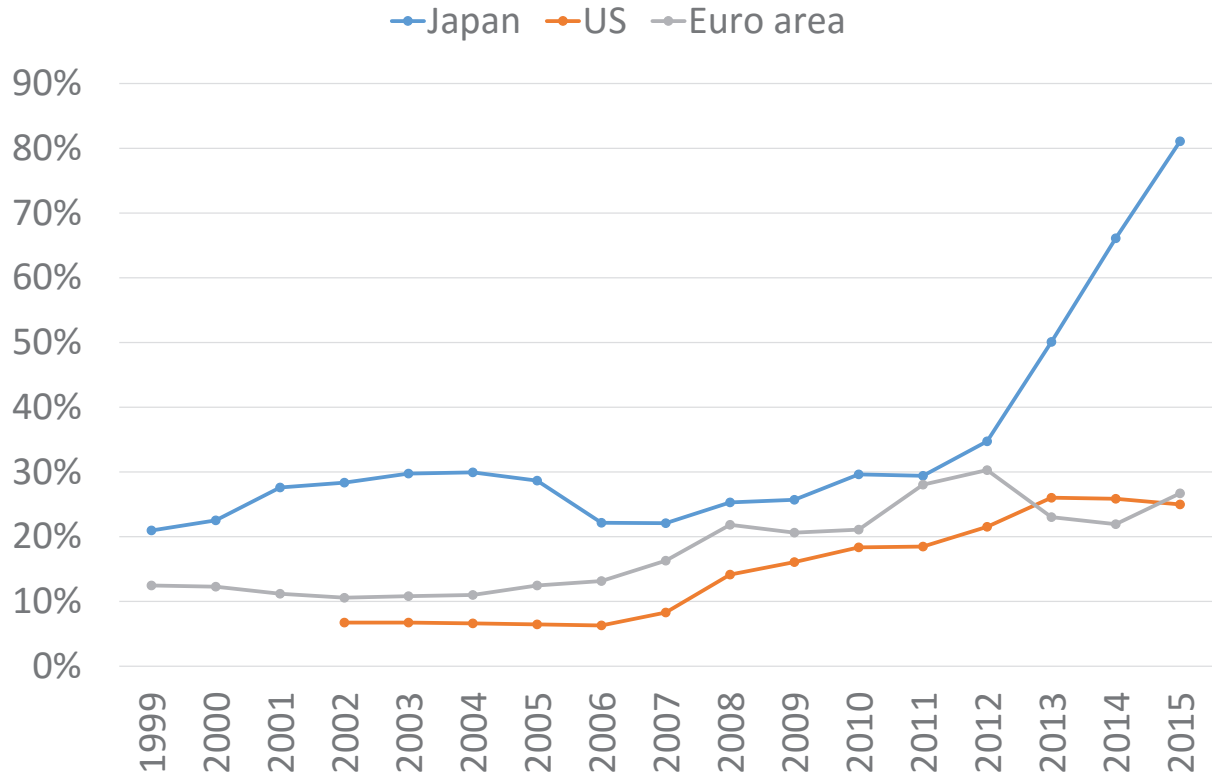
Notes: The sample period is the fiscal years between 1959 and 2013. We use the `ghansen` command in Stata 13. Each set of parentheses in the second and third columns provide the year of a structural break.

Table 6: Estimation of cointegration coefficients in Eq. (20)

	$b_1$	$b_2$
SOLS	1.121	-.090
DOLS	1.029	-.133
(s.e.)	(.043)	(.010)

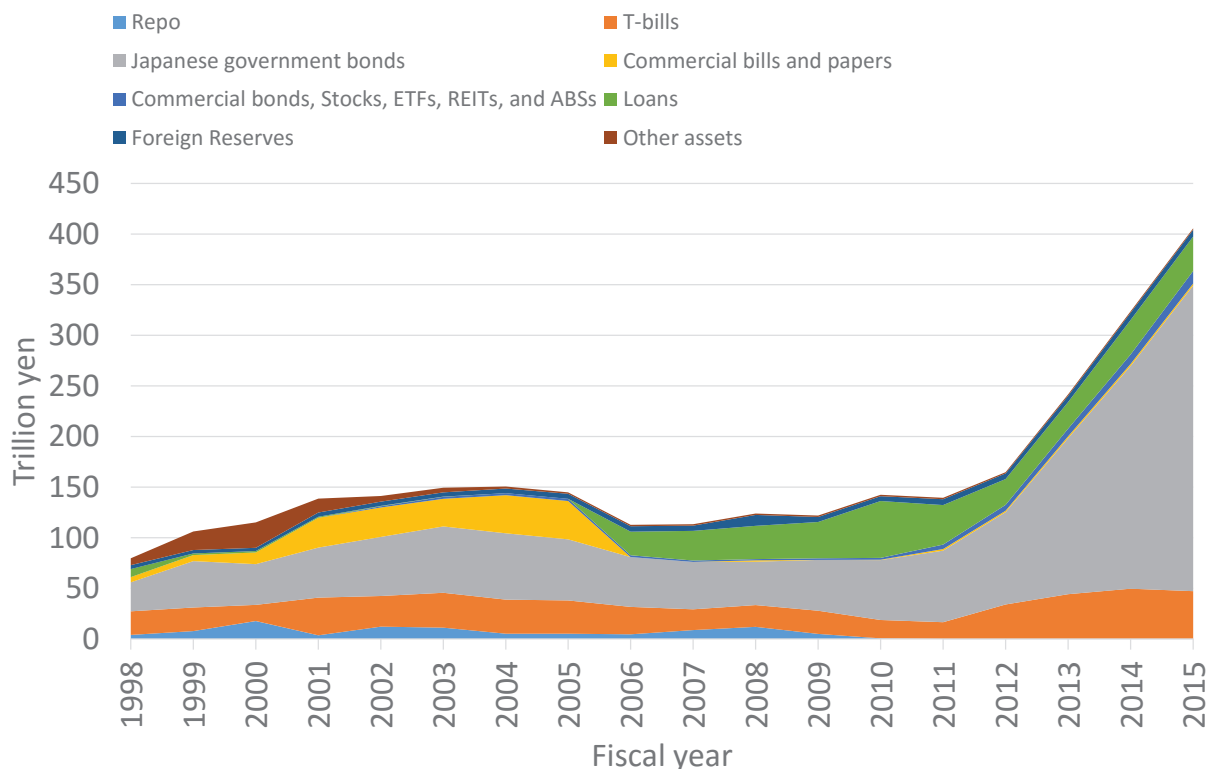
Notes: The sample period is the fiscal years between 1959 and 2013. We use the `ivreg2` command in Stata 13. The last row shows the standard errors of the coefficients estimated by DOLS. The number of lags for DOLS is set to two years.

Figure 1: Ratio of the central bank's total assets to GDP



Sources: The central bank's total assets: the Bank of Japan, the Federal Reserve Bank of St. Louis (FRED), and the European Central Bank; GDP: Economic and Social Research Institute (Cabinet Office, the Government of Japan), the Bureau of Economic Analysis, and the European Central Bank.

Figure 2: Composition of the BoJ's assets



Source: The Bank of Japan.

Note: “T-bills” for 1998-2000 includes only Financing Bills, as the Bank of Japan Accounts do not report the total short-term government debt before 2001. From 2001 onward, “T-bills” equals the total short-term government debt, which consists of Financing Bills and Treasury Bills. Currently, both of them are called T-bills in the market. “Commercial bills and papers” are replaced by “Loans” in 2006, as the BoJ has introduced the Funds-Supplying Operations against Pooled Collateral on behalf of the Paper-Based Bill Purchasing Operations in that year. “Loans” also include lending facilities, such as the Loan Support Program.



Figure 3: Balance sheet items of the BoJ in the benchmark scenario

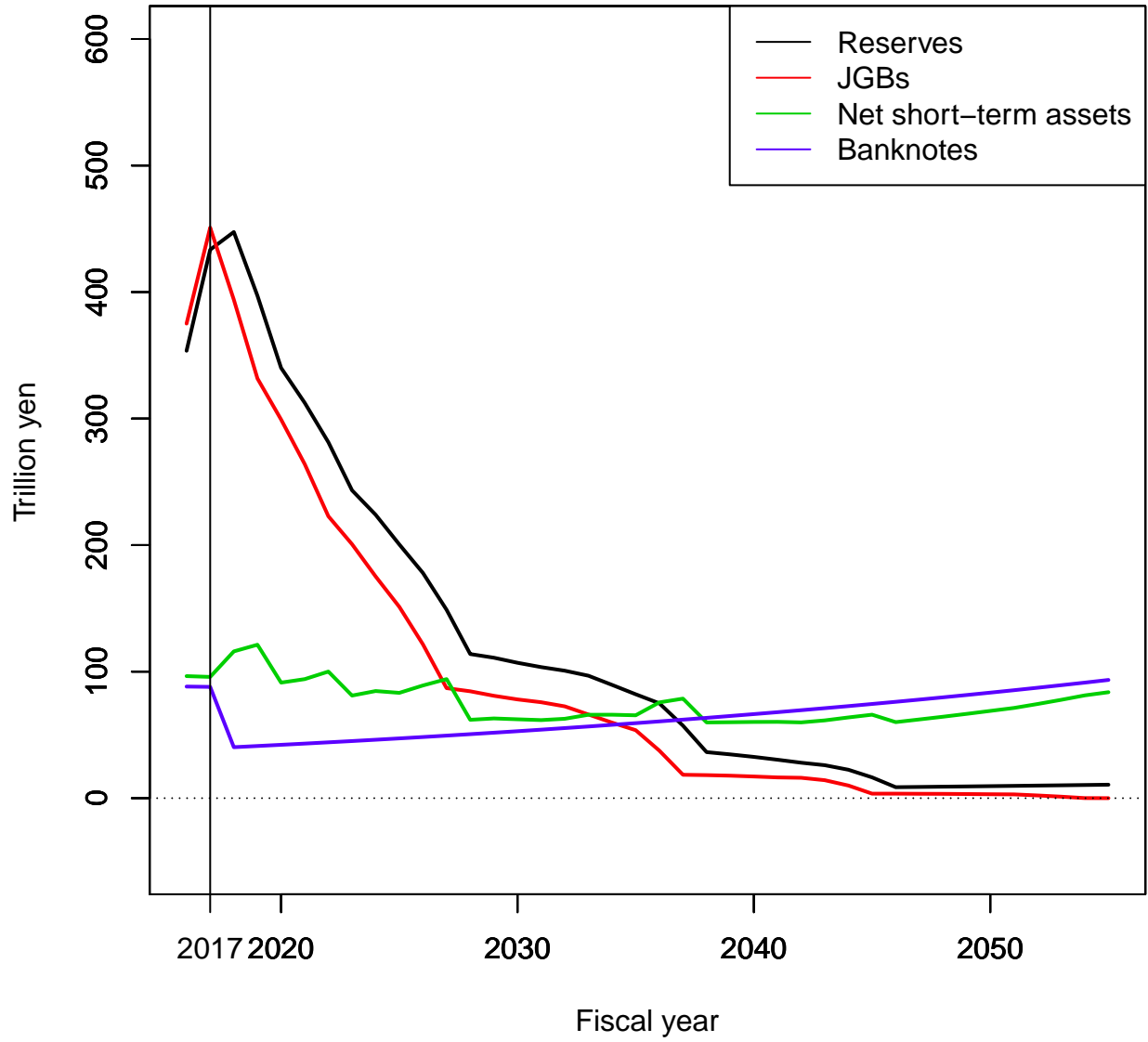
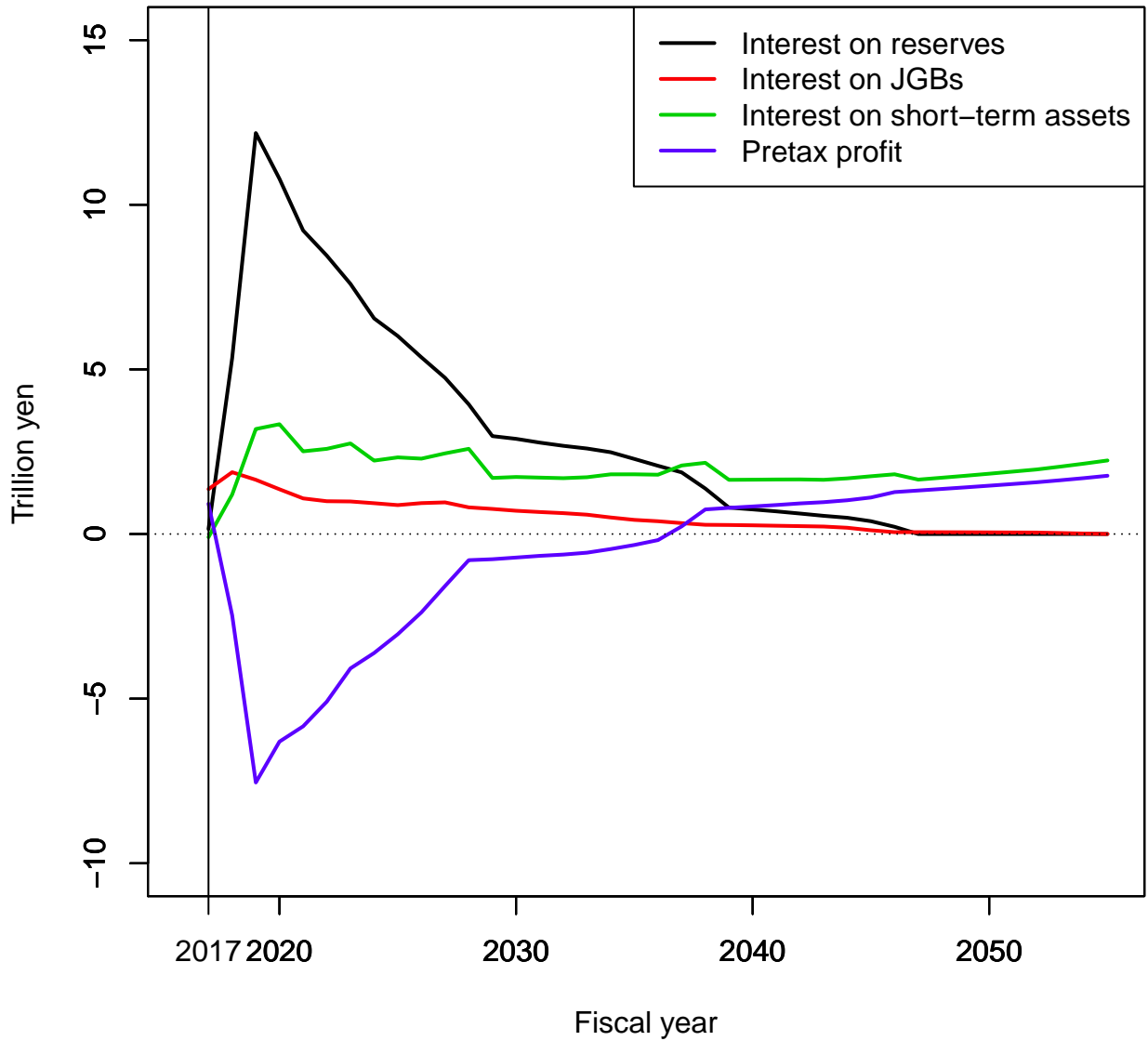
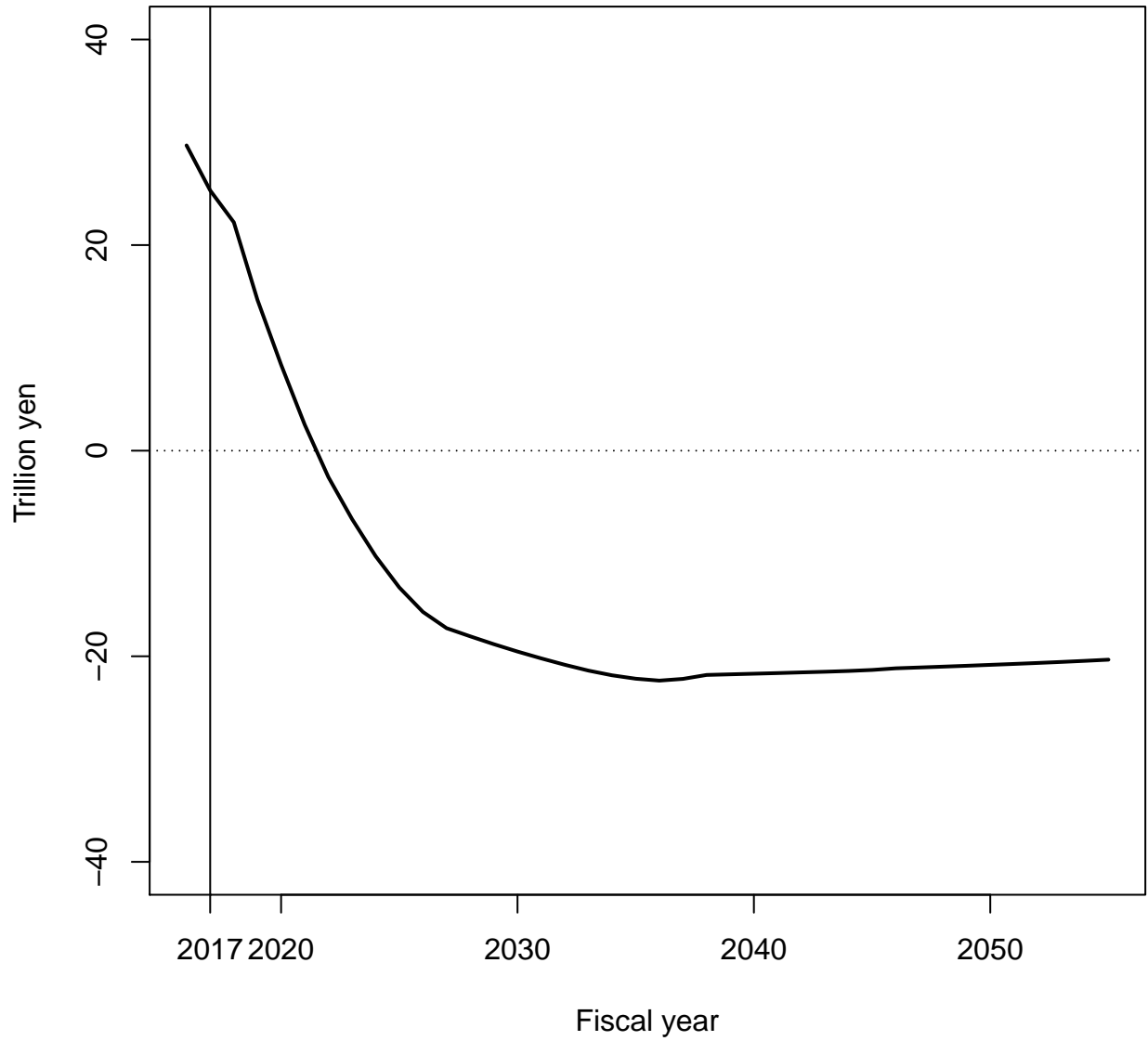


Figure 4: Major components of the BoJ's accounting profit in the benchmark scenario



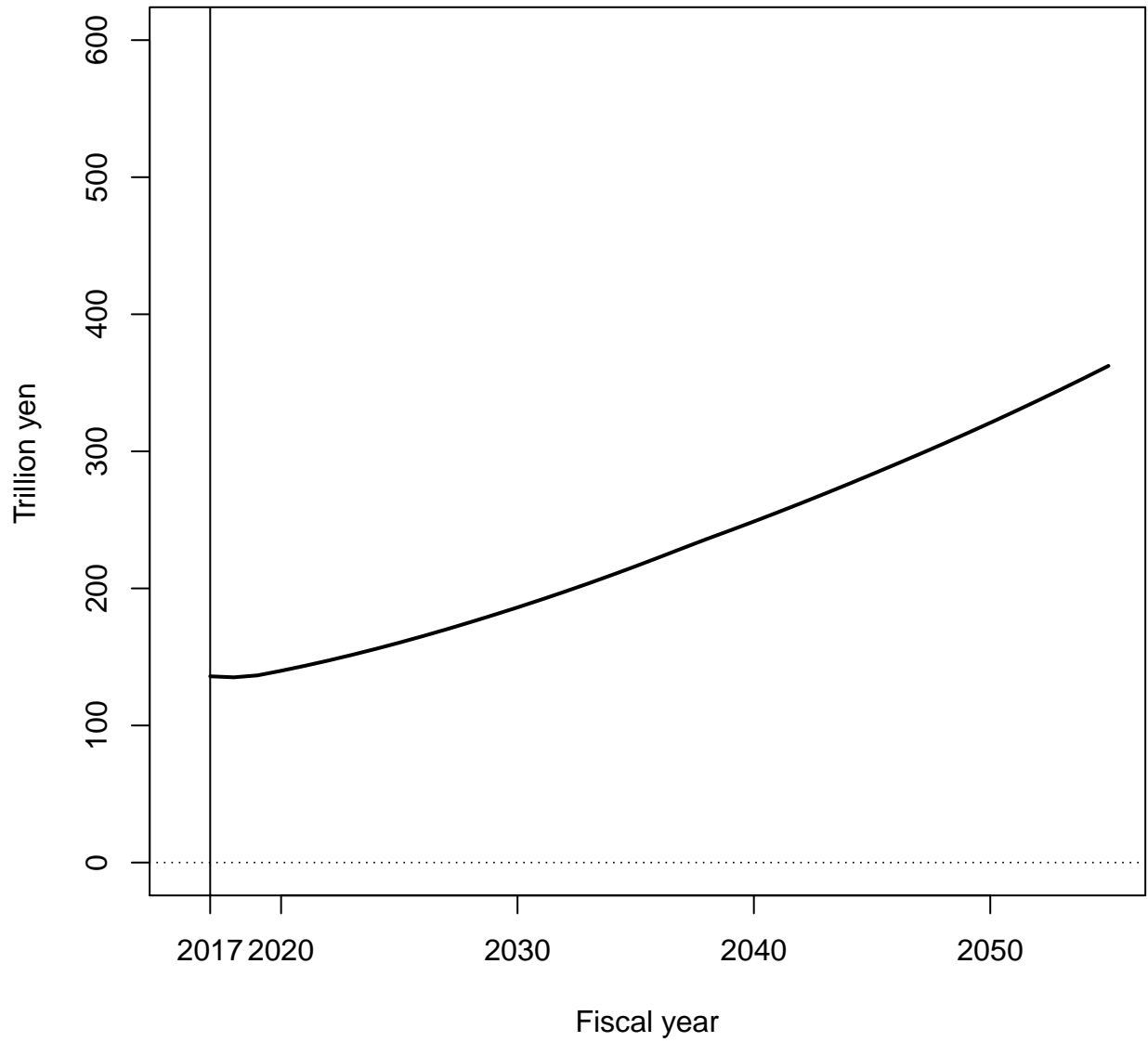
Note: “Interest rate on JGBs” includes the amortization of the discount under the amortized cost method in each year.

Figure 5: BoJ's accounting net assets in the benchmark scenario



Note: The value of the BoJ's net assets equals the difference between the BoJ's accounting assets and liabilities.

Figure 6: BoJ's net worth in the benchmark scenario



Note: The value of the BoJ's net worth equals the sum of the BoJ's accounting net assets and the presented discounted value of future profit.

Figure 7: BoJ's balance sheet when QQE lasts until 2019

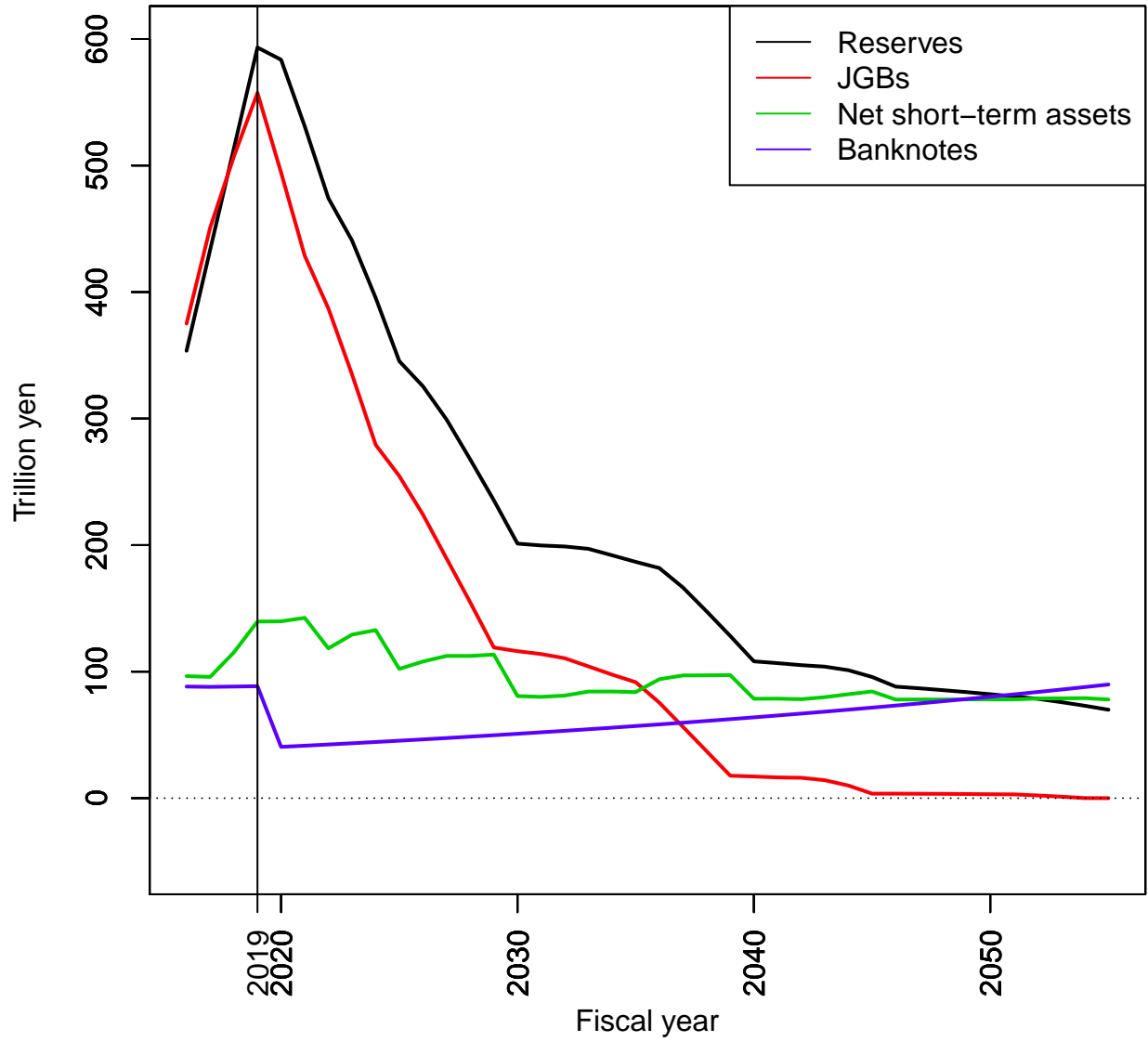
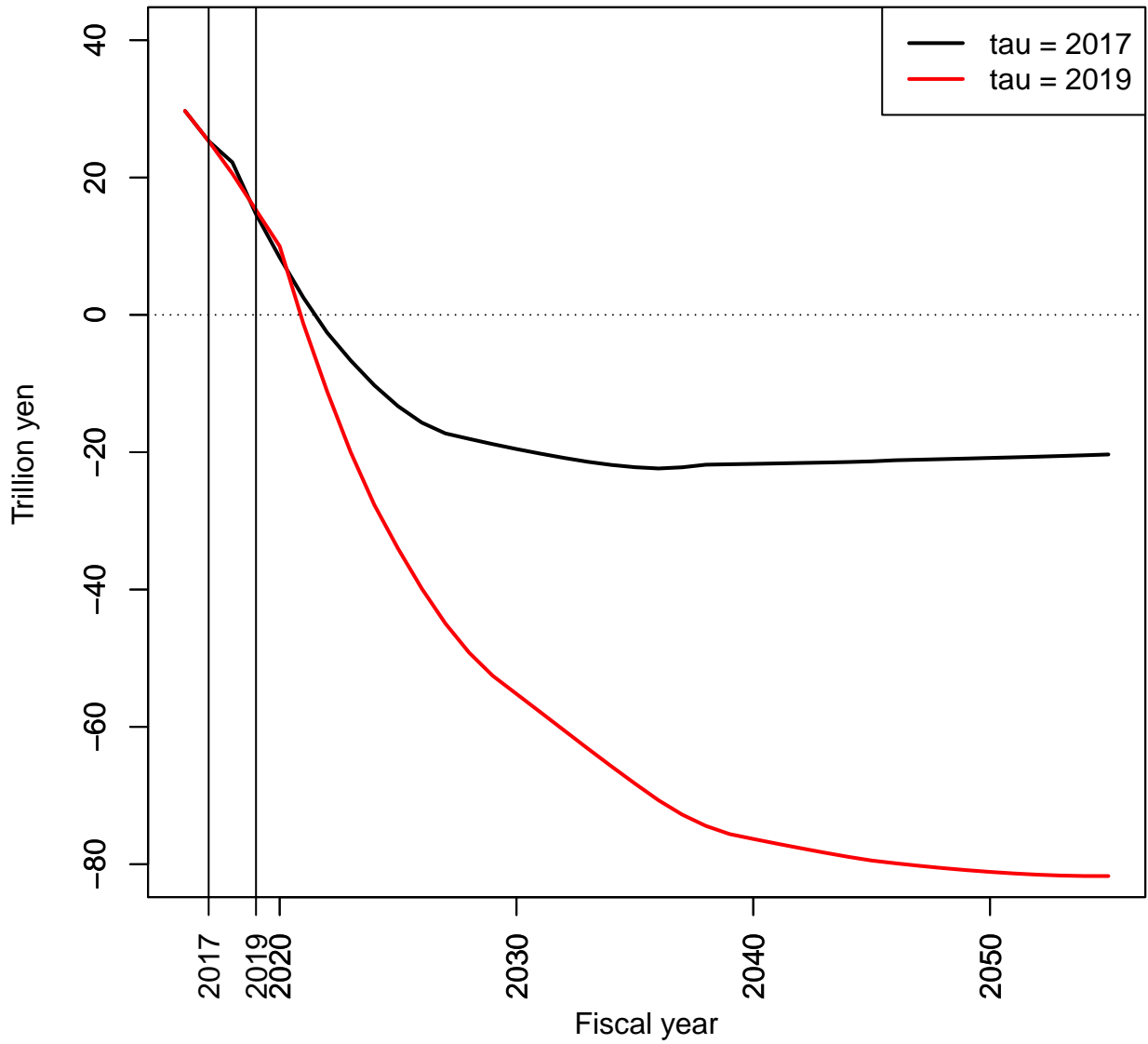
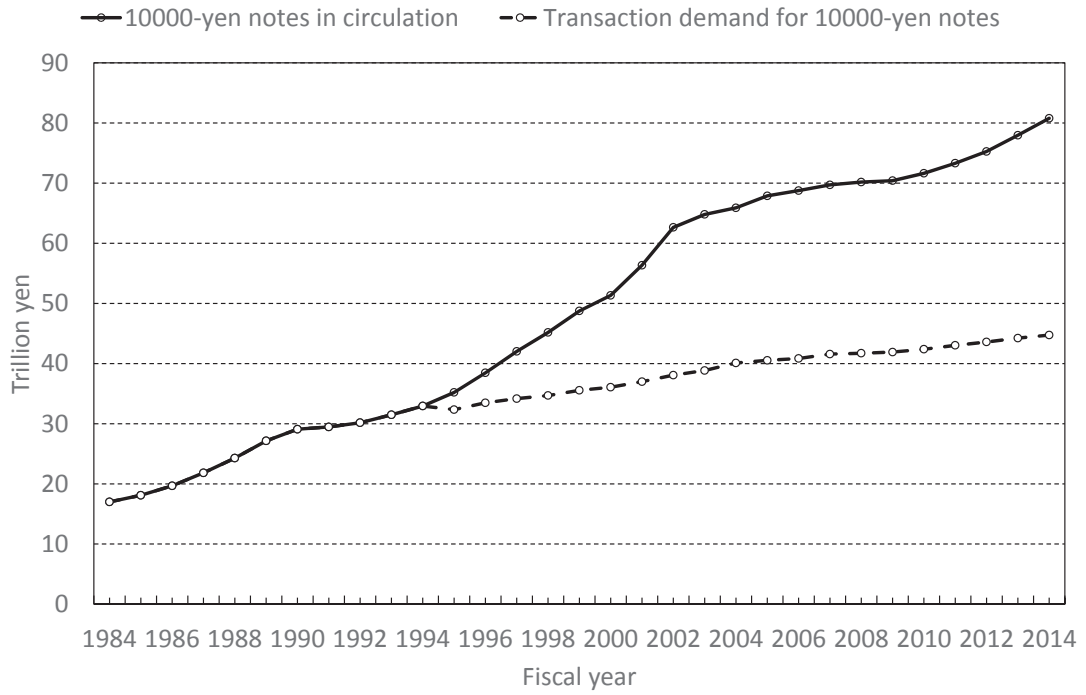


Figure 8: Sensitivity of BoJ's accounting net assets to the duration of QQE



Notes: The solid line is the benchmark; the red line is where the BoJ achieves the inflation target in fiscal year 2019.

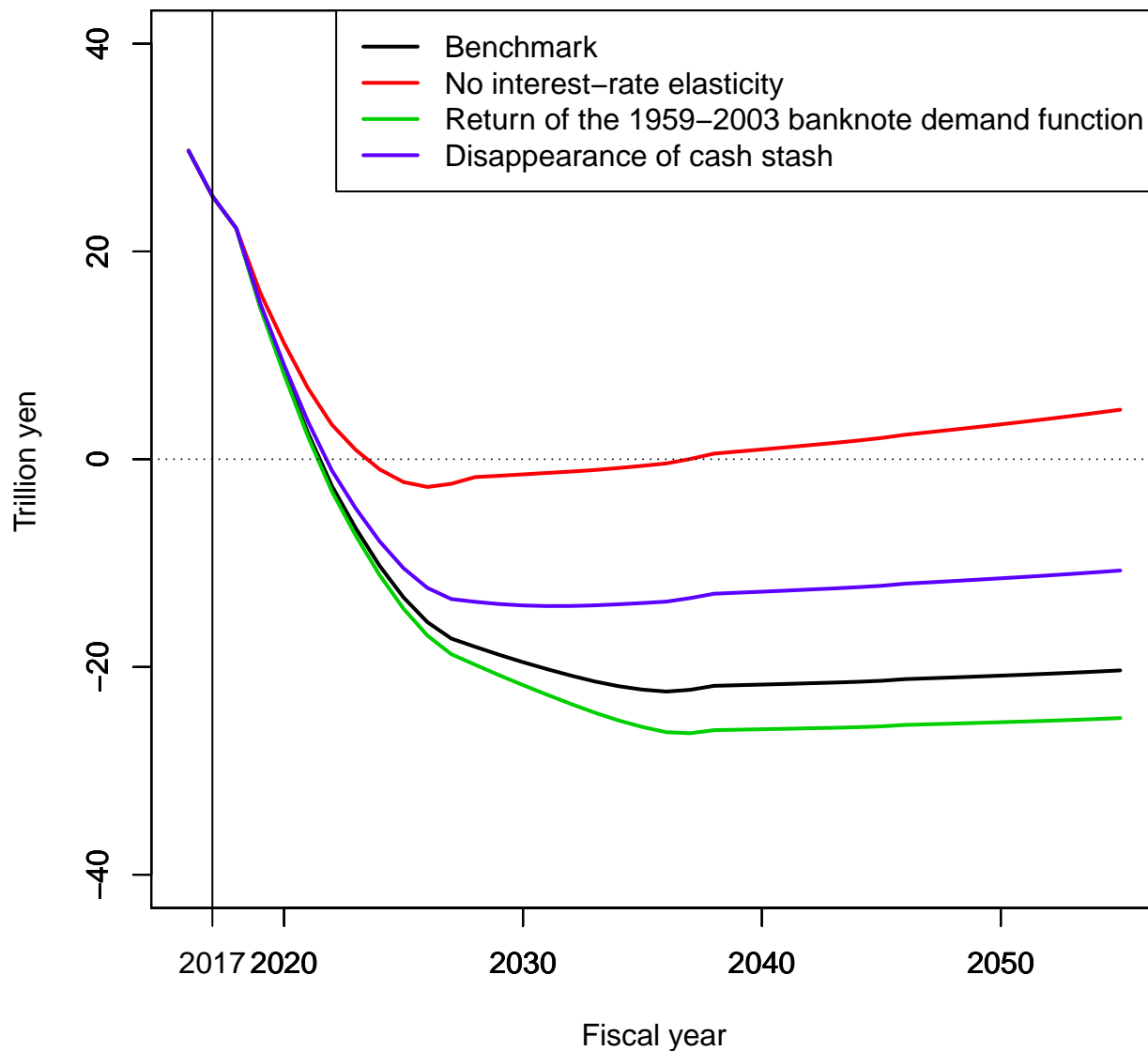
Figure 9: 10,000 yen notes in circulation and transaction demand for 10,000 yen notes



Sources: The Bank of Japan and the authors' calculation.

Note: "Transaction demand for 10000 yen notes" is the hypothetical balance of 10,000 yen notes that grows at the same rate as 1,000 yen notes in circulation from 1995.

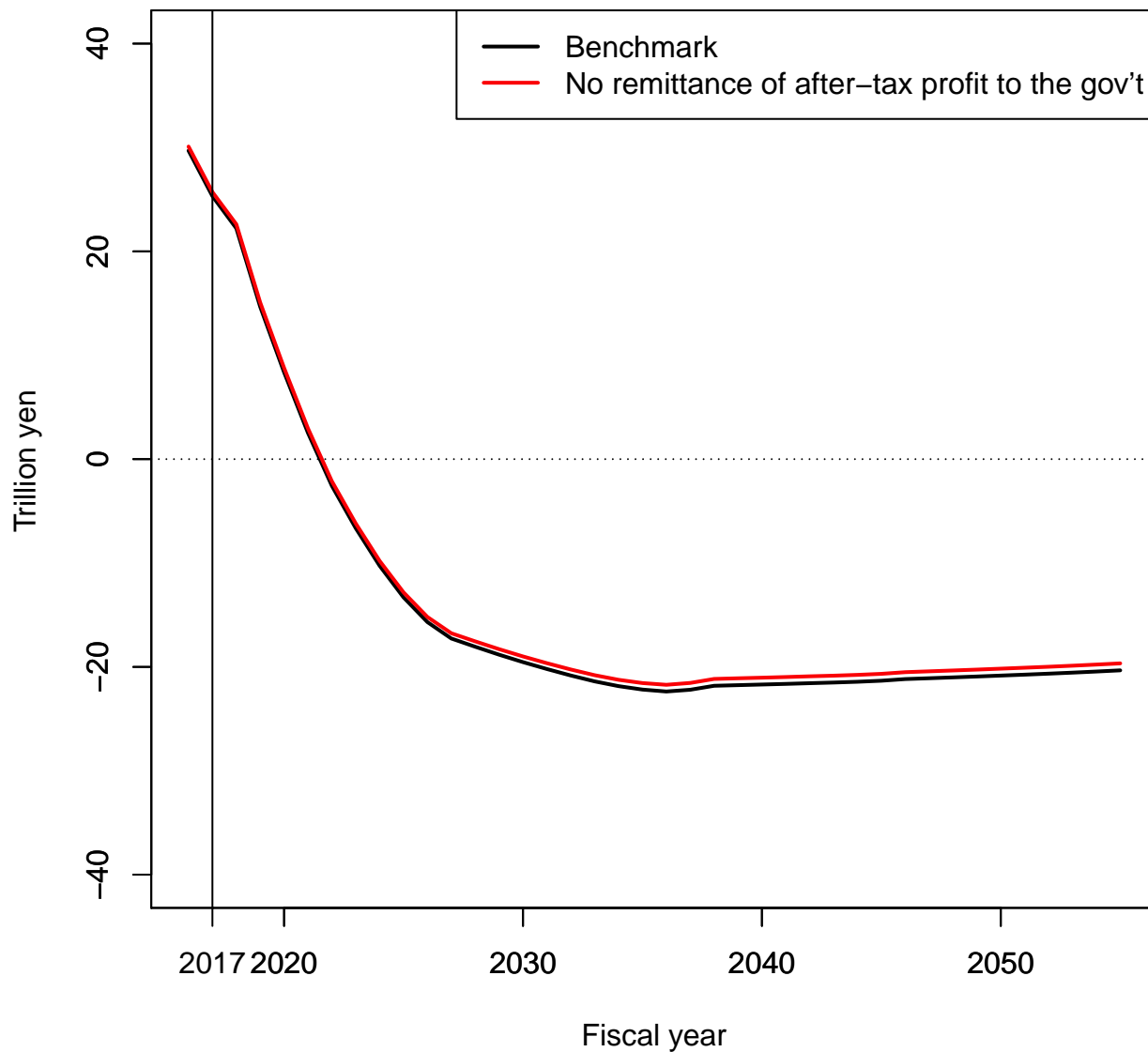
Figure 10: Sensitivity of BoJ's accounting net assets to interest-rate elasticity of banknote demand



Notes: The black line is the benchmark; the red line is where the banknotes-to-GDP ratio ( $\mu_t$ ) remains the same before and after the exit from QE; the green line is where the real banknote demand takes its pre-2004 form after the exit from QE; and the blue line is where the non-transaction demand for 10,000 yen banknotes disappears immediately after the BoJ raises the short-term interest rate after the exit from QE.



Figure 11: Sensitivity of BoJ's accounting net assets to retained earnings before the end of QQE



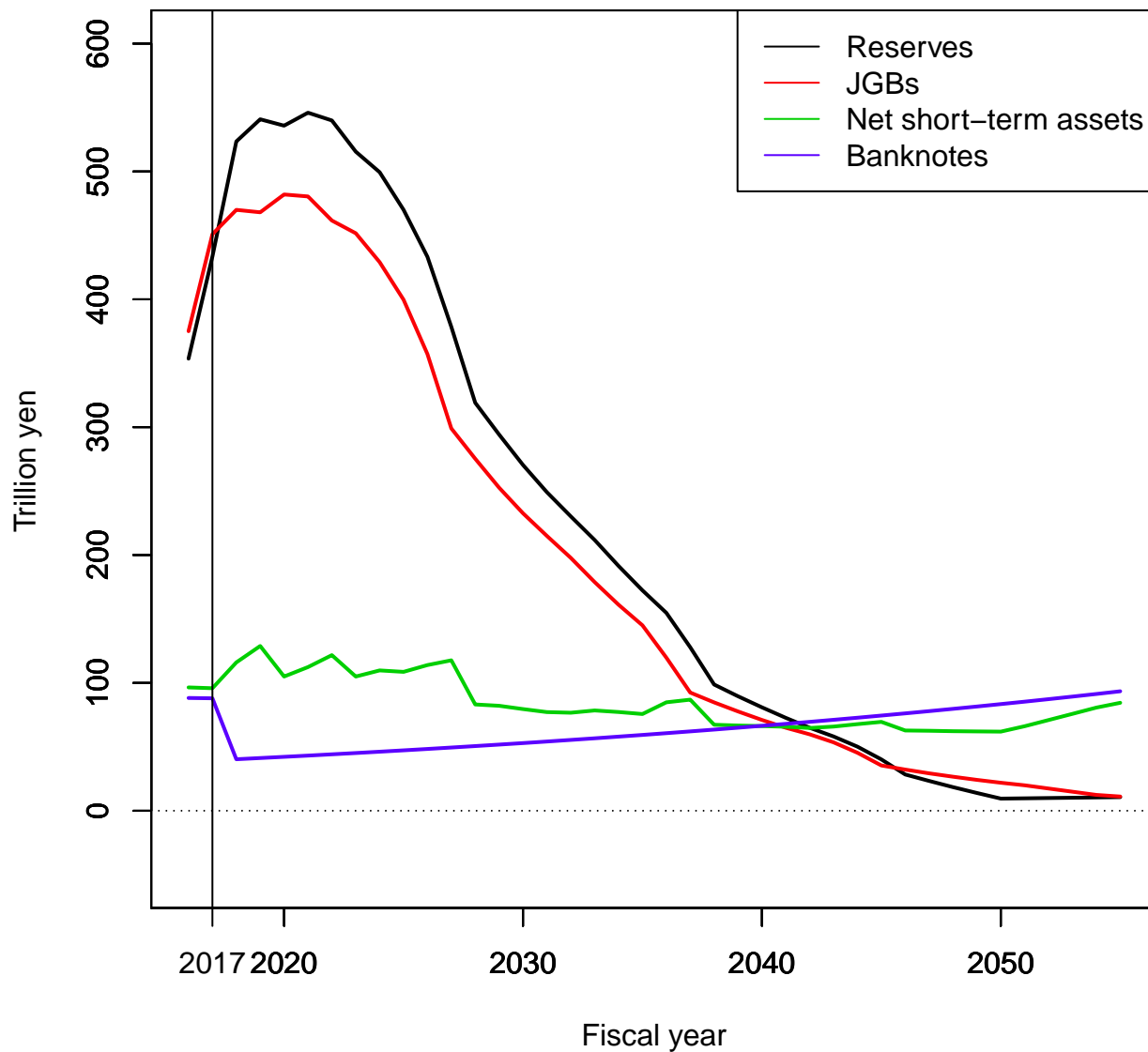
Notes: The solid line is the benchmark; the red line is where the BoJ retains all the after-tax profit until it achieves the inflation target in 2017.

Figure 12: Term spread in Japan



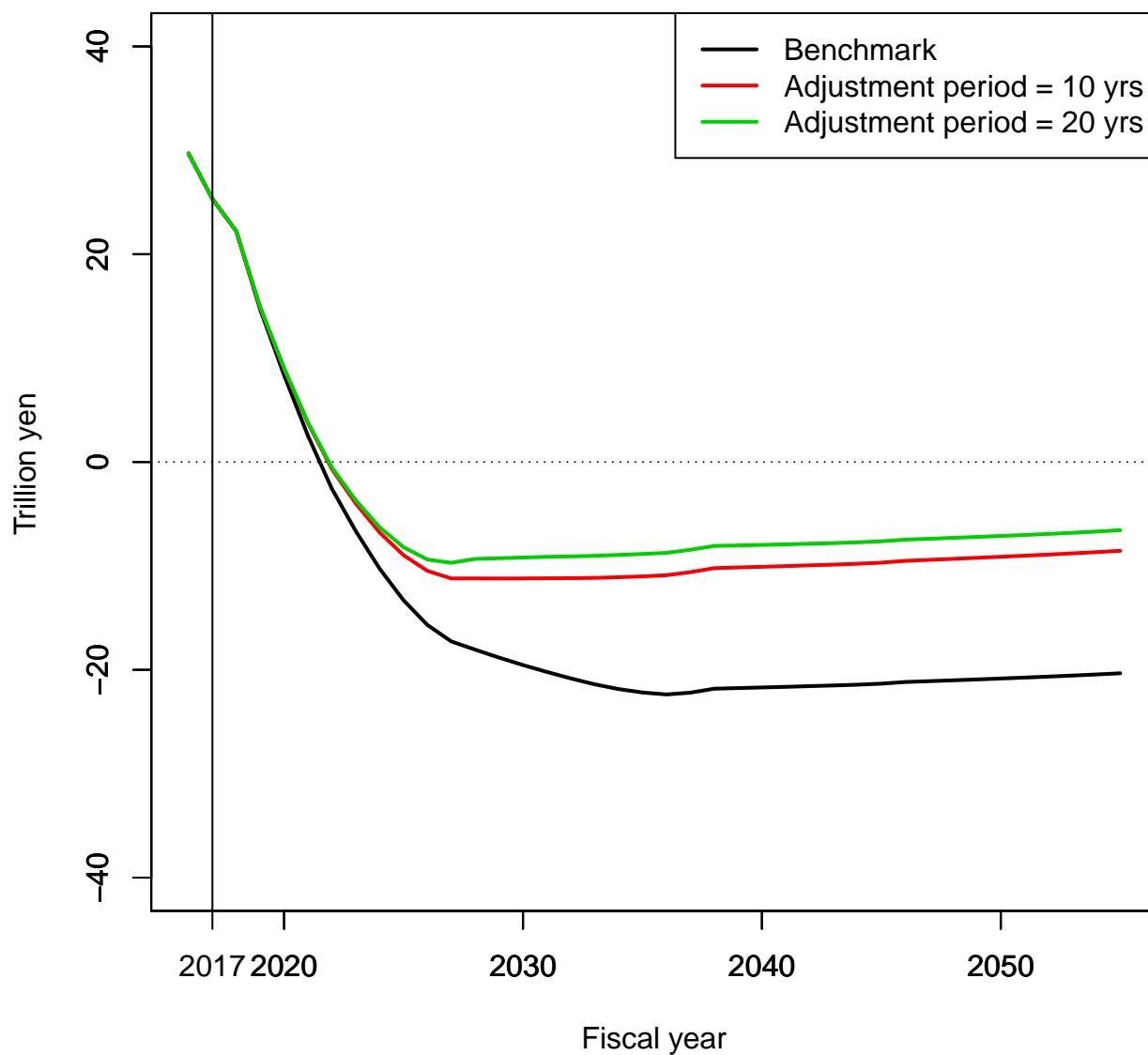
Source: The Bank of Japan and the Ministry of Finance.

Figure 13: BoJ's balance sheet when new purchases of JGBs are gradually decreased over the 10 years after the end of QQE



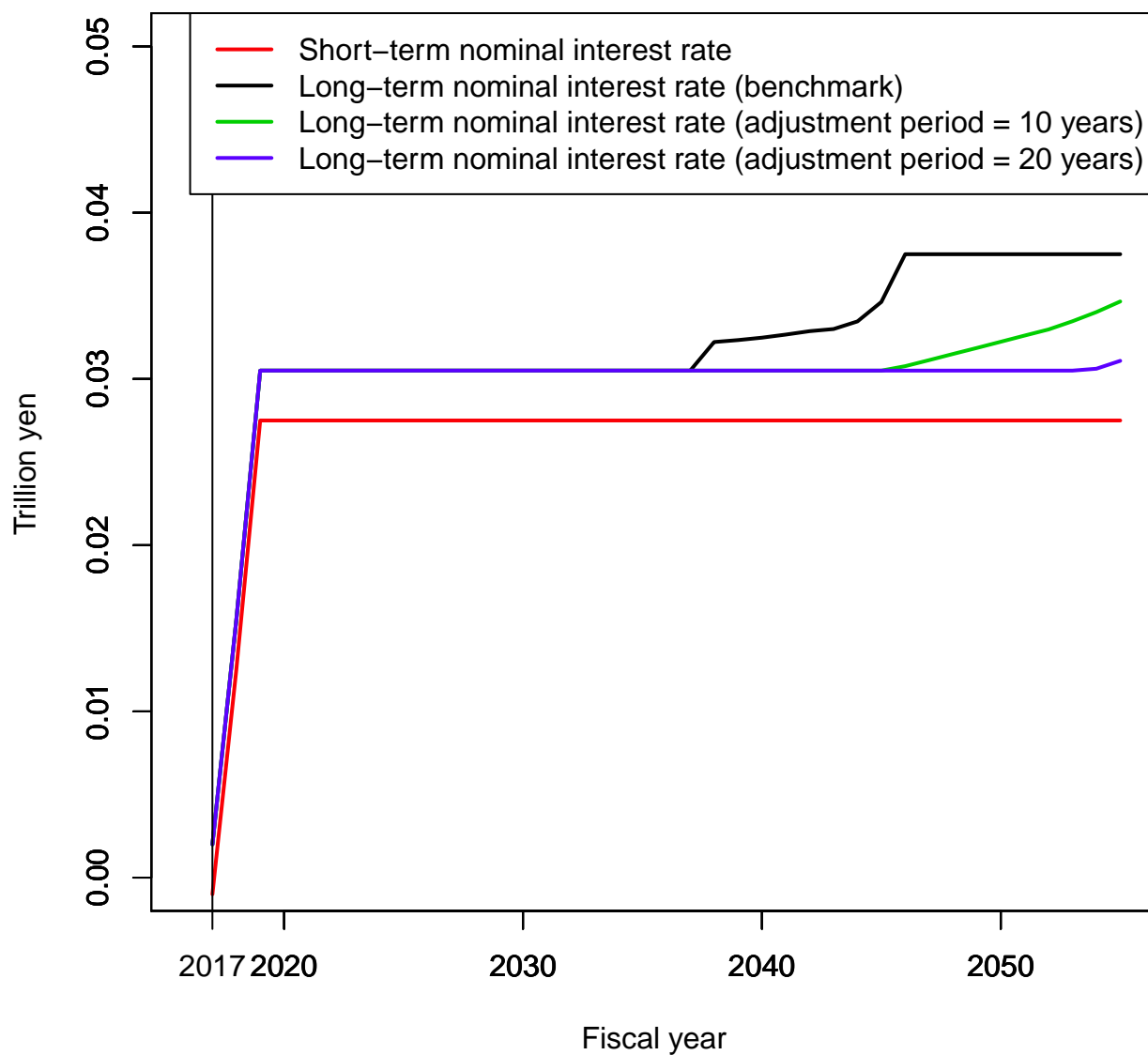
Note: The amount of new purchases of JGBs each year after the end of QQE is linearly reduced so that it becomes zero after 10 years.

Figure 14: Sensitivity of BoJ's accounting net assets to the adjustment period for the new purchases of JGBs after the end of QQE



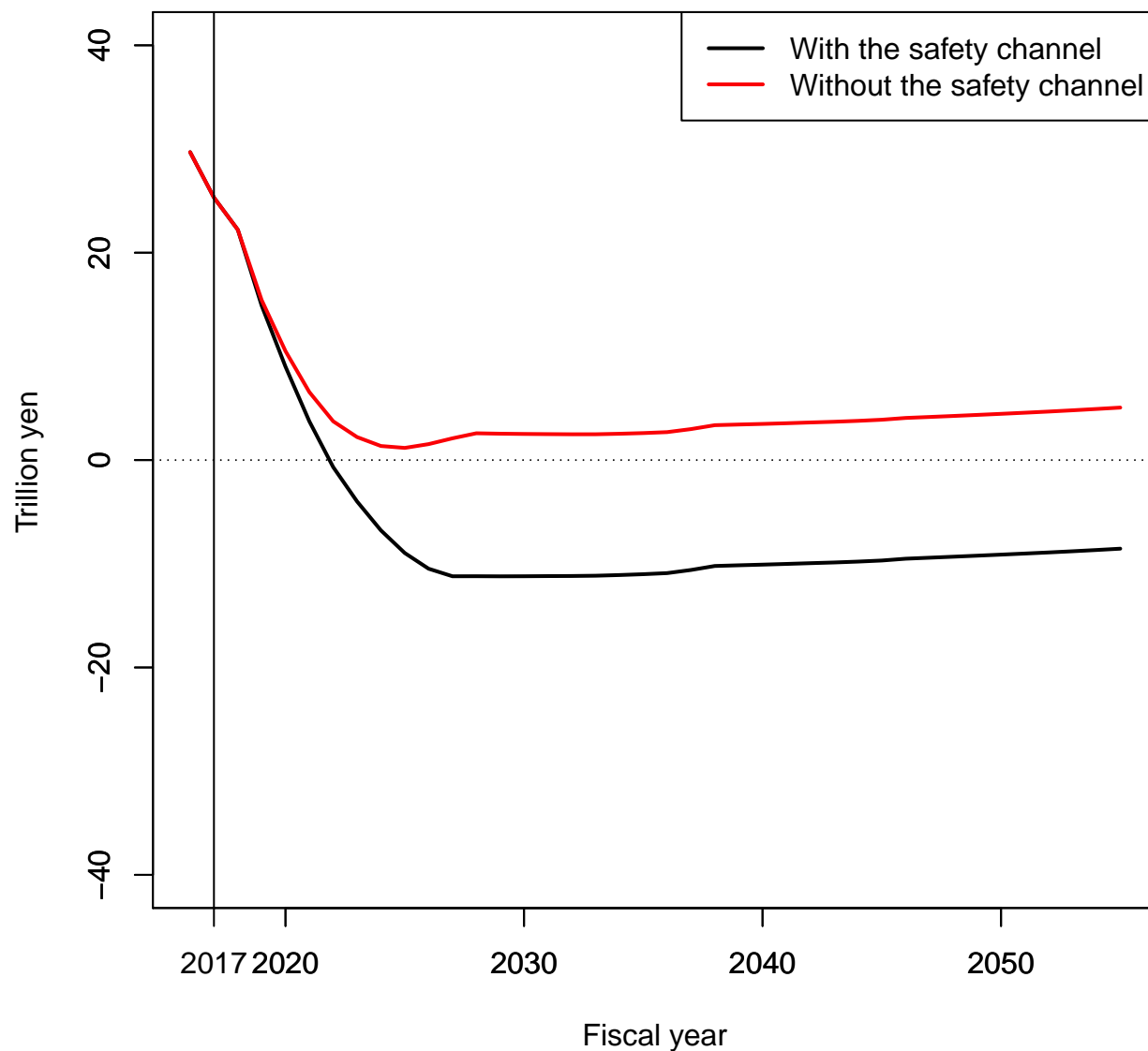
Note: The amount of new purchases of JGBs each year after the end of QQE is linearly reduced so that it becomes zero after the adjustment period.

Figure 15: The path of the long-term nominal interest rate with the adjustment of new purchases of JGBs after the end of QQE



Note: The amount of new purchases of JGBs each year after the end of QQE is linearly reduced so that it becomes zero after the adjustment period.

Figure 16: Sensitivity of BoJ's accounting net assets to the safety channel for the long-term nominal interest rate



Note: For both cases, the amount of new purchases of JGBs each year after the end of QQE is linearly reduced so that it becomes zero 10 years after the end of QQE.