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A Dynamic Theory of Concessions and War*

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Abstract

Why do some countries engage in temporary wars and others in total war? In this paper, we develop a dynamic theory of concessions and war in order to shed light on this question. In our framework, an aggressive country can forcibly extract concessions from a non-aggressive country via war. Alternatively, it can avoid war and allow the non-aggressive country to make concessions on its own. Both countries suffer from limited commitment, and under peace, the non-aggressive country may receive a private shock which deems concessions too costly. We show that the realization of war sustains concessions along the equilibrium path. The aggressive country punishes failed concessions by requesting larger and larger concessions, and their failure eventually leads to a temporary war. The aggressive country forgives the non-aggressive country by re-engaging in peace after the war because of the coarseness of public information. In the long run, temporary wars can be sustained only if countries are patient, if the cost of war is large, and if cost of concessions is low. Otherwise, the aggressive country cannot continue to forgive the non-aggressive country, and countries converge to total war (permanent war).

Keywords: Repeated Games, Asymmetric and Private Information, Contract Theory, International Political Economy, War

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1 Introduction

Many countries engage in short and infrequent wars. Since gaining independence, India and Pakistan have fought three wars, each lasting less than six months.[1] To this day, the possibility of future violence between these two countries remains, as evidenced by the recent standoff at the end of 2001. This behavior contrasts with numerous historical examples such as World War II in which countries fought a total war. This latter style of conflict is prolonged, uninterrupted, and results in the annihilation of one side. Why do some countries engage in temporary wars and others in total war? In this paper, we develop a dynamic theory of concessions and war in order to shed some light on this question.

The subject of war, first formalized in an economic framework in the seminal work of Schelling (1966), is the original impetus for important advances in the field of game theory. While there is renewed theoretical interest in the subject of war in economics (e.g. Besley and Persson, 2007, Esteban and Ray, 2008, Schwarz and Sonin, 2004), there is a lack of a formal framework for investigating the dynamics of war.[2] In this paper, we apply the modern tools from the theory of repeated games developed by Abreu, Pearce, and Stacchetti (1986,1990) to the classical subject of war. Specifically, we present a dynamic theory of war in which countries suffer from limited commitment and asymmetric information, two frictions which hamper their ability to peacefully negotiate. Our main conceptual result is a dynamic theory of escalation, temporary wars, and total war. On the theoretical side, our framework additionally allows us to derive novel results on the role of information in repeated games.[3]

In our model, one country, which we refer to as the aggressive country, is dissatisfied with the status quo and seeks concessions from its rival, which we refer to as the non-aggressive country. In every period, the aggressive country can either forcibly extract

[1]According to the Correlates of War Database, the First Kashmir War lasted 169 days, the Second Kashmir War lasted 50 days, and the Bangladesh War lasted 15 days. A similar observation can be made with respect to the Arab-Israeli wars, with the Palestine War lasting 143 days, the Six Day War lasting 6 days, and the Yom Kippur War lasting 19 days.

[2] Additional examples of recent papers on war include but are by no means limited to Alesina and Spolaore (2005), Baliga and Sjostrom (2004), Caselli and Coleman (2006), Chassang and Padro-i-Miguel (2007), Dixit (1987), Esteban and Ray (1994,1999), Fearon (1995), Hirschleifer (1995), Hirshleifer, Boldrin, and Levine (2008), Jackson and Morelli (2008), Powell (1999), Shavell and Spier (2002), Siu (2008), and Skarpedas (1992).

[3]The theory of dynamic games with imperfect information is introduced by Green and Porter (1984) and more recently developed by Sannikov (2007a,2007b). Our theoretical framework is very close to applied models of dynamic optimal contracts with private information, for example Atkeson (1991), DeMarzo and Fishman (2007), Golosov, Kocherlakota, and Tsyvinski (2003), Hauser and Hopenhayn (2004), Phelan and Townsend (1991), Spear and Srivastava (1987), and Thomas and Worrall (1990).

these concessions via war, or it can let the non-aggressive country peacefully make the concessions on its own. While peaceful concession-making is clearly less destructive than war, there are two limitations on the extent to which peaceful bargaining is possible. First, there is limited commitment. Specifically, the non-aggressive country cannot commit to making a concession once it sees that the threat of war has subsided. Moreover, the aggressive country cannot commit to peace in the future in order to reward concession-making by the non-aggressive country today. Second, there is imperfect information. The aggressive country does not have any information regarding the non-aggressive country's ability to make a concession, and the non-aggressive country can use this to its advantage. Specifically, since there is always a positive probability that concessions are too costly to make, the non-aggressive country may wish to misrepresent itself as being unable to make a concession whenever it is actually able to do so.

There are many applications of our framework. As an example, consider an aggressive country seeking commercial and political concessions from a non-aggressive country such as the control of terrorists operating from within the non-aggressive country's borders. The aggressive country can temporarily force the non-aggressive country into submitting to its demands by war. Alternatively, it can wait for the non-aggressive country to honor the terms of peace on its own, for example by preventing terror attacks. Nevertheless, with some probability, the non-aggressive country's government is incapable of fulfilling the terms of peace for exogenous reasons. This may occur, for instance, if the non-aggressive country's government is too weak or if it experiences significant domestic resistance to fulfilling the terms of peace. If concessions fail, however, the aggressive country cannot gauge the extent of the setbacks faced by the non-aggressive country's government, and it cannot distinguish between the intentional and unintentional failure of concessions. Thus, it cannot tell if the non-aggressive country is lying about the true reasons for its failure to cooperate.

We consider the efficient sequential equilibrium in which countries follow history-dependent strategies so as to characterize the rich dynamic path of concessions and war. In order to answer our motivating question, we distinguish between temporary war and total war, which we define as the permanent realization of war. In our model, war is the unique static Nash equilibrium, so that total war is equivalent to the repeated static Nash equilibrium in which countries refrain from ever peacefully negotiating.

There are three main results in our paper. Our first result is that wars are necessary along the equilibrium path. This insight adds to the theory of war by showing how the realization of war serves as a *punishment* for the failure to engage in successful peaceful bargaining in the past. In our framework, both the aggressive and non-aggressive country recognize that war is ex-post inefficient, though it improves ex-ante efficiency by providing incentives for concession-making by the non-aggressive country. Our intuition for the realization of war is linked to the insights achieved by previous work on the theory of dynamic games which shows that the realization of inefficient outcomes (such as price wars) can sustain efficient outcomes along the equilibrium path (e.g., Green and Porter, 1984 and Abreu, Pearce, and Stacchetti, 1986,1990). An important technical distinction of our work from this theoretical work is that information in our environment is coarse. Specifically, though the aggressive country is always certain that the non-aggressive country is cooperating whenever concessions succeed, the aggressive country receives no information if concessions fail, and it cannot deduce the likelihood that the non-aggressive country is genuinely unable to make a concession. Therefore, there is a chance that the aggressive country is making a mistake by going to war. This technical distinction is important for our next results.

Our second result is that temporary war can occur along the equilibrium path. While the aggressive country must fight the non-aggressive country in order to sustain concessions, it need not engage in total war; it can forgive the non-aggressive country for the first few failed concessions by providing the non-aggressive country with another chance at peace after the first round of fighting. This insight emerges because of the coarseness of information in our environment. There is a large chance that the aggressive country is misinterpreting the failure to make a concession as being due to lack of cooperation, and consequently, it is not necessary for the aggressive country to punish initial failed concessions with the most extreme punishment of total war since it may be making an error. More specifically, the equilibrium begins in the following fashion: Periods of peace are marked by escalation in which failure to make concessions by the non-aggressive country leads the aggressive country to request bigger and bigger concessions. Both countries strictly prefer this scenario to one in which initial failures to make a concession are punished by war since war is destructive and represents a welfare loss for both countries. With positive probability, the non-aggressive country is incapable of making concessions for several periods in sequence so that requested concessions become larger and larger, and the only way for the aggressive country to provide incentives for such large concessions to be made is to fight the non-aggressive country if these concessions fail. Consequently, some initial concessions fail, there is an initial temporary war, and this culminates with the aggressive country forgiving the non-aggressive country and giving peace another chance.

Our final result is that countries can engage in temporary wars in the long run only under special conditions, and countries necessarily converge to total war if these conditions are not satisfied. More specifically, temporary wars can be sustained in the long run equilibrium if countries are sufficiently patient, if the cost of war is sufficiently large, and if cost of concessions is sufficiently low. If countries are patient and if war is very costly relative to peace, then total war is an extremely costly punishment which need not be exercised to elicit peaceful concessions, particularly since these are not so costly for the non-aggressive country to make. In the long run, no matter how many concessions fail, the aggressive country continues to forgive the non-aggressive country after a round of fighting, and it continues to provide the non-aggressive country with another chance at peace. In contrast, if countries are impatient, if the cost of war is low, or if the cost of making concessions is high, then countries must converge to total war. In this scenario, even the most extreme punishment of total war is not unpleasant enough for the non-aggressive country since it does not suffer so much under war and it does not place much value on the future. Moreover, the cost of making a peaceful concession for the non-aggressive country is so large that it eventually requires an extreme punishment for failure to meet its obligation. Consequently, even though temporary wars occur along the equilibrium path through the process of escalation, eventually it becomes impossible for the aggressive country to continue to forgive the non-aggressive country and total war becomes a necessity.

Our paper makes two contributions. First, it is an application of a dynamic imperfect information game with history dependent strategies to war. This is important since the study of war is a dynamic issue in which countries have long memories–particularly in long lasting conflicts–and since the literature on war has recognized the importance of limited commitment and imperfect information. In contrast to the current work on war, we provide an explanation for war which combines these two frictions in a dynamic setting in which countries follow history dependent strategies and in which neither peace nor war is an absorbing state. This allows the model to feature escalation, temporary wars, and total war.[4]

Second, our paper is an application of a dynamic imperfect information game in an environment with a coarse information structure. Much of the existing literature on dynamic games assumes a rich information structure, and this leads to a *Bang-Bang* characterization of efficient equilibria.[5] In the context of war and diplomacy, this information structure and its equilibrium implications may not be appropriate. First, countries often have very little information about their enemy's behavior and intentions, particularly when their enemy is not cooperating. Second, even though temporary wars occur in actuality, in many environments in which total war represents the worst possible outcome, the *Bang-Bang*

[4]See Footnote 2 for references.

[5]See Footnote 3 for references.

characterization of efficient equilibria implies that temporary wars do not occur.[6] In this paper, we show that under a coarse information structure, the *Bang-Bang* property need not hold since the prospect for error is large, and this allows us to generate temporary wars in equilibrium. Nevertheless, we show that the *Bang-Bang* property must hold in the long run under some conditions in which countries converge to total war.[7]

The paper is organized as follows. Section 2 describes the model. Section 3 defines efficient sequential equilibria. Section 4 characterizes the equilibrium and provides our main results. Section 5 illustrates the mechanics of the model and relates it to historical examples. Section 6 concludes. The Appendix contains proofs and additional material not included in the text.

2 Model

We consider an environment in which an aggressive country seeks political or economic concessions from a non-aggressive country. In every period, the aggressive country can enforce these concessions by war, or it can alternatively let the non-aggressive country make these concessions unilaterally under peace. With some positive probability, the non-aggressive country is incapable of making concessions because they are too costly. This may happen, for instance, because the non-aggressive country's government experiences severe domestic opposition to concession-making. Nevertheless, this cost of concession-making is not observed by the aggressive country, so that the non-aggressive country can always lie about the true reasons for the failure of concessions.

More formally, there are two countries $i = \{1, 2\}$ and time periods $t = \{0, ..., \infty\}$. Country 1 is the aggressive country and country 2 is the non-aggressive country. In every date t, country 1 publicly chooses $W_t = \{0, 1\}$. If $W_t = 1$, war takes place, each country i receives w_i, and the period ends. Alternatively, if $W_t = 0$, peace occurs, and country 2 publicly makes a concession to country 1 of size $x_t \in [0, \overline{x}]$. Country 1 receives x_t and country 2 receives $-x_t - c(x_t, s_t)$ for $c(x_t, s_t)$ which represents country 2's *private* additional cost of making a concession x_t which is a function of the state $s_t = \{0, 1\}$. s_t is observed by country 2 but not by country 1. Let $c(x_t, s_t) = \overline{c} > 0$ if $x_t > 0$ and $s_t = 0$ and let $c(x_t, s_t) = 0$ otherwise. s_t is stochastic and determined as follows. If $W_t = 0$, then

[6] That is if total war is the min-max. This characterization applies only to efficient equilibria. Efficient equilibria as opposed to other often-examined equilibria such as Markovian equilibria or trigger strategy equilibria are a useful selection device in our setting since rival countries have long memories of their past interaction which can lead to escalation.

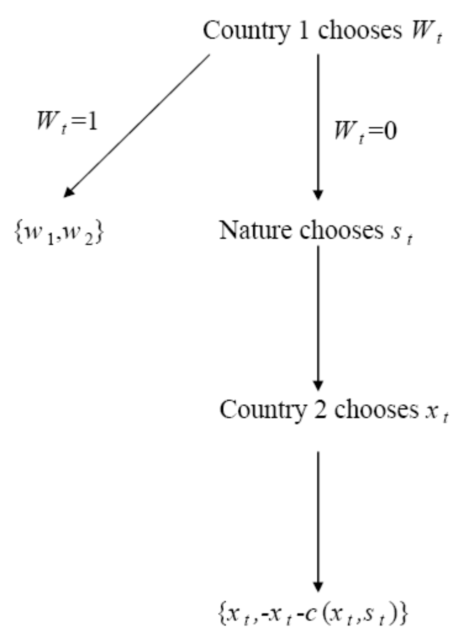
[7] See Fudenberg and Levine (2007) and Sannikov and Skrzypacz (2007) for an additional discussion of the characteristics of equilibria under different information structures.

prior to the choice of x_t, nature chooses s_t with $\Pr\{s_t = 1\} = \pi \in (0,1)$.

Concessions by country 2 are more costly if $s_t = 0$, but this cannot be verified by country 1. For example, imagine if $\bar{c}$ is very high–as we will do–and imagine if this implies that concessions cannot be positive if $s_t = 0$. Then, if $W_t = 0$ and if country 1 receives no concessions (i.e., $x_t = 0$), country 1 cannot tell if country 2 could not make concessions since the cost was too high (i.e., $s_t = 0$) or if country 2 could make concessions but chose to not cooperate (i.e., $s_t = 1$).

We do not allow country 2 to choose to go to war or to receive concessions from country 1 only as a matter of parsimony. Under this additional refinement, the characterization of the equilibrium is identical to the one presented here, and all of our results are left unchanged.[8] Moreover, all of our results and intuitions generalize to an environment in which concessions are binary with $x_t \in \{0, \overline{x}\}$.[9] The game is displayed in Figure 1.

Figure 1: Game



[8]Such a model is isomorphic to the one here since country 1 always makes zero concessions. Details available upon request.

[9]This may be more appropriate for some applications. Note that being able to randomize over concession-making becomes crucial here. Details available upon request.

Let $u_i(W_t, x_t, s_t)$ represent the payoff to i at t.[10] Each country i has a period zero welfare

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u_i(W_t, x_t, s_t), \ \beta \in (0,1).$$

Assumption 1 (inefficiency of war) $\exists x \in [0, \overline{x}]$ s.t. $\pi x > w_1$ and $-\pi x > w_2$.

Assumption 2 (military power of country 1) $w_1 > 0$.

Assumption 1 captures the fact that war is destructive, since both countries can be made better off if war does not take place and country 2 makes a concession to country 1 in state 1. Assumption 2 illustrates why country 1 is the aggressive country, since country 1's military power w_1 exceeds the economic resources under its control of size 0. The fact that w_1 exceeds w_2 (which is negative) is without loss of generality, and it is due to the normalization of both countries' resources to 0 which is purely for notational simplicity.[11]

Assumption 2 has an important implication. Specifically, in a one-shot equilibrium $W = 1$ is the unique static Nash equilibrium. This is because conditional on $W = 0$, country 2 chooses $x = 0$. Thus, by Assumption 2, country 1 chooses $W = 1$. Because the possibility of war precedes the possibility of peace, country 2 cannot commit to making concessions.[12] Consequently, in a static equilibrium, country 1, which is dissatisfied with the lack of concessions (by Assumption 2), will choose to enforce concessions via war rather than to provide country 2 with a chance at peace.

Since the static Nash equilibrium is inefficient (by Assumption 1), one can imagine that in a dynamic framework, country 1 may be able to enforce concessions from country 2 by rewarding successful concessions today by refraining from war in the future. Nevertheless, there are two obstacles to this arrangement which are important to consider. First, country 1 cannot commit to unconditionally refraining from war in the future, since it also suffers from limited commitment. Thus, whenever country 1 refrains from fighting at some date, it must be promised sufficient concessions in the future as a reward. Second, country 1 does not observe the state s_t and the cost of concessions $c(\cdot,\cdot)$ which may be very large.

Assumption 3 (high cost of concessions) $\overline{c} > -\beta w_2 / (1-\beta)$.

[10] Specifically, $u_1(W_t, x_t, s_t) = W_t w_1 + (1 - W_t) x_t$ and $u_2(W_t, x_t, s_t) = W_t w_2 - (1 - W_t)(x_t + c(x_t, s_t))$.

[11] Country 2 can be more powerful and control more resources than country 1 and vice versa as long as country 1's military power exceeds its economic power.

[12] If the opportunity to engage in war followed peace, then country 2 would not be constrained by limited commitment since concession-making could be induced even in a static Nash equilibrium.

In a dynamic environment, Assumption 3 implies that if $s_t = 0$, then country 2's concessions are so prohibitively costly that even the highest reward for a positive concession and the highest punishment for zero concessions together cannot induce a positive concession by country 2.[13] Therefore, concessions must be zero if $s_t = 0$. Consequently, if concessions fail (i.e., $x_t = 0$), country 1 cannot determine if this is unintentional because their cost is too high (i.e., $s_t = 0$) or if this is intentional because their cost is low (i.e., $s_t = 1$). This means that if country 1 goes to war in response to a failed concession, there is a chance that it is making a mistake since the concession's failure is unintentional.

More formally, information in our environment is coarse. Though country 1 is always certain that country 2 is cooperating whenever concessions succeed, country 1 receives no information if concessions fail, and it cannot deduce the likelihood that country 2 is genuinely unable to make a concession. As we will discuss in Section 4.3, this detail is important as it will lead to temporary wars.

3 Efficient Sequential Equilibria

In this section, we present our recursive method for the characterization of the efficient sequential equilibria between the two countries. We provide a formal definition of these equilibria in the Appendix. The important feature of a sequential equilibrium is that each country dynamically chooses its best response given the strategy of its rival at every public history.[14]

As there are many sequential equilibria, we characterize the efficient sequential equilibria to this game, which are the set of equilibria which maximize the period 0 welfare of country 1 subject to providing country 2 with some minimal period 0 welfare v_0 (or vice versa). In contrast to Markovian equilibria or trigger strategy equilibria, these equilibria are allowed to feature rich history dependent dynamics such as escalation, and this is arguably a more accurate description of warring countries which are often motivated by long memories of their past interactions.[15] The most important feature of these equilibria due to the original insight achieved by Abreu (1988) is that they are sustained by the worst punishment. More specifically, all public deviations from equilibrium actions lead countries to the worst punishment off the equilibrium path, which in our environ-

[13] This is because the discounted difference between the largest possible reward (permanent peace with continuation value of 0) and the largest possible punishment (permanent war with continuation value of $w_2(1-\beta)$) is not sufficiently large relative to $\bar{c}$.

[14] Because country 1's strategy is public by definition, any deviation by country 2 to a non-public strategy is irrelevant (see Fudenberg, Levine, and Maskin, 1994).

[15] This is also the approach pursued in the related work mentioned in Footnote 3.

ment corresponds to the repeated static Nash equilibrium, which we refer to as *total war*. Therefore, we define

$$\underline{U}_i = \frac{w_i}{1-\beta},$$

the payoff to country i from total war.

Note that in characterizing this equilibrium, we take into account that it may be efficient for country 1 to probabilistically choose to go to war. Therefore, we will say that in every period, country 1 chooses a probability of war, and the continuation game will depend in part on whether war has taken place.[16]

As is the case in many incentive problems, an efficient sequential equilibrium can be represented in a recursive fashion, and this is a useful simplification for characterizing equilibrium dynamics.[17] Specifically, at any public history, the entire public history of the game is subsumed in the continuation value to each country, and associated with these two continuation values is a continuation sequence of actions and continuation values. More specifically, let v represent the continuation value of country 2 at a given history. Associated with v is $J(v)$, which represents the highest continuation value achievable by country 1 in a sequential equilibrium conditional on country 2 achieving a continuation value of v.[18] More formally:

$$J(v) = \max_{W, v^W, x, v^H, v^L} W\left[w_1 + \beta J\left(v^W\right)\right] + (1-W)\left[\pi\left(x + \beta J\left(v^H\right)\right) + (1-\pi)\beta J\left(v^L\right)\right] \tag{1}$$

s.t.

$$v = W\left[w_2 + \beta v^W\right] + (1-W)\left[\pi\left(-x + \beta v^H\right) + (1-\pi)\beta v^L\right], \tag{2}$$

$$J\left(v^W\right), J\left(v^H\right), J\left(v^L\right) \geq \underline{U}_1, \tag{3}$$

$$v^W, v^H, v^L \geq \underline{U}_2, \tag{4}$$

$$-x + \beta v^H \geq \beta v^L, \tag{5}$$

$$v^H = v^L \text{ if } x = 0, \tag{6}$$

$$W \in [0,1] \text{ and } x \in [0, \overline{x}]. \tag{7}$$

(1) represents the continuation value to country 1 written in a recursive fashion at a

[16] This is possible by the introduction of a random public signal so that countries play correlated public strategies. All of our proofs take into account that country 2 can also randomize over its concessions, but we ignore this for expositional simplicity in the text. See Appendix for more details.

[17] This is consequence of the insights from the work of Abreu, Pearce, and Stacchetti (1986,1990).

[18] If country 1 were receiving any continuation value below $J(v)$ at a given public history, then the equilibrium would not be efficient.

given history. With some abuse of notation, W represents the probability of war today. v^W represents the continuation value promised to country 2 for tomorrow conditional on war taking place today. If war does not take place, then concessions are zero if $s = 0$ (by Assumption 3), and concessions are equal to x if $s = 1$. We refer to x as the *requested* concession. Moreover, conditional on peace today, the continuation value promised to country 2 for tomorrow is v^H if $s = 1$ and v^L if $s = 0$.

Equation (2) represents the promise keeping constraint which ensures that country 2 is achieving a continuation value of v. Constraint (6) ensures that the continuation equilibrium is a function of public information. Constraints (7) ensure that the allocation is feasible.

Constraints (3) – (5) represent the incentive compatibility constraints of this game. Without these constraints, the solution to the problem starting from an initial v_0 is simple: Countries refrain from war forever. Constraints (3) – (5) captures the inefficiencies introduced by the presence of *limited commitment* and *imperfect information* which ultimately lead to the possibility of war. Constraint (3) captures the fact that at any history, country 1 cannot commit to refraining from total war which provides a continuation welfare of $\underline{U}_1$. Constraint (4) captures the fact that at any history, country 2 cannot commit to concession-making, as it can stop concessions forever and ensure itself a continuation value of at least $\underline{U}_2$. Therefore, country 2 cannot commit to making concessions and country 1 cannot commit to rewarding country 2 by refraining from war. Constraints (3) and (4) together capture the constraint of limited commitment. Under perfect information, they imply that if countries are sufficiently patient, permanent peace can be sustained by the off-equilibrium threat of total war. Constraint (5) captures the additional constraint of imperfect information: Country 1 does not observe the state s. If $s = 1$ and requested concessions $x > 0$ can be made, country 2 can always choose to pretend that $s = 0$ and make zero concessions without detection by country 1. Constraint (5) ensures that country 2's punishment from this deviation (βv^L) is weakly exceeded by the equilibrium path reward from making the concession ($-x + \beta v^H$).

Figure 2 depicts $J(v)$ as a function of v for $v \in \left[\underline{U}_2, \overline{U}_2\right]$ for some $\overline{U}_2 \geq \underline{U}_2$ which represents country 2's highest sequential equilibrium continuation value. The y-axis represents $J(v)$ and the x-axis represents v. All of the points underneath $J(v)$ and above the x-axis represent the space of sequential equilibrium continuation values.

There are three important features of Figure 2.[19] First, $J(\underline{U}_2) = \underline{U}_1$. This is a consequence of (4), country 2's inability to commit to concessions. If country 2 could

[19]See the Appendix for a formalism and proof of these facts.

commit to concessions, then country 1 would choose $W = 0$ and request a high enough level of concessions from country 2 so as to provide it with a continuation value of $\underline{U}_2$.[20] This would clearly be less destructive than total war by Assumption 1. However, under limited commitment, country 2 can always deviate from such an arrangement by making zero concessions today and guaranteeing itself a continuation value of at least $\underline{U}_2$ starting from tomorrow, so that its welfare today from the deviation is $\beta\underline{U}_2$ which exceeds its equilibrium welfare $\underline{U}_2$ (since w_2 is negative). Therefore, because country 2 cannot commit to concessions, country 1 must engage in total war in order to provide a continuation value of $\underline{U}_2$ to country 2.

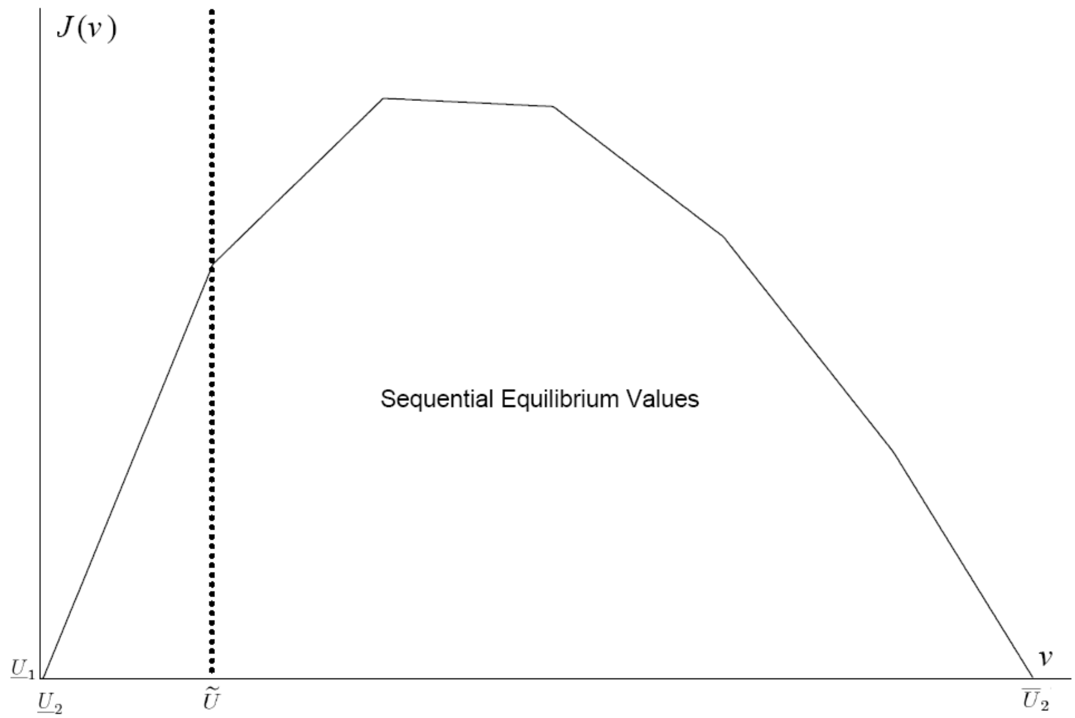
The second important feature of Figure 2 is that $J\left(\overline{U}_2\right) = \underline{U}_1$. This is a consequence of (3), country 1's inability to commit to peace. If country 1 could commit to peace, the the highest continuation value to country 2 would be associated with permanent peace and zero concessions, yielding a continuation value of 0 to both countries. However, under limited commitment, country 1 can always deviate from such an arrangement by engaging in total war and guaranteeing itself a continuation value of $\underline{U}_1$ which exceeds 0 by Assumption 2. Therefore, because country 1 cannot commit to peace, the present discounted value of concessions must always be positive whenever country 1 is refraining from war, and this is embedded in the fact that $J\left(\overline{U}_2\right) = \underline{U}_1 > 0$.

The third important feature of Figure 2 is that $J(v)$ is inverse U-shaped. The increasing portion of $J(v)$ is a consequence of the fact that country 1 is made better off by the increase in country 2's value since this implies a lower incidence of war and an increase in the size of the surplus to be shared by the two countries. The decreasing portion of $J(v)$ is a consequence of the fact that beyond a certain point, an increase in country 2's value entails a decrease in the size of the concessions made from country 2 to country 1, which means that country 1's value declines. Along this downward portion, it is not possible to make one country strictly better off without making the other country strictly worse off. As such, any efficient sequential equilibrium must begin on the downward sloping potion of $J(v)$. Nevertheless, as we will see, the presence of imperfect information embedded in constraint (5) implies that it is not possible for the two countries to remain along the downward sloping portion of $J(v)$ forever.[21]

[20] That is, assuming that $-\pi\overline{x} < w_2$ so that large enough concessions are feasible.

[21] In fact, if (5) is ignored, then the two countries can sustain a permanently peaceful equilibrium along the downward sloping portion of $J(v)$.

Figure 2: $J(v)$



4 Analysis

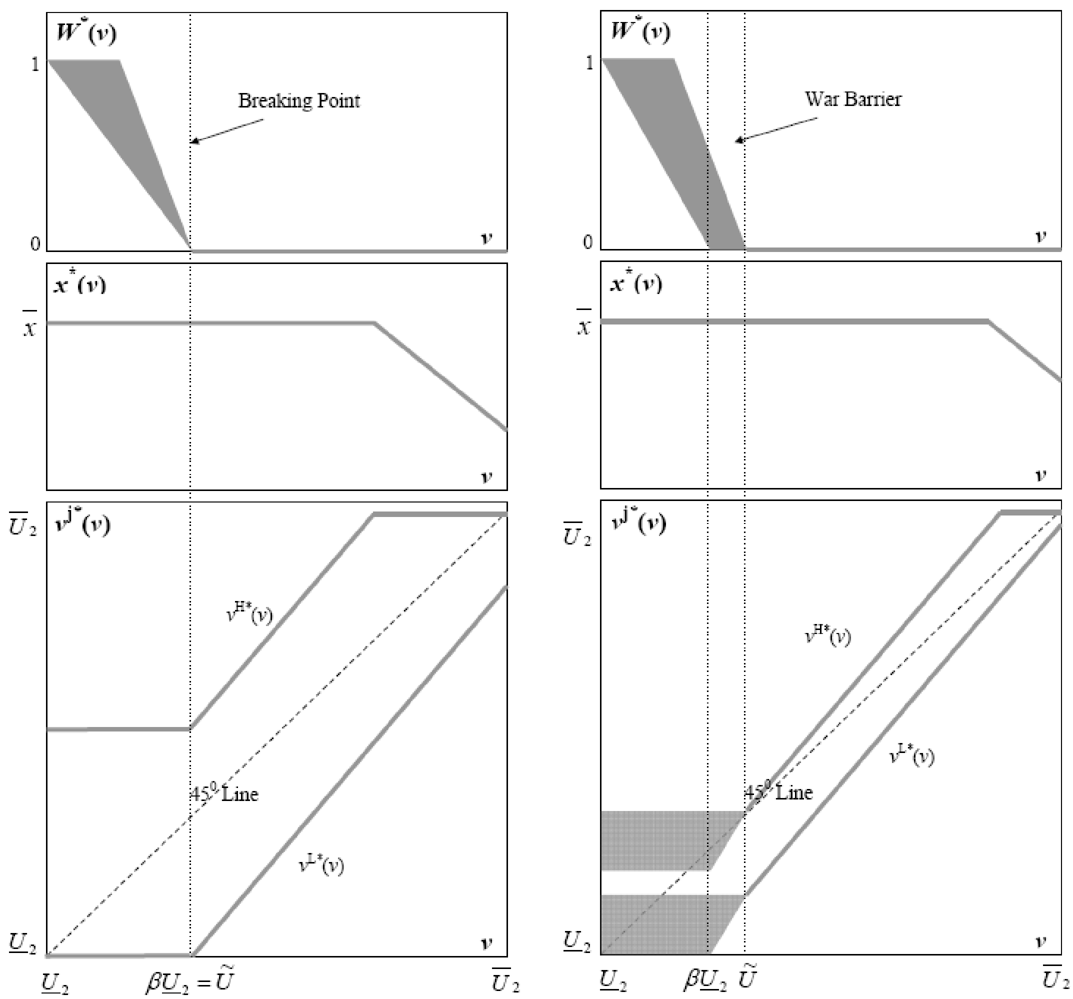
We can denote the solution to $(1) - (7)$ by

$$\left\{W^*(v), v^{W*}(v), x^*(v), v^{H*}(v), v^{L*}(v)\right\}.$$

While this solution need not be unique, all of the solutions will have several common characteristics which we will discuss. As a preview of the solution, we display $W^*(v), x^*(v), v^{H*}(v)$, and $v^{L*}(v)$ graphically in Figure 3, and we exclude $v^{W*}(v)$ due to space constraints. For reasons which will become clear, we distinguish between two cases: $w_2 > -\beta\overline{x}$ and $w_2 < -\beta\overline{x}$.[22]

[22]We do not display the knife-edge case with $w_2 = -\beta\overline{x}$ due to space restrictions.

Figure 3: Recursive Solution



Case 1: $w_2 > -\beta\overline{x}$

Case 2: $w_2 < -\beta\overline{x}$

For the remainder of our discussion, we will assume that countries are sufficiently patient that peace is incentive compatible for a positive mass of continuation values.[23]

Assumption 4 $\beta > -w_1/\left(\pi w_2\right).$

We discuss this solutions and its equilibrium implications in the following sections. In doing so, we pay specific attention to the equilibrium realization of temporary wars and to the long run properties of the equilibrium. In particular, whereas total war corresponds

[23] The precise implications of Assumption 4 are described in the Appendix.

to the permanent realization of war associated with continuation value $\underline{U}_2$, a *temporary war* at t corresponds to a situation in which

$$\Pr\{W_{t+k} = 0 | W_t = 1\} > 0 \text{ for some } k > 0,$$

so that the realization of war is not absorbing and a return to peace can take place.[24] In our analysis, we show that the process of escalation can lead to temporary wars. Nonetheless, while temporary wars are possible along the equilibrium path, we show that the long run realization of temporary wars requires a certain set of conditions.

4.1 Peace Sustained by War

We argue in this section that the realization of peace along the equilibrium path is sustained by the realization of war in the future if concessions fail under peace. A critical feature of this argument is the fact that above a certain continuation value $\widetilde{U}$, war ceases to occur, and below this continuation value, war takes place. Intuitively, war is a way for country 1 to punish country 2 for its failure to cooperate, and it is associated with a low continuation value. This is expressed in the below lemma.

Lemma 1 *If $v \geq \widetilde{U}$, then $W^*(v) = 0$.*

This result is displayed in the top panel of Figure 3 which depicts $W^*(v)$ as a function of v. The probability of war becomes positive below continuation value $\widetilde{U}$. A heuristic description for the proof of Lemma 1 is as follows. Countries can effectively randomize over the realization of war, and this ability to randomize implies a linearity in $J(v)$ for the continuation values which are generated by the positive probability of war. Since $W^*(\underline{U}_2) = 1$, it follows that there is an interval $\left[\underline{U}_2, \widetilde{U}\right)$ over which the probability of war is positive and over which $J(v)$ is linear. This is displayed in Figure 2.

Proposition 1 ***(necessity of war)*** *The solution admits* $\Pr\{W_{t+k} = 1 | W_t = 0\} > 0\ \forall t$ *for some $k > 0$.*

The proposition states that if peace occurs today, then war must be expected with some probability in the future. The intuition behind Proposition 1 is related to the key insights developed in theory of dynamic games in Green and Porter (1984) and Abreu, Pearce, and Stacchetti (1986,1990). The inefficient event of war must be realized ex-post

[24] We will only use $W^*(v)$ to refer to the probability of war in the recursive solution, whereas $W_t = \{0, 1\}$ continues to correspond to the stochastic realization of war in period t.

in order to sustain peace ex-ante. Without war, country 2 makes zero concessions, and by Assumption 2, country 1 cannot be satisfied by zero concessions. Therefore, any periods of peace are necessarily followed by periods of war, since concessions can always fail with positive probability.

More formally, if $W^*(v) = 0$ so that peace takes place today, then (2) and (5) imply that

$$v^{L*}(v) \leq v/\beta < v, \tag{8}$$

where we have used the fact that $v < 0$. This last point follows from the fact that a continuation value of 0 is associated with permanent peace which does not constitute a sequential equilibrium since it is not incentive compatible for country 1 (by Assumption 2). Therefore, continuation values always have a probability $1 - \pi$ of declining conditional on peace taking place today. This is displayed in the bottom panel of Figure 3 which shows that for $v \geq \widetilde{U}$, $v^{L*}(v) < v$. Therefore, the existence of imperfect information embedded in the constraint (5) implies that war must occur along the equilibrium path, since v can always decline beyond $\widetilde{U}$ so as to induce the positive probability of war.

There are two important implications of (8). First, since $\widetilde{U} \geq \beta v^{L*}\left(\widetilde{U}\right)$ and $v^{L*}\left(\widetilde{U}\right) \geq \underline{U}_2$, it follows that $\widetilde{U} \geq \beta\underline{U}_2 > \underline{U}_2$. Below $\beta\underline{U}_2$ assured peace ceases to be incentive compatible since there does not exist a severe enough punishment to provide country 2 sufficient incentives for large enough concession-making. Whether $\widetilde{U}$ equals or exceeds $\beta\underline{U}_2$ is an important to detail to keep in mind for our discussion of long run temporary wars in Section 4.4.

The second important implication of (8) relates to the shape of $J(\cdot)$ in Figure 2. The efficient equilibrium which begins on the downward sloping portion of $J(\cdot)$ inevitably transitions to the upward sloping portion of $J(\cdot)$. Once the two countries arrive at the upward sloping portion of $J(\cdot)$, they recognize that it is necessary for them to engage in an inefficient interaction in order to sustain the efficient interaction which has taken place in the past. Moreover, countries realize that *attempted* cooperation has in fact occurred in the past: War is by no means ex-post necessary, though it is ex-ante required for the enforcement of peace.

4.2 Escalation

What is the size of the requested concession $x^*(v)$? How does country 1 reward or punish country 2 for the success or failure of concessions, respectively? In this section, we answer these questions, and in doing so, we show that our equilibrium features escalation.

Rather than punishing initial failed concessions with war, country 1 punishes country 2 by requesting larger and larger concessions.[25]

Proposition 2 ***(escalation)*** *If* $v \geq \widetilde{U}$*, then*

$$x^*(v) = \min\left\{\beta\overline{U}_2 - v, \overline{x}\right\},\ v^{H*}(v) = \min\left\{\frac{v+\overline{x}}{\beta}, \overline{U}_2\right\},\ \textit{and } v^{L*}(v) = v/\beta.$$

The content of Proposition 2 is displayed in the middle and bottom panels of Figure 3. As the continuation value v declines, the requested concession $x^*(v)$ increases until it reaches the maximum $\overline{x}$. Moreover, the reward for a successful concession $v^{H*}(v)$ and the punishment for an unsuccessful concession $v^{L*}(v)$ are both weakly increasing in the continuation value v. There are two important results embedded in Proposition 2. First, it must be the case that country 1 is either (i) demanding the highest feasible concession or (ii) promising the highest incentive compatible reward in the future. Second, equation (5), the incentive compatibility constraint due to imperfect information, must bind.

The intuition for the first result is as follows. Though war needs to occur, country 1 would like to postpone its realization since it destroys surplus and harms both countries. An important way in which country 1 can postpone the realization of war is by requesting as high concessions from country 2 today so as to reward their success with as high a reward as possible. This works since a higher reward is associated a longer duration of peace which benefits both countries going forward.

More formally, let us fix $v^{L*}(v)$, taking into account that since $v \geq \widetilde{U}$, it must be that $W^*(v) = 0$. To get a heuristic insight for the proof, imagine if $x^*(v) < \overline{x}$ and $v^{H*}(v) < \overline{U}_2$. Then necessarily, a perturbation which increases $x^*(v)$ by ϵ and $v^{H*}(v)$ by ϵ/β for some $\epsilon > 0$ which is arbitrarily small continues to satisfy $(2)-(7)$. Moreover, the change in the welfare of country 1 is $\pi\left(\epsilon + \beta J\left(v^{H*}(v) + \epsilon/\beta\right) - J\left(v^{H*}(v)\right)\right)$ is positive as long as the slope of $J(\cdot)$ is strictly above -1, which is the case in our framework. To see why, note that in an environment which ignores incentive compatibility constraints $(3)-(5)$, the slope of $J(\cdot)$ would be -1. This is because a transfer of 1 unit of welfare from country 1 to country 2 would occur through a reduction in concessions which the two countries equally value. In contrast, under $(3)-(5)$, the transfer of 1 unit of welfare from country 1 to country 2 occurs through a reduction in concessions *as well as a reduction in the probability of future war, which is beneficial to both countries.* Therefore, the implied

[25] An equivalent version of our model in which concessions are binary features escalation in the form of an increased probability of requested concessions.

reduction in size of the concession is not as large so that the slope of $J(\cdot)$ exceeds -1.[26]

An additional rationale for this result can be generated by imagining an environment which ignores the upper bound on x as well as the lower bound on $J(v)$ in (3). It can be shown that in such a setting, the efficient solution always requires country 1 to reward country 2's first successful concession with permanent peace. This is efficient since it allows country 1 to extract as much as possible in the current period while maximizing the duration of peace in the future, which is beneficial to both countries. In our environment, this insight drives either the upper bound on x or the lower bound on $J(v)$ to bind.

The second result embedded in Proposition 2 is that (5) binds. The intuition for this is that lower levels of $v^{L*}(v)$ are associated with a welfare loss to both countries due to the increased incidence of war, and consequently, it is optimal to maximize $v^{L*}(v)$ so as to increase the duration of peace. The technical reason for this is that above $\widetilde{U}$, $v^{H*}(v)$ and $v^{L*}(v)$ are never on the same line segment of $J(v)$. Thus, optimality and the concavity of $J(\cdot)$ requires that $v^{L*}(v)$ be as high as possible subject to the satisfaction of (5).

Proposition 2 has an important implication for the time path of concessions. As we will discuss in the next couple of sections, an important feature of $\widetilde{U}$ is that $\frac{\widetilde{U}+\overline{x}}{\beta} \geq \widetilde{U}$, which effectively means that $v^{H*}(v) > v$ for all $v > \widetilde{U}$, so that a successful concession is rewarded with an increase in continuation value. Therefore, if a concession today is successful, the continuation value tomorrow increases, and by consequence, requested concessions tomorrow weakly decrease. Therefore, a reward for successful concessions is a reduction in future requested concessions. In contrast, if a concession today is unsuccessful, the continuation value tomorrow decreases, and by consequence, any concession requested tomorrow must weakly increase. This incentive scheme enforces concessions along the equilibrium path, since country 2 will always make a concession when it is able to, since failure to do so can result in an increase in future requested concessions. Therefore, along the equilibrium path, a long sequence of failed concessions by country 2 can cause requested concessions by country 1 to incrementally increase, and by Proposition 1, war eventually becomes necessary to enforce these ever-increasing concessions.

As an example, consider an environment in which $\overline{x}$ is arbitrarily large, so that the constraint that $x \leq \overline{x}$ never binds. In this scenario, Proposition 2 dictates that $v^{H*}(v) = \overline{U}_2$, $x^*\left(\overline{U}_2\right) = (\beta - 1)\,\overline{U}_2$, and $v^{L*}(v) \geq \widetilde{U}$ if $v \geq \widetilde{U}/\beta$. Now imagine that along the equilibrium path country 1 requests a concession x_t from country 2, where associated

[26] Formally, in the absence of $(3)-(5)$, a reduction of $1/\pi$ unit of concessions from country 2 to country 1 provides an additional unit of welfare for country 2. In the presence of $(3)-(5)$, there is a reduction in the future probability of war which is incentive compatible since country 2 now makes lower concessions. Thus, the relevant reduction in concessions need not be as large as $1/\pi$.

with this concession is a continuation value $v_t \geq \widetilde{U}/\beta$ promised to country 2. It follows from Proposition 2 that $x_{t+1} = x^* \left(\overline{U}_2\right) + x_t/\beta$ if the concession at t is unsuccessful and that $x_{t+1} = x^* \left(\overline{U}_2\right)$ if it is successful.[27] If v_{t+1} in the former scenario exceeds $\widetilde{U}/\beta$, then it follows that a subsequent failed concession at $t+1$ causes country 1 to request a concession $x_{t+2} = x^* \left(\overline{U}_2\right) + x_{t+1}/\beta$ at $t+2$, and so on. Therefore, in this simple example, country 1 always requests a base concession of size $x^* \left(\overline{U}_2\right)$ plus accrued missed concessions from the past, adjusting for discounting.

Why do failed concessions lead to escalation as opposed to immediate war? This is because war is costly to both countries, whereas larger concessions are only costly to country 2 and beneficial to country 1. Therefore, a sequence of initial missed concessions does not automatically lead to war, but to escalation. Country 1 forgives country 2 for the first missed concessions by requesting larger and larger concessions. More specifically, it requests compensation for previously missed concessions, to the extent allowed by the upper bound $\overline{x}$. Nevertheless, there is a limit to which punishing country 2 with an increase in requested concessions can work, since beyond a certain point, requested concessions become so large that country 1 must punish their failure with the realization of war.

4.3 Temporary Wars

We have argued in the previous section that concessions can be sustained by the threat of escalation through the request of even larger concessions. Nonetheless, as is clear in Proposition 1, there is a limit to which such a punishment mechanism is sustainable, since eventually concessions become so large that their failure must be punished by war. In this section, we show that the initial realization of war can be a temporary war. In doing so, we highlight how the coarseness of information in our model is critical for the generation of temporary wars, and we relate this insight to the previous work on dynamic games which has assumed a rich information structure.

As a reminder, a temporary war at t takes place if $\Pr\{W_{t+k} = 0 | W_t = 1\} > 0$ for some $k > 0$. Therefore, conditional on the probability of war $W^*(v) > 0$, a temporary war starting from continuation value v occurs if $v^{W*}(v) > \underline{U}_2$. Since $v^{W*}(v)$ exceeds the continuation value associated with total war $\underline{U}_2$, it is necessary that it is associated with the future stochastic realization of peace.

Lemma 2 *If $v \in \left(\underline{U}_2, \widetilde{U}\right)$, $W^*(v) > 0$ and $v^{W*}(v) > \underline{U}_2$ is a solution.*

[27] The derivation of x_{t+1} follows from the fact that $v_{t+1} = v_t/\beta$ if $s_t = 0$, $x_{t+1} = \beta\overline{U}_2 - v_{t+1}$, and $x_t = \beta\overline{U}_2 - v_t$.

Lemma 2 implies that any path to a continuation value in the interval $\left(\underline{U}_2, \widetilde{U}\right)$ can be associated with a temporary war. This interval represents a set of continuation values in which country 2 is punished, but not maximally with total war. The technical reason for why these continuation values are associated with a temporary war is as follows. Consider a solution which does not feature a temporary war so that $W^*(v) > 0$ and $v^{W*}(v) = \underline{U}_2$ associated with some $W^*(v) < 1$.[28] Consider a perturbation which increases the probability of war $W^*(v)$ by ϵ and increases the continuation value $v^{W*}(v)$ by an amount (which is a function of ϵ) so as to leave (2) satisfied for some $\epsilon > 0$ arbitrarily small. This continues to satisfy $(2)-(7)$, and the linearity of $J(\cdot)$ in the interval $\left[\underline{U}_2, \widetilde{U}\right]$ implies that this perturbation yields the same welfare to country 1 as the original allocation. The intuition for this is as follows. One obvious method of punishing country 2's failed concessions is to engage in total war with low probability. An alternative method is to engage in a temporary war with high probability. Both of these methods are equivalent from an efficiency perspective, and they deliver the same continuation value to country 2. Consequently, conditional on the two countries arriving to a history in the interval $\left(\underline{U}_2, \widetilde{U}\right)$, there is no need for country 1 to punish country 2's failed concessions with total war.

This idea is displayed in the top panel of Figure 3 which shows that in the interval $\left(\underline{U}_2, \widetilde{U}\right)$, $W^*(v)$ is a correspondence. It can take on low values if $v^{W*}(v)$ is chosen to be low, whereas it can take on high values if $v^{W*}(v)$ is chosen to be high. Intuitively, country 1 has flexibility in the intertemporal allocation of war. It can occur with high probability today, but with lower probability in the future, or alternatively, it can occur with low probability today, but with high probability in the future.

For example, consider an equilibrium which begins at an initial point v_0 on the downward sloping portion of $J(\cdot)$. Imagine if this point is not equal to $\beta^T \underline{U}_2$ for some integer T.[29] If the first concession fails, the equilibrium transitions to a lower continuation value v_0/β by Proposition 2. If $v_0/\beta > \widetilde{U} \geq \beta \underline{U}_2$, then country 1 will request a second concession, and in the event that it also fails, the equilibrium transitions to an even lower continuation value v_0/β^2. This can continue to happen with positive probability until the continuation value reaches the interval $\left(\underline{U}_2, \widetilde{U}\right)$. In this interval, country 1 can engage country 2 in total war with some low probability. Alternatively, a temporary war can take place which higher probability. This example leads to the following result that temporary wars can always occur along the equilibrium path starting from some initial continuation

[28]If $W^*(v) = 1$, then $v = \underline{U}_2$ by definition.

[29]This is important, since as a reminder, (8) requires $\widetilde{U} \geq \beta \underline{U}_2$.

value v_0.[30]

Proposition 3 ***(temporary wars)*** *Temporary wars are a solution for some v_0.*

Why does total war not strictly dominate temporary wars in country 1's decision to punish country 2 for failed concessions? This question is particularly relevant given that the previous work on the theory of dynamic games due to Abreu, Pearce, and Stacchetti (1986,1990) has established the necessity of the *Bang-Bang* property in the characterization of efficient equilibria. In our context, the *Bang-Bang* property implies that continuation values only travel to extreme points of $J(\cdot)$. Since all points on $J(\cdot)$ generated by probabilistic war are located on a line in the interval $\left[\underline{U}_2, \widetilde{U}\right)$ (see Figure 2), this implies that, if the *Bang-Bang* property held in our context, any realization of war would be associated with the continuation value $\underline{U}_2$ and total war. Thus, according to previous work, one would predict that in our framework, escalation and the failure to make concessions should lead directly to total war. In our context, escalation to total war is efficient, yet temporary wars are *not inefficient* as suggested by previous work. Temporary wars can always occur, and there is an important reason for this.

Information in our environment is coarse, and as a consequence, the *Bang-Bang* property–which is necessary in environments in which information is sufficiently rich–need not hold. More specifically, though country 1 is always certain that country 2 is cooperating whenever concessions succeed, country 1 receives no information if concessions fail, and it cannot deduce the likelihood that country 2 is genuinely unable to make a concession. Therefore, there is a chance that country 1 is making a mistake by going to war. Thus, total war does not dominate temporary wars as a punishment device since there is a limit on the information which is available to country 1 when it decides on the extent of war. Our model therefore suggests that imperfect and coarse information in the practice of international relations is an important friction which can explain the existence of temporary wars.

This situation would be significantly different, for instance, if country 1 could observe a sufficient amount of information in periods in which concessions fail. For example, imagine if a continuous public signal y were revealed whenever concessions are zero, where this signal y is informative about the state s with higher values of y being more likely if $s = 1$ (i.e., the cost of concessions is low).[31] In this situation, escalation would always

[30] The only v_0 where this may not be true involve equilibria which begin on a point $\beta^T \underline{U}_2$ for some integer T.

[31] Specifically, y has full support conditional on s and it satisfies the monotone likelihood ratio property. I thank Andrew Atkeson for pointing out this example.

be followed by the stochastic realization of total war, with total war being more likely if concessions fail contemporaneously with the realization of a high signal y. Country 1 would effectively use extreme rewards and punishments to provide incentives to country 2 while simultaneously utilizing the information in y to optimally reduce the probability of error in going to war. Our model highlights why this mechanism fails to work once information becomes coarse. Country 1 may make a mistake in going to total war, so that it does not strictly benefit from using such an extreme punishment.

4.4 Long Run Temporary Wars vs. Total War

Our model generates temporary wars along the equilibrium path, and a natural question concerns the extent to which such temporary wars can be sustained in the long run. This is particularly relevant for understanding conflicts in the world which have lasted a significant length of time but have not culminated in total war. We argue that even though temporary wars can occur along the equilibrium path, they can only be sustained in the long run if countries are sufficiently patient (β is high), if the cost of war is sufficiently large (w_2 is low), and if cost of concessions is sufficiently low ($\overline{x}$ is low).

Theorem 1 ***(long run total war)*** *If $w_2 \geq -\beta\overline{x}$, then all solutions for all v_0 converge to total war.*

Theorem 2 ***(long run temporary wars)*** *If $w_2 < -\beta\overline{x}$, then long run temporary wars are a solution for all v_0.*

Theorem 1 implies that convergence to total war is necessary if $w_2 \geq -\beta\overline{x}$, meaning if the cost of war w_2 is low relative to the discounted cost of the maximal concession $-\beta\overline{x}$. Together with our discussion in Section 4.3, Theorem 1 effectively states that even though temporary wars occur along the equilibrium path, eventual convergence to total war is necessary. In contrast, by Theorem 2, convergence to total war is not necessary if $w_2 < -\beta\overline{x}$. In this situation, temporary wars can be sustained forever. Though convergence to total war constitutes an efficient equilibrium, convergence to total war is not necessary for efficiency. If $w_2 \geq -\beta\overline{x}$, then the *Bang-Bang* property described in Section 4.3 necessarily holds in the long run, whereas if $w_2 < -\beta\overline{x}$, the *Bang-Bang* property can continue to fail even in the long run.

The intuition for the first case is as follows. If w_2 is high and β is low, then the cost of total war to country 2 is low relative to the cost of the maximal concession of size $\overline{x}$. As a consequence, it is necessary for country 1 to use the most extreme punishment to induce concessions from country 2, since the weaker punishment of temporary war cannot induce these large concessions. The welfare of the two countries converges to a minimum point, and the two countries sacrifice their welfare in the long run in exchange for efficient incentive provision along the equilibrium path. Note that if $w_2 \geq -\overline{x}$, convergence to total war takes place even as β approaches 1.[32]

In contrast, if $w_2 < -\beta\overline{x}$, then the cost of total war to country 2 is extremely high relative to the cost of the maximal concession of size $\overline{x}$. As a consequence, it is not necessary for country 1 to use the most extreme punishment to induce concessions from country 2 since the weaker punishment of temporary wars is sufficiently painful. Therefore, the welfare of the two countries can remain above the minimum point, and the two countries can converge to the linear segment of $J(v)$ in Figure 2 in the range $\left(\underline{U}_2, \widetilde{U}\right]$. The sacrifice of long run welfare is not necessary for efficiency since it does not improve incentive provision along the equilibrium path.

An implication of our model is that an increase in $\overline{x}$ from below $-w_2/\beta$ to above $-w_2/\beta$ leads countries from long run temporary wars to long run total war. Therefore, the transformation *decreases* welfare in the long run for the two countries. An important point to bear in mind is that the transformation *increases* welfare along the equilibrium path. The reason is that if $\overline{x}$ increases, it becomes easier for the two countries to postpone the realization of war, since escalation as opposed to war can be more easily used by country 1 to provide inducements to country 2. Rather than fighting at a particular date, country 1 can request even larger concessions from country 2 and leave war for a later date. This raises welfare along the equilibrium path by prolonging the incidence of peace. Nevertheless, this is made at the cost of total war in the long run which becomes necessary to induce the increase in concessions under peace.

An insight into the proofs of Theorems 1 and 2 emerges if one considers the value of $\widetilde{U}$ relative to $\beta\underline{U}_2$. As a reminder, $\widetilde{U}$ is the continuation value above which assured peace strictly dominates probabilistic war. Moreover, $\beta\underline{U}_2$ is the continuation value below which assured peace ceases to be incentive compatible (see Section 4.1). If $w_2 \geq -\beta\overline{x}$, then $\widetilde{U} = \beta\underline{U}_2$, whereas if $w_2 < -\beta\overline{x}$, $\widetilde{U} > \beta\underline{U}_2$.

More specifically, if $w_2 \geq -\beta\overline{x}$, then if assured peace is incentive compatible it is strictly optimal. This is because country 1 can induce a sufficient amount of suffering on country 2 via concessions relative to the suffering it can induce under war. This ensures that successful concessions by country 2 can be rewarded with a lower incidence of war in the future. This benefits both countries, which is why assured peace dominates proba-

[32] It is nevertheless the case that the probability of total war also approaches zero since $v^{L*}(v)$ approaches v and the decrease in continuation value approaches zero (see Proposition 2).

bilistic war. Formally, this is implied by the fact that by Proposition 2, $v^{H*}(v) \geq v$ for all v which weakly exceed $\beta \underline{U}_2$, so that the incidence of war is reduced whenever concessions succeed. Since assured peace takes place whenever it is incentive compatible, the duration of peace is prolonged as much as possible along the equilibrium path. Nonetheless, this comes at a cost in the long run. Eventually, a long stream of concessions fail, and this leads to inevitable total war.

In contrast, if $w_2 < -\beta \overline{x}$, then even if assured peace is incentive compatible, it need not be strictly optimal, specifically if v is between $\beta \underline{U}_2$ and $\widetilde{U}$. Country 1 is limited in the amount of suffering it can induce on country 2 via concessions relative to war, and therefore successful concessions by country 2 cannot be rewarded by a lower incidence of war in the future. Consequently, assured peace does not strictly dominate probabilistic war, and there is no sense in which the two countries maximize the duration of peace along the equilibrium path at the cost of total war in the long run.

5 Examples and Discussion

Using Figure 3, we generate sample paths for concessions and war to illustrate the mechanics of our model and to relate it to historical examples. We show that escalation to total war in environments in which total war is necessary involves both countries crossing a *breaking point* after every temporary war. At the breaking point, country 1 presents country 2 with a last chance at peace before engaging in total war. In contrast, we show that in environments in which temporary wars can be sustained in the long run, the two countries avoid a breaking point altogether and instead eventually cross a *war barrier*. Beyond the war barrier, the process of escalation described in Section 4.2 ceases. Country 1 asks for the maximal concession whenever peace takes place, temporary wars can occur even after successful concessions, and unsuccessful concessions can go unpunished by war.

5.1 Case 1: $w_2 > -\beta \overline{x}$

We first consider the case in which total war is necessary in the long run. In Figures 4 and 5, requested concessions are on the y-axis, time is on the x-axis, and the vertical dotted lines coincide with shocks to the cost of concessions s_t and decisions to go to war W_t.[33] In both figures, we let $v_0 = \overline{U}_2$ so that the initial requested concession is minimal.

[33] Formally, "Successful Concessions" refers to $s_t = 1$ and "Unsuccessful Concessions" refers to $s_t = 0$. "Requested Concessions" corresponds to x_t conditional on $s_t = 1$.

Figure 4 represents a potential solution which does not feature temporary wars.[34] Concessions requested by country 1 increase whenever they fail through the process of escalation (see Section 4.2). If country 2's concessions are successful, then country 1 forgives country 2 and reduces requested concessions in the future. However, eventually a long sequence of concessions fail, and country 1 must engage in probabilistic war. In the example, the first incidence of war is permanent, so that total war directly follows escalation.

In contrast, Figure 5 depicts a solution in which countries do not immediately engage in total war. There are three important features in this figure. First, in contrast to Figure 4, initial escalation leads to a temporary war as opposed to total war. Second, the temporary war ends at a *breaking point*, which corresponds to the continuation value associated with $\widetilde{U}$ in the left panel of Figure 3. At this point, country 1 gives country 2 a second chance at peace and requests a concession. If this concession fails, then total war ensues. Alternatively, if this concessions succeeds, then peace ensues. Finally, the fact that every temporary war ends at the breaking point implies that total war is unavoidable. By Proposition 1, any period of peace leads to total war or a temporary war in the future with positive probability. Therefore, even if country 2 satisfies country 1 at the breaking point, it cannot avoid the breaking point in the future where again the danger of total war will be imminent. Eventually, concessions will fail at the breaking point and total war will take place, as it does in Figure 5.

[34] This would be the case for a solution satisfying the *Bang-Bang* property.

Figure 4: Escalation and Total War

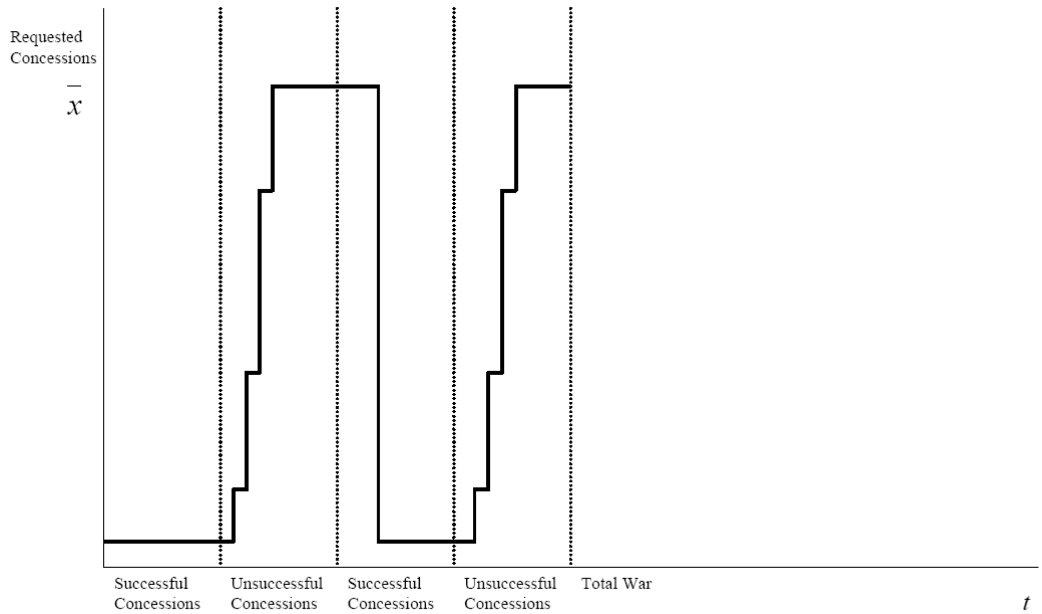
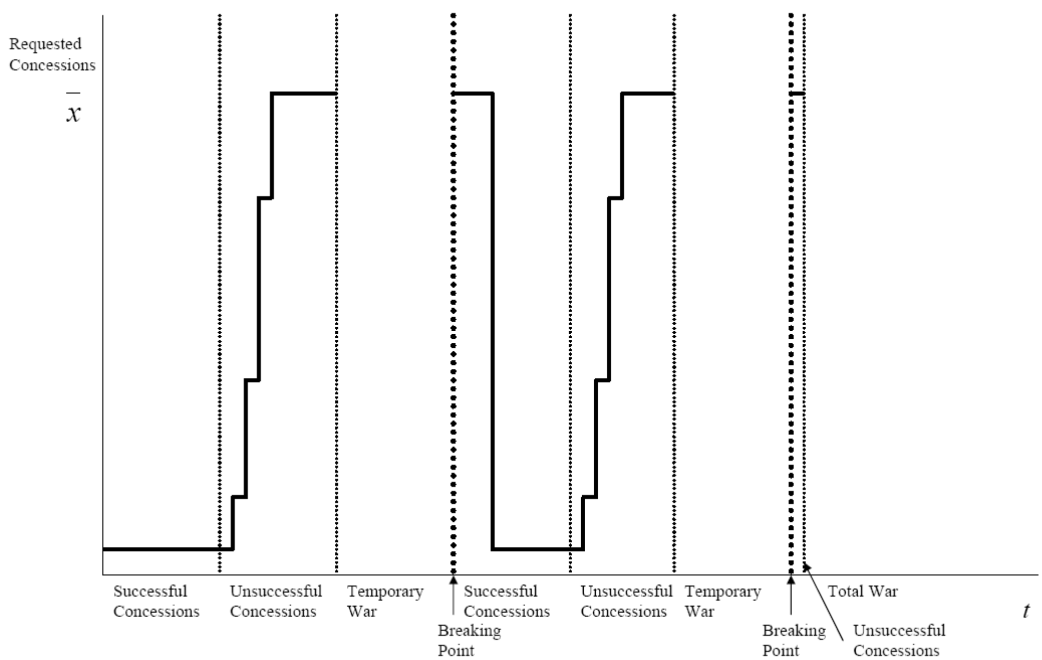


Figure 5: Escalation, Temporary Wars, and Total War



5.2 Case 2: $w_2 < -\beta\overline{x}$

We now consider the case in which total war is not necessary in the long run. Figures 6 and 7 under case 2 are analogous to Figure 4 and 5, respectively, under case 1. The primary difference between Figures 4 and 6 is that in Figure 6, once a sufficiently large number of concessions have failed, country 1 ceases to reward successful concessions by country 2 with a reduction in requested concessions in the future. This is because such forgiveness is not possible: There is no possible concession today which can be large enough so as to allow country 1 to forgive country 2 for past failed concessions going forward.

Figure 7 depicts a solution which features temporary wars. Prior to the first temporary war, equilibrium dynamics in Figure 7 resemble those of Figure 5 since they feature escalation. However, in Figure 7, the first temporary war emerges after the two countries have moved beyond the war barrier, which corresponds to the continuation values between $\beta\underline{U}_2$ and $\widetilde{U}$ in the right panel of Figure 3. Beyond this barrier, country 1 always requests the maximal concession from country 2. As in Figure 6, no concession by country 2 can be large enough so as to allow country 1 to forgive country 2 for past failed concessions going forward. Moreover, even immediately after a successful concession, country 1 can engage country 2 in a temporary war with some probability. Analogously, there is always a probability that a failed concession can lead to peace.[35] What effectively happens beyond the war barrier is that country 1 is forced to punish country 2 for failed concessions which took place prior to crossing the war barrier. This punishment involves requesting the largest possible concession whenever peace occurs and fighting for a sufficient number of periods going forward. The ability of country 1 to reallocate war over time implies that temporary wars can occur forever, since total war is not required for the enforcement of incentives.[36]

[35] It is nevetheless the case that if concessions fail at t, then the "discounted average probability of war" starting from $t+1$, $(1-\beta)\,\mathbf{E}_t \sum_{k=t+1}^{\infty} \beta^{k-t-1} W_k$, is higher.

[36] These facts are displayed in Figure 3b which shows that the probability of war and continuation values as a function of v are correspondences for continuation values below $\widetilde{U}$.

Figure 6: Escalation and Total War

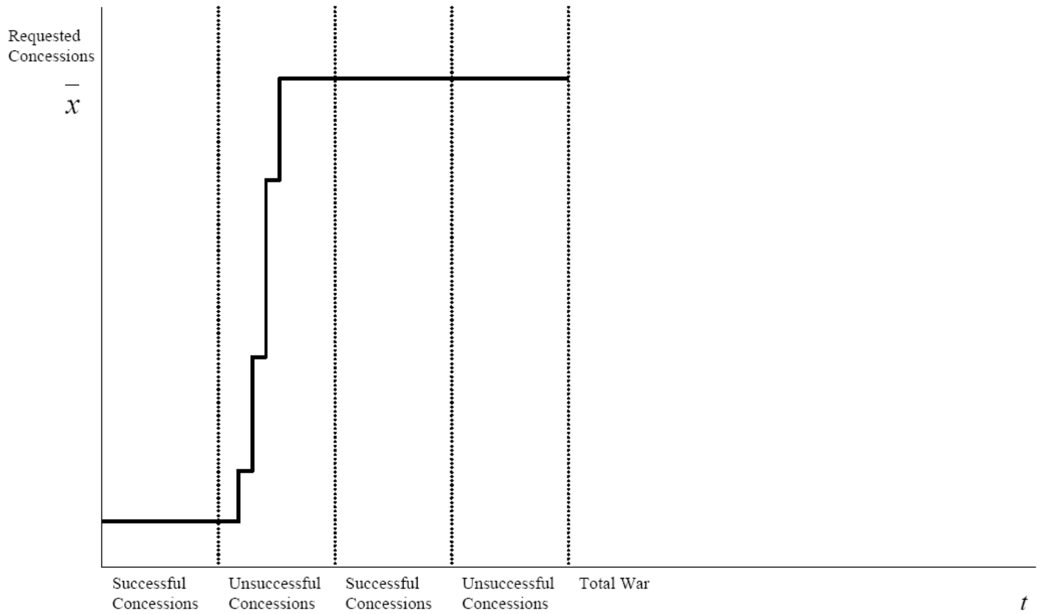
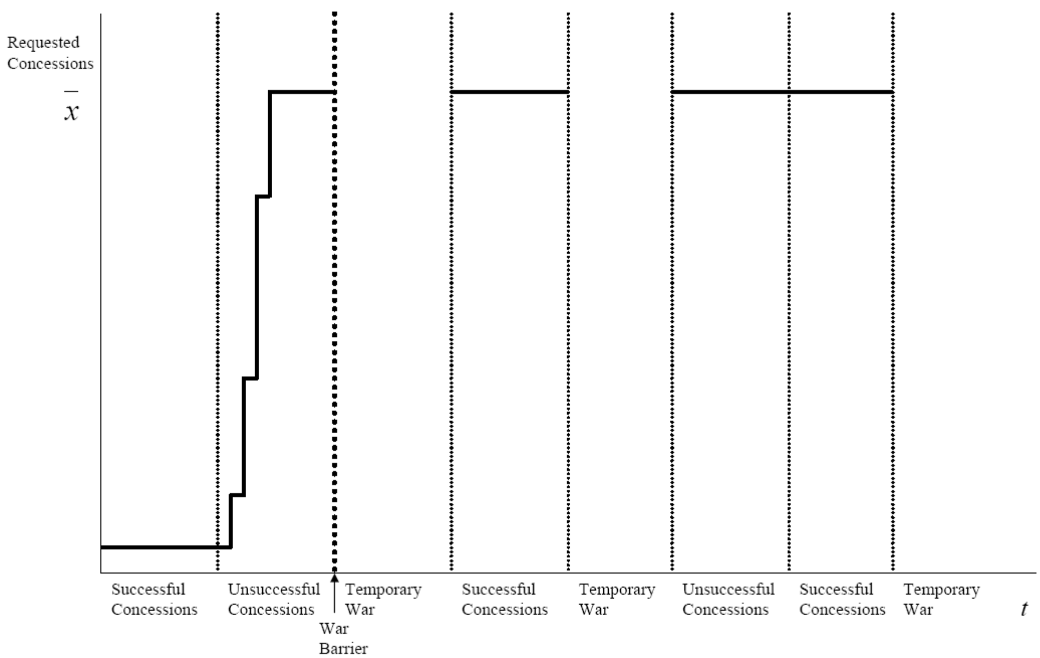


Figure 7: Escalation and Temporary Wars



5.3 Discussion

There are several insights in our model which are useful for interpreting conflicts around the world. In this section, we briefly discuss them using two examples.

The main insight from our model is that imperfect information can generate temporary wars. Consider for example, the case of the Second Palestinian Intifada during which the Israeli government (country 1) urged the Palestinian Authority leader Yasir Arafat (country 2) to crack down on terrorists.[37] During this episode, terrorist acts persisted, yet it was unclear to the Israelis whether Arafat could not control the militants ($s_t = 0$), or whether he privately endorsed the militants while publicly apologizing for their behavior ($s_t = 1$). Raanan Gissin, a spokesman for Israeli Prime Minister Ariel Sharon, argued that:

> "Arafat is responsible since he encourages terrorists to commit suicide acts." (BBC News, January 31, 2002)

Arafat defended himself by claiming to have no control over the militants:

> "There are those who claim that I am not a partner in peace...I condemn terrorism. I condemn the killing of innocent civilians...But condemnations do not stop terrorism. To stop terrorism, we must understand that terrorism is simply the symptom, not the disease."(New York Times, February 3, 2002)

These apologies, however, did not deter the Israelis from engaging in various bouts of violence against the Palestinians. Our model explains the realization of these temporary wars by suggesting them to be the outcome of informational frictions between the Israelis and the Palestinians. Israelis could not gauge the verity of the Palestinian Authority's apologies when cooperation failed, and even if these apologies were known to be sincere, it was necessary for the Israelis to engage in violence in order to provide the incentives for such sincerity. Moreover, our model explains why we may not observe Israeli violence directly deterring future terror, since Israeli violence was meant to provide incentives for cooperation in the past. This appears in line with what is observed in the data. For example, Jaeger and Paserman (2005) find that there is little evidence that both sides of the conflict reacted in a regular and predictable way to violence against them, though there is a tendency for Israeli attacks to have *followed* the Palestinian's inability to cooperate. Moreover, this imperfect predictability of violence in the data is particularly

[37] One can imagine a binary concession $x_t = \{0, \overline{x}\}$ with 0 representing a "terrorist act" and $\overline{x}$ representing a "crack down on terrorists."

fitting given our characterization of the equilibrium beyond the war barrier in Section 5.2 which involves a noisy realization of war in which war can sometimes follow cooperation and peace can sometimes follow non-cooperation.

An additional insight from our model concerns the extent to which these temporary wars can be sustained without culminating in total war. Our model predicts that it is possible for two countries to engage in temporary wars on the path to total war. However, for total war to be avoided, countries need to be sufficiently patient, and war must be sufficiently costly relative to the cost of the contested concessions. Given this rationale, the model predicts that current conflicts which have not escalated to total war–such as the Israeli-Palestinian conflict or the Indian-Pakistani conflict–should persist as they have if we believe that the sides of the conflict view total war as exceptionally costly and unnecessary for the maintenance of peaceful cooperation. There are some hints that may be the case. For example, according to the International Centre for Peace Initiatives (2004), the recent standoff between India and Pakistan (December 2001- October 2002) cost Pakistan almost two percent of GDP, a very large amount. Moreover, the World Bank (2003) estimates that two years of the Second Intifada cost the Palestinians about a year's worth of total output, an astronomical amount. These large costs associated with temporary war suggest that temporary wars are a strong enough enforcement mechanism, and escalation to total war is unnecessary.

Our model also highlights particular dynamics which are present in situations which culminate in total war. Specifically, consider the time path of requested concessions in Figure 5. Every time that a concession is missed, the aggressive country requests an even larger concession from the non-aggressive country. This type of escalation is present in many situation in international relations. Consider for example the events which preceded the First Barbary War (1801-1805) which are described in detail in Lambert (2005). The Barbary States of North Africa (country 1) requested tribute from the United States (country 2) in exchange for a safe passage of American ships through the Mediterrranean. The United States failed to make successful payments on multiple occasions, in large part because of the federal government's inability to effectively raise revenues from the states. This resulted in the Pacha of Tripoli requesting ever-increasing concessions, which is in line with the predictions of our model. Eventually, failure to make concessions resulted in the First Barbary War which cultimated with the forgiveness of past American debts and a continuation of the peaceful relationship between the Barbary States and the United States. With respect to our model, the culmination of the First Barbary War can be represented as taking place at a *breaking point*, which is a new chance at peace after an initial temporary war. Eventually, however, the United States continued to miss payments, and this resulted in the Second Barbary War (1815)–total war in our model–in which the United States emerged victorious and which culminated with an end to the Mediterranean tribute system.

In sum, our model provides a dynamic framework for analyzing various types of conflicts. It allows us to predict which conflicts can persist while avoiding total war and which ones cannot. Moreover, the mechanism of the model provides us with particular set of rich dynamics which appear to be in line with some examples in the world, and which are useful for understanding the nature and dynamics of conflict going forward.

6 Conclusion

We have analyzed a dynamic model of concessions and war to determine whether and how temporary wars between two countries can occur. In doing so, we have characterized the dynamics of escalation and highlighted how imperfect information generates temporary wars. Moreover, we present conditions which are necessary for the two countries to avoid total war and engage in temporary wars in the long run. Our examples show that countries which escalate to total war reach a breaking point at the culmination of every temporary war. In contrast, countries which are able to avoid total war find themselves having passed a war barrier beyond which forgiveness in the form of lower demands by the aggressive country become impossible. Our analysis sheds light on some historical examples, and it provides us with a framework for predicting which conflicts can be sustained without convergence to total war.

There are some important caveats in interpreting our results. First, in choosing to focus on the role of diplomatic concessions, we have ignored the fact that military concessions such as disarmament could also serve to avert conflict by altering the payoff from war. Second, we have implicitly assumed that there is a single good over which the two countries bargain. One can imagine a natural extension of this framework in which each country controls different goods, so that bilateral concessions are necessary to sustain peace. In such a setting, both countries could potentially have an incentive to engage in war. Finally, in the interest of parsimony, we have ignored issues regarding military strategy during war, and we have abstracted from the mechanism by which total war is resolved by defining it as equivalent to permanent war. A thorough investigation of the implications of these issues for our results would be interesting for future research.

7 Appendix

7.1 Notation

The following simplify notation:

$$\begin{aligned}\Upsilon^{+}(v,\epsilon) &= \frac{J(v+\epsilon)-J(v)}{\epsilon}\\ \Upsilon^{-}(v,\epsilon) &= \frac{J(v)-J(v-\epsilon)}{\epsilon}\end{aligned}$$

$$v^{\max} = \min_{v}\left\{v \in \left[\underline{U}_2, \overline{U}_2\right] \text{ s.t. } v = \arg\max_{v} J(v)\right\}$$

$$\begin{aligned}v^{F*}(v) &= w_2 + \beta v^{W*}(v)\\ v^{P*}(v) &= \pi\left(-x^{*}(v) + \beta v^{H*}(v)\right) + (1-\pi)\beta v^{L*}(v)\end{aligned}$$

7.2 Notes and Proofs for Section 3

7.2.1 Equilibrium Definition

In this section we formally define and characterize the set of equilibria described in Section 3. We begin by formally defining randomization. Let $z_t \in [0,1]$ represent an i.i.d. random variable independent of s_t and all actions drawn from a continuous c.d.f. $G(\cdot)$ at the beginning of every period t. z_t is observed by both countries and can be used as a randomization device which can improve efficiency by allowing country 1 to probabilistically go to war.

We consider equilibria in which each country conditions its strategy on past public information. Let $h_t = \{z^{t-1}, W^{t-1}, x^{t-1}\}$, the history of public information at t prior to the realization of z_t.[38] Define a strategy $\sigma = \{\sigma_1, \sigma_2\} = \left\{\{W_t(h_t, z_t)\}_{t=0}^{\infty}, \left\{\{x_t(h_t, z_t, s_t)\}_{s_t=0,1}\right\}_{t=0}^{\infty}\right\}$. σ is feasible if $\forall t \geq 0$ and $\forall (h_t, z_t)$,

$$\left\{W_t(h_t, z_t), \{x_t(h_t, z_t, s_t)\}_{s_t=0,1}\right\} \in \left\{\{0,1\}, [0, \overline{x}]^2\right\}.$$

Given σ, define the equilibrium continuation value for country i at (h_t, z_t) as

$$U_i\left(\sigma|_{h_t,z_t}\right) = \mathbf{E}\left\{\begin{array}{c} u_i\left(W_t(h_t, z_t), x_t(h_t, z_t, s_t), s_t\right) + \\ \beta\mathbf{E}\left\{U_i\left(\sigma|_{h_{t+1},z_{t+1}}\right)|h_t, z_t, W_t(h_t, z_t), x_t(h_t, z_t, s_t)\right\}\end{array} |h_t, z_t\right\} \tag{9}$$

[38] Without loss of generality, we let $x_t = 0$ if $W_t = 1$.

for $\sigma|_{h_t,z_t}$ which is the continuation of a strategy after (h_t, z_t) has been realized. Define $U_i\left(\sigma|_{h_t,z_t}\right)|_{s_t}$ as the term inside the first expectation operator on the right hand side of (9). Let $\Sigma_i|_{h_t,z_t}$ denote the entire set of feasible continuation strategies for i after (h_t, z_t) has been realized.

Definition 1 *σ is a sequential equilibrium if it is feasible and if* $\forall\left(h_t, z_t\right)$

$$\begin{aligned} U_1\left(\sigma|_{h_t,z_t}\right) &\geq U_1\left(\sigma_1'|_{h_t,z_t}, \sigma_2|_{h_t,z_t}\right)\ \forall \sigma_1'|_{h_t,z_t} \in \Sigma_1|_{h_t,z_t} \text{ and} \\ U_2\left(\sigma|_{h_t,z_t}\right)|_{s_t} &\geq U_1\left(\sigma_1|_{h_t,z_t}, \sigma_2'|_{h_t,z_t}\right)|_{s_t}\ \forall \sigma_2'|_{h_t,z_t} \in \Sigma_2|_{h_t,z_t} \text{ for } s_t = \{0,1\}. \end{aligned}$$

In a sequential equilibrium, each country dynamically chooses its best response given the strategy of its rival. Because country 1's strategy is public by definition, any deviation by country 2 to a non-public strategy is irrelevant (see Fudenberg, Levine, and Maskin, 1994).

In order to build a sequential equilibrium allocation, let $q_t = \left\{z^{t-1}, s^{t-1}\right\}$, the *exogenous* equilibrium history of public signals and states prior to the realization of z_t.[39] Define an equilibrium allocation as a function of the exogenous history:

$$\alpha = \left\{W_t\left(q_t, z_t\right), \left\{x_t\left(q_t, z_t, s_t\right)\right\}\right\}_{t=0}^{\infty}.$$

Note that along the equilibrium path of a sequential equilibrium, even though country 1 does not necessarily know q_t, both countries have common knowledge the continuations of α since actions are a function of past public information. Let $\mathcal{F}$ denote the set of feasible allocations α with continuations which are measurable with respect to past public information. Let $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\}$ represent the expected continuation value to country 2 at $t+1$ conditional on q_t, z_t, and $s_t = 1$, and let $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 0\right\}$ be analogously defined for $s_t = 0$.

Proposition 4 *$\alpha \in \mathcal{F}$ is a sequential equilibrium allocation if and only if* $\forall\left(q_t, z_t\right)$, $x_t\left(q_t, z_t, s_t = 0\right) = 0$,

$$U_i\left(\alpha|_{q_t,z_t}\right) \geq \underline{U}_i \text{ for } i = 1, 2 \text{ and} \tag{10}$$

$$\begin{array}{c} -x_t\left(q_t, z_t, s_t = 1\right) + \\ \beta\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\} \end{array} \geq \beta\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 0\right\} \tag{11}$$

$$\text{if } W_t\left(q_t, z_t\right) = 0.$$

[39]Without loss of generality, let s_t be revealed even if $W_t = 1$.

Proof. Step 1. If α is a sequential equilibrium allocation, then $x_t(q_t, z_t, s_t = 0) = 0$ $\forall (q_t, z_t)$. If instead $x_t(q_t, z_t, s_t = 0) > 0$, consider a deviation by country 2 at $(q_t, z_t, s_t = 0)$ to $x'_k(q_k, z_k, s_k) = 0$ $\forall k \geq t$ and $\forall (q_k, z_k, s_k)$ which yields a minimum continuation value of $\beta \underline{U}_2$. Since $x_t(q_t, z_t, s_t = 0)$ is bounded from below by 0 so that $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\}$ is bounded from above by 0, if this deviation is weakly dominated, then it must be that $-\bar{c} \geq \beta \underline{U}_2$ for $x > 0$, but this violates Assumption 3. **Step 2.** The necessity of (10) for $i = 1$ follows from the fact that country 1 can choose $W'_k(q_k, z_k) = 1$ $\forall k \geq t$ and $\forall (q_k, z_k)$ and this delivers continuation value $\underline{U}_1$. The necessity of (10) for $i = 2$ follows from the fact that country 2 can choose $x'_k(q_k, z_k, s_k) = 0$ $\forall k \geq t$ and $\forall (q_k, z_k, s_k)$, and this delivers a minimum continuation value $\underline{U}_2$. The necessity of (11) follows from the fact that conditional on $W_t(q_t, z_t) = 0$, country 2 can unobservably deviate to $x'_t(q_t, z_t, s_t = 1) = x_t(q_t, z_t, s_t = 0) = 0$ and follow the equilibrium strategy associated with $(q_t, z_t, s_t = 0)$ thereafter. **Step 3.** For sufficiency, consider an allocation in which $x_t(q_t, z_t, s_t = 0) = 0$ $\forall (q_t, z_t)$ which also satisfies (10) and (11), and construct the following off-equilibrium strategy. Any observable deviation results in a reversion to the repeated static Nash equilibrium. We only consider single period deviations since $\beta < 1$ and since $U_i(\alpha)$ is bounded for $i = 1, 2$. If $W_t(q_t, z_t) = 1$, a deviation to $W'_t(q_t, z_t) = 0$ is strictly dominated by (10) since $\beta \underline{U}_1 < \underline{U}_1$. If $W_t(q_t, z_t) = 0$, a deviation to $W'_t(q_t, z_t) = 1$ is weakly dominated by (10). If $W_t(q_t, z_t) = 0$, any deviation to $x'_t(q_t, z_t, s_t = 1) > 0$ is weakly dominated by a deviation to $x'_t(q_t, z_t, s_t = 1) = x_t(q_t, z_t, s_t = 0) = 0$ since $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 0\right\} \geq \underline{U}_2$. A deviation to $x'_t(q_t, z_t, s_t = 1) = 0$ is weakly dominated by (11). Any deviation to $x'_t(q_t, z_t, s_t = 0) \neq x_t(q_t, z_t, s_t = 1)$ is strictly dominated since $\bar{c} > 0$. Since $\mathbf{E}\left\{U_2\left(\alpha|_{q_{t+1},z_{t+1}}\right)|q_t, z_t, s_t = 1\right\} \leq 0$, by Assumption 3 and (10), a deviation to $x'_t(q_t, z_t, s_t = 0) = x_t(q_t, z_t, s_t = 1)$ is strictly dominated. ■

We can now formally define the efficient sequential equilibrium. Let $U_i(\alpha)$ represents the period 0 continuation value to i implied by α prior to the realization of z_0. Define Λ as the set of sequential equilibrium allocations.

Definition 2 *$\alpha \in \Lambda$ is an efficient sequential equilibrium allocation if $\nexists \alpha' \neq \alpha$ s.t. $\alpha' \in \Lambda$, $U_i(\alpha') > U_i(\alpha)$, and $U_{-i}(\alpha') \geq U_{-i}(\alpha)$ for $i = 1, 2$.*

We can write our program as maximizing the welfare of country 1 subject to providing country 2 with a minimum welfare of v_0:

$$\max_{\alpha} U_1(\alpha) \text{ s.t. } U_2(\alpha) \geq v_0 \text{ and } \alpha \in \Lambda. \tag{12}$$

7.2.2 Recursive Representation

In this section, we make the formal arguments required to produce Figure 2 and to represent the equilibrium recursively. Define $V = \{\{U_1(\alpha), U_2(\alpha)\} | \alpha \in \Lambda\}$ as the set of period zero continuation values for both countries. By the stationarity of the game, $\{U_1(\alpha|_{q_t,z_t}), U_2(\alpha|_{q_t,z_t})\} \in V \; \forall (q_t, z_t)$. The presence of the public signal z to be used as a randomization device implies that V is convex. Let $J(v)$ represent the value of $U_1(\alpha)$ at the solution to (12) subject to the additional restriction that $U_2(\alpha) = v$ for some $v \geq v_0$.

Lemma 3 *V is convex and compact.*

Proof. Step 1. Consider two continuation value pair $\{U_1', U_2'\} \in V$ and $\{U_1'', U_2''\} \in V$ with corresponding allocations α' and α''. It must be that

$$\{U_1^\kappa, U_2^\kappa\} = \{\kappa U_1' + (1-\kappa) U_1'', \kappa U_2' + (1-\kappa), U_2''\} \in V \; \forall \kappa \in (0,1).$$

Define $\alpha^\kappa = \{\alpha^\kappa|_{q_0,z_0}\}_{z_0 \in [0,1]}$ as follows:

$$\alpha^\kappa|_{q_0,z_0} = \begin{cases} \alpha'|_{q_0,z_0} & \text{if } z_0 = 0 \\ \alpha'|_{q_0,\frac{z_0}{\kappa}} & \text{if } z_0 \in (0,\kappa) \\ \alpha''|_{q_0,\frac{z_0-\kappa}{1-\kappa}} & \text{if } z_0 \in [\kappa, 1] \end{cases},$$

where $\alpha'|_{q_0,\frac{z_0}{\kappa}}$ for $z_0 \in (0,\kappa)$ is identical to $\alpha'|_{q_0,z_0}$ with the exception that $\frac{z_0}{\kappa}$ replaces z_0 in all information sets q_t, and $\alpha''|_{q_0,\frac{z_0-\kappa}{1-\kappa}}$ for $z_0 \in [\kappa, 1]$ is analogously defined. α^κ achieves $\{U_1^\kappa, U_2^\kappa\}$, and since $\alpha', \alpha'' \in \Lambda$, then $\alpha^\kappa \in \Lambda$. **Step 2.** V is bounded since $U_i(\alpha)$ is bounded for $i = 1, 2$. **Step 3.** To show that V is closed, consider a sequence $V_j' \in V$ such that $\lim_{j\to\infty} V_j' = V'$. There exists a corresponding sequence of allocations α_j' which converges to α_∞' since $U_i\left(\alpha_j'\right)$ is continuous in α_j'. Since every element of α_j' at (q_t, z_t) is contained in $\{0,1\} \times [0, \overline{x}]^2$, and since (10) and (11) are weak inequalities, then Λ is closed and $\alpha_\infty' \in \Lambda$. Since $\beta \in (0,1)$, then by the Dominated Convergence Theorem, $U_i(\alpha_\infty') = U_i'$ for $i = 1, 2$. Therefore $V' \in V$. ■

Lemma 4 $J(\underline{U}_2) = J\left(\overline{U}_2\right) = \underline{U}_1$*.*

Proof. Step 1. It is not possible that $J(\cdot) < \underline{U}_1$ since this violates (10) for $i = 1$. **Step 2.** Imagine if $J(\underline{U}_2) > \underline{U}_1$ and consider the associated α. By Assumptions 1 and 2 and equation (10) for $i = 2$, equation (11) implies that $U_2(\alpha|_{q_0,z_0}) \geq \beta \underline{U}_2 > \underline{U}_2$ if $W_0(q_0, z_0) = 0$. Since $U_2(\alpha|_{q_0,z_0}) \geq \underline{U}_2$, then $U_2(\alpha|_{q_0,z_0}) = \underline{U}_2$ and $W_0(q_0, z_0) = 1$

$\forall (q_0, z_0)$. This requires $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} = \underline{U}_2 \; \forall (q_0, z_0)$ and therefore $U_2(\alpha|_{q_1,z_1}) = \underline{U}_2 \; \forall (q_1, z_1)$. Forward induction on this argument implies that $W_t(q_t, z_t) = 1 \; \forall (q_t, z_t)$ so that $J(\underline{U}_2) = \underline{U}_1$. **Step 3.** Imagine if $J(\overline{U}_2) > \underline{U}_1$ and consider the associated α. Since $U_2(\alpha|_{q_0,z_0}) \leq \overline{U}_2$, then $U_2(\alpha|_{q_0,z_0}) = \overline{U}_2 \; \forall (q_0, z_0)$ in order that $U_2(\alpha) = \overline{U}_2$. If $W_0(q_0, z_0) = 1$, then $\overline{U}_2 = w_2 + \beta \mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} \leq w_2 + \beta \overline{U}_2$, which means that $\underline{U}_2 = \overline{U}_2$ and by step 2, $J(\overline{U}_2) = \underline{U}_1$. Now consider $W_0(q_0, z_0) = 0$. It must be that $x_0(q_0, z_0) = 0$ and $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\} = \overline{U}_2$, otherwise it is possible to reduce $x_0(q_0, z_0)$ or increase $\mathbf{E}\{U_2(\alpha|_{q_1,z_1})|q_0,z_0\}$ while maintaining (10) and (11) and strictly increasing $U_2(\alpha)$. This means that $\overline{U}_2 = \beta \overline{U}_2$, but this violates the fact that $\overline{U}_2 < 0$, since $\overline{U}_2 = 0$ is not incentive compatible. This is because by Assumption 1, $J(\overline{U}_2) + \overline{U}_2 \leq 0$, so that if $\overline{U}_2 = 0$, then $J(\overline{U}_2) \leq 0$, which violates (10) for $i = 1$. ■

Given the presence of the random public signal z, $(1)-(7)$ can be rewritten generally as:

$$J(v) = \max_{\{W_z, v_z^W, x_z, v_z^H, v_z^L\}_{z\in[0,1]}} \int_0^1 \begin{pmatrix} W_z \left[w_1 + \beta J\left(v_z^W\right)\right] + \\ (1-W_z)\left[\pi\left(x_z + \beta J\left(v_z^H\right)\right) + (1-\pi)\beta J\left(v_z^L\right)\right] \end{pmatrix} dG_z \tag{13}$$

s.t.

$$v = \int_0^1 \left(W_z\left[w_2 + \beta v_z^W\right] + (1-W_z)\left[\pi\left(-x_z + \beta v_z^H\right) + (1-\pi)\beta v_z^L\right]\right) dG_z, \tag{14}$$

$$J\left(v_z^W\right), J\left(v_z^H\right), J\left(v_z^L\right) \geq \underline{U}_1 \; \forall z \in [0,1], \tag{15}$$

$$v_z^W, v_z^H, v_z^L \geq \underline{U}_2 \; \forall z \in [0,1], \tag{16}$$

$$-x_z + \beta v_z^H \geq \beta v_z^L \; \forall z \in [0,1], \tag{17}$$

$$v_z^H = v_z^L \text{ if } x_z = 0 \; \forall z \in [0,1], \tag{18}$$

$$W_z \in \{0,1\} \;\; \forall z \in [0,1], \text{ and } x_z \in [0, \overline{x}] \;\; \forall z \in [0,1]. \tag{19}$$

Let $\alpha^*(v)$ represent the argument which solves $(13)-(19)$, which consists of

$$\left\{W_z^*(v), v_z^{W*}(v), x_z^*(v), v_z^{H*}(v), v_z^{L*}(v)\right\}_{z\in[0,1]}.$$

Since $\alpha^*(v)$ may not be unique, we define the set of solutions for a particular v. Given the concavity of (13) and the convexity of $(14)-(19)$, a solution always exists which only features randomization between war and peace:

$$\left\{v_z^{W*}(v), x_z^*(v), v_z^{H*}(v), v_z^{L*}(v)\right\} = \left\{v^{W*}(v), x^*(v), v^{H*}(v), v^{L*}(v)\right\} \;\forall z, \tag{20}$$

so that one can let $W^*(v) = \int_0^1 W_z^*(v)\, dG_z$. In the text, we focus on the solution which satisfies (20), but all of our proofs correspond to the entire set of solutions for a given v which we denote by $\Psi(v)$.

Definition 3 $\Psi(v) = \{\alpha^*(v) \,|\alpha^*(v) \;\; \textit{solves} \;\; (13) - (19)\}$.

7.2.3 Implications of Assumption 4

Assumption 4 implies that $J(v) > \underline{U}_1$ for some v. To see why, construct the following equilibrium. If $s_{t-1} = 1$, then $W_t = 0$ and $x_t = x = w_1/\pi + \epsilon$ if $s_t = 1$ and $x_t = 0$ otherwise for $\epsilon > 0$ arbitrarily small so as to continue to satisfy $-w_1 - \pi\epsilon > w_2$. If $s_{t-1} = 0$, both countries revert to the repeated static Nash equilibrium forever. Let $W_0 = 0$. By Assumption 1, country 1's welfare strictly exceeds $\underline{U}_1$ and country 2's welfare weakly exceeds $\underline{U}_2$ so that (10) is satisfied. To check (11), let $U_2|_{s=1}$ represent the continuation value to country 2 conditional on successful concessions yesterday. The stationarity of the equilibrium implies

$$U_2|_{s=1} = -\pi x + \beta\left(\pi U_2|_{s=1} + (1-\pi)\,\underline{U}_2\right),$$

so that (11) which requires $-x + \beta U_2|_{s=1} \geq \beta \underline{U}_2$ becomes $-x \geq \beta w_2$ which is guaranteed by Assumption 4.

7.3 Notes and Proofs for Section 4

7.3.1 Proof of Lemma 1

We rewrite and prove Lemma 1 and also provide an important related lemma to be used for deriving future results. Lemma 1 can be rewritten formally as follows:

$\exists \widetilde{U} \in \left(\underline{U}_2, \overline{U}_2\right)$ s.t. $\forall v \geq \widetilde{U}$ and $\forall \alpha^*(v) \in \Psi(v)$, $W_z^*(v) = 0 \;\forall z$.

Step 1. Consider $\alpha^*(v) \in \Psi(v)$ for $v > \underline{U}_2$ for which $W^*(v) \in (0,1)$. Since a perturbation to which satisfies (20) satisfies (14) – (19), optimality and the concavity of $J(\cdot)$ require

$$J(v) = W^*(v)\, J\left(v^{F*}(v)\right) + \left(1 - W^*(v)\right) J\left(v^{P*}(v)\right). \tag{21}$$

By (21) and the concavity of $J(\cdot)$, if $v^{F*}(v) < v^{P*}(v)$, then $\Upsilon^+\left(v^{F*}(v), \epsilon\right) = \Upsilon^-\left(v^{P*}(v), \epsilon\right)$ for $\epsilon > 0$ arbitrarily small. **Step 2.** By (14), if $W^*(v) = 1$ for $v > \underline{U}_2$, then $v^{W*}(v) =$

$\frac{v-w_2}{\beta} > v$, so that $\frac{J(v)-J(\underline{U}_2)}{v-\underline{U}_2} = \frac{\left(J\left(\frac{v-w_2}{\beta}\right)-J(\underline{U}_2)\right)}{\frac{v-\underline{U}_2}{\beta}}$. By the concavity of $J(\cdot)$ and Assumption 4,

$$J\left(\frac{v-w_2}{\beta}\right) = \underline{U}_1 + m\left(\frac{v-w_2}{\beta} - \underline{U}_2\right) \text{ for } m > 0. \tag{22}$$

Define $\widetilde{U}$ as

$$\Upsilon^+\left(\widetilde{U},\epsilon\right) < m \text{ and } \Upsilon^-\left(\widetilde{U},\epsilon\right) = m. \tag{23}$$

By (22), $\widetilde{U} > \max_v v^{F*}(v) \geq \underline{U}_2$. By the concavity of $J(\cdot)$ and Assumption 4, $\widetilde{U} \leq v^{\max} < \overline{U}_2$. If instead $\nexists v > \underline{U}_2$ s.t. $W^*(v) = 1$, then $\max_v v^{F*}(v) = \underline{U}_2$. By (17), $v^{P*}(v) \geq \beta v^{L*}(v) \geq \beta\underline{U}_2 > \underline{U}_2$. Therefore, $W^*(v) \in (0,1)$ and (21) applies $\forall v \in (\underline{U}_2, \beta\underline{U}_2)$, so that (22) holds $\forall v \in \left(\underline{U}_2, w_2 + \beta^2\underline{U}_2\right)$, and $\widetilde{U} \in \left(\underline{U}_2, \overline{U}_2\right)$ by Assumption 4 and Lemma 4. **Step 3.** $\forall\alpha^*(v) \in \Psi(v)$ s.t. $\forall v \geq \widetilde{U}$, $W^*(v) = 0$. The possibility that $W^*(v) = 1$ is ruled out since $\max_v v^{F*}(v) < \widetilde{U}$. If $W^*(v) \in (0,1)$, then since $v^{F*}(v) < v < v^{P*}(v)$, $\Upsilon^+\left(v^{F*}(v),\epsilon\right) = \Upsilon^-\left(v^{P*}(v),\epsilon\right)$ from step 1. However, this contradicts (23) and the concavity of $J(\cdot)$. **Q.E.D.**

Lemma 5 *The following properties hold:*

1. *If $w_2 \geq -\overline{x}/\beta$, then $\widetilde{U} = \beta\underline{U}_2$, and*

2. *If $w_2 < -\overline{x}/\beta$ then $\widetilde{U} > \beta\underline{U}_2$.*

Proof. Step 1. Imagine if $\widetilde{U} < \beta\underline{U}_2$. This violates (16) and (17) which require $\widetilde{U} \geq \beta v^{L*}\left(\widetilde{U}\right) \geq \beta\underline{U}_2$. **Step 2.** $J(v+\epsilon) > J(v) - \epsilon$ for $\epsilon > 0$. If instead $\exists v$ s.t. $\Upsilon^-(v,\epsilon) \leq -1$, then by the concavity of $J(\cdot)$ and Lemma 4, $\exists\widehat{U} \in \left[v^{\max}, \overline{U}_2\right)$ s.t. $\Upsilon^+\left(\widehat{U},\epsilon\right) \leq -1$ and $\Upsilon^-\left(\widehat{U},\epsilon\right) > -1$, where $\widehat{U} \geq \widetilde{U}$ so that $W^*\left(\widehat{U}\right) = 0$ by step 3 of the proof of Lemma 1. Consider $\alpha^*\left(\widehat{U}\right) \in \Psi\left(\widehat{U}\right)$ which satisfies (20) and under which (17) binds, which is always weakly optimal by the weak concavity of the program and convexity of the constraint set. If $x^*\left(\widehat{U}\right) > 0$, then optimality requires that

$$J\left(\widehat{U}+\epsilon\right) \geq \pi\left(x^*\left(\widehat{U}\right) - \epsilon + \beta J\left(v^{H*}\left(\widehat{U}\right)\right)\right) + (1-\pi)\beta J\left(v^{L*}\left(\widehat{U}\right) + \frac{\epsilon}{\beta}\right) \tag{24}$$

since a perturbation to $x'\left(\widehat{U}+\epsilon\right) = x^*\left(\widehat{U}\right) - \epsilon$, $v^{H\prime}\left(\widehat{U}+\epsilon\right) = v^{H*}\left(\widehat{U}\right)$, and $v^{L\prime}\left(\widehat{U}+\epsilon\right) = v^{L*}\left(\widehat{U}\right) + \frac{\epsilon}{\beta}$ satisfies (14) – (19) for $v = \widehat{U}+\epsilon$. Subtraction of $J\left(\widehat{U}\right)$ from both sides of (24) yields $\Upsilon^+\left(\widehat{U},\epsilon\right) \geq -\pi + (1-\pi)\Upsilon^+\left(\frac{\widehat{U}}{\beta},\frac{\epsilon}{\beta}\right) > -1$, which is a contradiction, where by step 3 of the proof of Lemma 4, $v < 0$, so that $\frac{v}{\beta} < v$. If instead $x^*\left(\widehat{U}\right) = 0$ so that $v^{H*}\left(\widehat{U}\right) = v^{L*}\left(\widehat{U}\right) = \frac{\widehat{U}}{\beta}$, then analogous arguments can be made with a perturbation to $x'\left(\widehat{U}+\epsilon\right) = x^*\left(\widehat{U}\right)$ and $v^{H\prime}\left(\widehat{U}+\epsilon\right) = v^{L\prime}\left(\widehat{U}+\epsilon\right) = \frac{v}{\beta}+\frac{\epsilon}{\beta}$, so that $\Upsilon^+\left(\widehat{U},\epsilon\right) \geq \Upsilon^+\left(\frac{\widehat{U}}{\beta},\frac{\epsilon}{\beta}\right) > -1$ which is also a contradiction. **Step 3.** $\forall v \in \left[\underline{U}_2,\overline{U}_2\right]$ and $\forall \alpha^*(v) \in \Psi(v)$, if $W_z^*(v) = 0$ then $x_z^*(v) = \overline{x}$ or $v_z^{H*}(v) = \overline{U}_2$ $\forall z$. If $x^*(v) < \overline{x}$ and $v^{H*}(v) < \overline{U}_2$, then consider a perturbation to $x_z'(v) = x^*(v)+\epsilon$, $v_z^{H\prime}(v) = v^{H*}(v)+\epsilon/\beta$, and $v_z^{L\prime}(v) = v^{L*}(v)$ $\forall z$. Such a perturbation satisfies $(14)-(19)$ and strictly improves welfare by step 2. **Step 4.** Consider if $w_2 \geq -\overline{x}/\beta$ and imagine if $\widetilde{U} > \beta\underline{U}_2$. Let $J^P(v)$ denote the value of the constrained program $(13)-(19)$ s.t. $W^*(v) = 0$. By the proof of Lemma 1, $J^P(v) \leq J(v)$ for $v < \widetilde{U}$ and $J^P(v) = J(v)$ for $v \geq \widetilde{U}$. Therefore,

$$m = \Upsilon^-\left(\widetilde{U},\epsilon\right) \leq \frac{J^P\left(\widetilde{U}\right) - J^P\left(\widetilde{U}-\epsilon\right)}{\epsilon} \tag{25}$$

for $\epsilon > 0$ arbitrarily small. By step 3, $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$, and by the same reasoning as step 2, (17) can bind. Therefore, if $\frac{\widetilde{U}+\overline{x}}{\beta} > \overline{U}_2$, then $\frac{J^P(\widetilde{U})-J^P(\widetilde{U}-\epsilon)}{\epsilon} = -\pi + (1-\pi)\Upsilon^-\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) < m$, but this contradicts (25). If $\frac{\widetilde{U}+\overline{x}}{\beta} \leq \overline{U}_2$, then $\frac{J^P(\widetilde{U})-J^P(\widetilde{U}-\epsilon)}{\epsilon} = \pi\Upsilon^-\left(\frac{\widetilde{U}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) + (1-\pi)\Upsilon^-\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) < m$, but this also contradicts (25). This establishes the first part of the lemma. **Step 5.** Consider if $w_2 < -\overline{x}/\beta$ and imagine if $\widetilde{U} = \beta\underline{U}_2$. This implies that (23) holds for $\widetilde{U} = \beta\underline{U}_2$. By the proof of Lemma 1, $W^*(\beta\underline{U}_2) = W^*(\beta\underline{U}_2+\epsilon) = 0$ for $\epsilon > 0$ arbitrarily small. By step 3, $x^*(v) = \overline{x}$ or $v^{H*}(v) = \overline{U}_2$ and (17) may bind for $v = \beta\underline{U}_2$ and $v = \beta\underline{U}_2+\epsilon$ by the same reasoning as step 2. Since $\frac{\widetilde{U}+\overline{x}}{\beta} < \beta\underline{U}_2 \leq \overline{U}_2$, then $\Upsilon^+\left(\widetilde{U},\epsilon\right) = \pi\Upsilon^+\left(\frac{\widetilde{U}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) + (1-\pi)\Upsilon^+\left(\frac{\widetilde{U}}{\beta},\frac{\epsilon}{\beta}\right) = m$, but this contradicts (23). ■

7.3.2 Proof of Proposition 1

Imagine if $W_t(q_t,z_t) = 0$ and imagine if $W_k(q_k,z_k) = 0$ $\forall (q_k,z_k)$ $\forall k > t$. Country 2 can deviate to $x_k'(q_k,z_k,s_k) = 0$ $\forall k \geq t$ and $\forall (q_k,z_k,s_k)$, which delivers a continuation value to country 2 equal to 0. Therefore, $U_2(\alpha|_{q_t,z_t}) \geq 0$ $\forall (q_t,z_t)$. Since $u_1(\cdot)+u_2(\cdot) \leq 0$ by Assumption 1, $U_1(\alpha|_{q_t,z_t}) + U_2(\alpha|_{q_t,z_t}) \leq 0$, which means that $U_1(\alpha|_{q_t,z_t}) \leq 0$ $\forall (q_t,z_t)$, but this violates (10) by Assumption 2. Consequently, $W_k(q_k,z_k) = 1$ for some (q_k,z_k) and $k > t$. **Q.E.D.**

7.3.3 Proof of Proposition 2

We prove here a more general proposition and a corollary which imply Proposition 2. More specifically, Proposition 5 provides conditions which are implied by efficiency, and these results are useful for the proofs of our theorems. The corollary to this proposition shows that in an equilibrium in which only features randomization over the realization of war (as the one described in the text), these conditions are not only necessary but sufficient for efficiency. This proposition and corollary imply Proposition 2 as well as the description of the equilibrium in Figure 3.

Proposition 5 *If* $\alpha^*(v) \in \Psi(v)$ *then it satisfies* $(14) - (19)$*,*

1. $W_z^*(v) = 0\ \forall z$ *if* $v \geq \widetilde{U}$*,*
2. $v_z^{W*}(v) \leq \widetilde{U}\ \forall z$*,*
3. $\pi\left(-x_z^*(v) + \beta v_z^{H*}(v)\right) + (1-\pi)\beta v_z^{L*}(v) \leq \widetilde{U}\ \forall z$ *if* $v \leq \widetilde{U}$*,*
4. $x_z^*(v) = \overline{x}$ *or* $v_z^{H*}(v) = \overline{U}_2\ \forall z$*, and*
5. (17) *binds* $\forall z$ *if* $v \geq \widetilde{U}$*.*

Proof. Step 1. The necessity of $(14) - (19)$ follows by definition. The necessity of $W_z^*(v) = 0\ \forall z$ if $v \geq \widetilde{U}$ follows from Lemma 1. The necessity of $x_z^*(v) = \overline{x}$ or $v_z^{H*}(v) = \overline{U}_2$ $\forall z$ follows from step 3 of the proof of Lemma 5. **Step 2.** Imagine if $v_z^{W*}(v) > \widetilde{U}$. Perturb the allocation as in step 1 of the proof of Lemma 1. By (22), $\Upsilon^-\left(v^{W*}(v), \epsilon\right) = m$, which by the concavity of $J(\cdot)$ implies $v^{W*}(v) \leq \widetilde{U}$. In order that this perturbation not strictly improve welfare, it is necessary that $\Upsilon^+\left(v_z^{W*}(v), \epsilon\right) = m\ \forall z$ which by (23) implies a contradiction. **Step 3.** Imagine if $\pi\left(-x_z^*(v) + \beta v_z^{H*}(v)\right) + (1-\pi)\beta v_z^{L*}(v) > \widetilde{U}$. Perturb the allocation as in step 1 of the proof of Lemma 1. By step 1 of the proof of Lemma 1, $\Upsilon^-\left(v^{P*}(v), \epsilon\right) = m$ and which by the concavity of $J(\cdot)$ implies $v^{P*}(v) \leq \widetilde{U}$. In order that this perturbation not strictly improve welfare, it is necessary that

$$\Upsilon^+\left(\pi\left(-x_z^*(v) + \beta v_z^{H*}(v)\right) + (1-\pi)\beta v_z^{L*}(v), \epsilon\right) = m\ \forall z$$

which by (23) implies a contradiction. **Step 4.** Imagine if (17) does not bind for some z if $v \geq \widetilde{U}$. If $x^*(v) < \overline{x}$, consider a perturbation to $x_z'(v) = x^*(v)$, $v_z^{H\prime}(v) = \frac{v + x^*(v)}{\beta} < v^{H*}(v)$, and $v_z^{L\prime}(v) = \frac{v}{\beta} > v^{L*}(v)\ \forall z$. Such a perturbation satisfies $(14) - (19)$ and weakly increases welfare by the concavity of $J(\cdot)$. However, the perturbed allocation is suboptimal by step 3 of the proof of Lemma 5 since $x_z'(v) < \overline{x}$ and $v_z^{H\prime}(v) < \overline{U}_2$. **Step**

5. If instead $x^*(v) = \overline{x}$, denote $v = U'$. If the perturbation of step 4 does not strictly improve welfare, then

$$\Upsilon^+(v,\epsilon) = \Upsilon^+\left(\frac{v+\overline{x}}{\beta},\epsilon\right) = \Upsilon^+\left(\frac{v}{\beta},\epsilon\right) \tag{26}$$

for $\epsilon > 0$ arbitrarily low and $v = U'$, and with some abuse of notation, define $\Upsilon^+\left(\frac{v+\overline{x}}{\beta},\epsilon\right) = \Upsilon^-\left(\frac{v+\overline{x}}{\beta},\epsilon\right)$ if $\frac{v+\overline{x}}{\beta} = \overline{U}_2$. By step 1 of the proof of Lemma 1 and steps 2 and 3 of the proof of Lemma 5, $\forall v \in \left[\widetilde{U}, U'\right)$, there exists a solution to $(13)-(19)$ s.t. $x^*(v) = \overline{x}$ for which (17) binds so that $v^{H*}(v) = \frac{v+\overline{x}}{\beta}$ and $v^{L*}(v) = \frac{v}{\beta}$. Therefore, $\forall v \in \left[\widetilde{U}, U'\right)$,

$$\Upsilon^+(v,\epsilon) = \pi\Upsilon^+\left(\frac{v+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) + (1-\pi)\Upsilon^+\left(\frac{v}{\beta},\frac{\epsilon}{\beta}\right). \tag{27}$$

However, if (26) is satisfied for $v = U'$, then it must be satisfied for $v = \frac{U'}{\beta}$ if $\frac{U'}{\beta} \geq \widetilde{U}$. This follows from the concavity of $J(\cdot)$ which implies $\Upsilon^+\left(\frac{U'}{\beta},\epsilon\right) \geq \Upsilon^+\left(\frac{\frac{U'}{\beta}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) \geq \Upsilon^+\left(\frac{U'+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) = \Upsilon^+\left(\frac{U'}{\beta},\epsilon\right)$ and from (27) which implies $\Upsilon^+\left(\frac{\frac{U'}{\beta}+\overline{x}}{\beta},\frac{\epsilon}{\beta}\right) = \Upsilon^+\left(\frac{\frac{U'}{\beta}}{\beta},\frac{\epsilon}{\beta}\right)$. By forward induction, $\exists N = \{0,1,2,...\}$ s.t. $\frac{U'}{\beta^N} \in \left[\widetilde{U}, \beta\widetilde{U}\right)$ and (26) holds for $v = \frac{U'}{\beta^N}$, and by step 2 of the proof of Lemma 1, $\Upsilon^+\left(\frac{U'}{\beta^N},\epsilon\right) < m$. However, since $\frac{\frac{U'}{\beta^N}}{\beta} < \widetilde{U}$, $\Upsilon^+\left(\frac{\frac{U'}{\beta^N}}{\beta},\frac{\epsilon}{\beta}\right) = m$, and (26) cannot hold for $v = \frac{U'}{\beta^N}$. ■

Corollary 1 *If $\alpha^*(v)$ satisfies Proposition 5's conditions and* (20)*, then $\alpha^*(v) \in \Psi(v)$.*

Proof. Step 1. If $v \geq \widetilde{U}$, then consider any solution which satisfies the conditions of Proposition 5. Since a perturbation as in step 1 of Lemma 1 satisfies (20), yields a unique solution, and weakly improves welfare, then $\alpha^*(v) \in \Psi(v)$. **Step 2.** If $v < \widetilde{U}$, consider $w_2 \geq -\overline{x}/\beta$. Satisfaction of the conditions entails $v^{F*}(v) = w_2 + \beta v^{W*}(v) \in \left[\underline{U}_2, \min\left\{v, w_2 + \beta\widetilde{U}\right\}\right]$ and $v^{P*}(v) = \beta\underline{U}_2 = \widetilde{U}$. Suboptimality implies that

$$J(v) > W^*(v)\left(w_1 + \beta J\left(v^{W*}(v)\right)\right) + (1 - W^*(v))\, J\left(\widetilde{U}\right), \tag{28}$$

for $W^*(v) = \frac{\widetilde{U}-v}{\widetilde{U}-v^{F*}(v)}$, but (28) contradicts (22). **Step 3.** If $v < \widetilde{U}$, consider $w_2 < -\overline{x}/\beta$. Satisfaction of the conditions entails $v^{F*}(v) = w_2 + \beta v^{W*}(v) \in \left[\underline{U}_2, w_2 + \beta\widetilde{U}\right]$

and $v^{P*}(v) \in \left[\beta \underline{U}_2, \widetilde{U}\right]$. Suboptimality and (22) imply that

$$\begin{aligned} J(v) &> W^*(v)\left(w_1 + \beta J\left(v^{W*}(v)\right)\right) \\ & \quad + (1 - W^*(v))\left(\pi\overline{x} + \beta\left(\underline{U}_1 + m\left(\frac{\widetilde{U} + \pi\overline{x}}{\beta} - \underline{U}_2\right)\right)\right), \end{aligned} \tag{29}$$

for $W^*(v) = \frac{v^{P*}(v) - v}{v^{P*}(v) - v^{F*}(v)}$ if $v^{F*}(v) \leq v \leq v^{P*}(v)$ and $W^*(v) = \frac{v^{F*}(v) - v}{v^{F*}(v) - v^{P*}(v)}$ if $v^{F*}(v) \geq v \geq v^{P*}(v)$. However, step 1 and (22) imply that,

$$J\left(\widetilde{U}\right) = \pi\overline{x} + \beta\left(\underline{U}_1 + m\left(\frac{\widetilde{U} + \pi\overline{x}}{\beta} - \underline{U}_2\right)\right),$$

and since $m = \frac{J(\widetilde{U}) - \underline{U}_1}{\widetilde{U} - \underline{U}_2}$, then by some algebra, $m = -\frac{\pi\overline{x} - w_1}{\pi\overline{x} + w_2}$, and (29) contradicts (22). ■

7.3.4 Proof of Lemma 2

By Proposition 5 and Corollary 1, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) > 0$ and $v^{W*}(v) > \underline{U}_2$ $\forall v \in \left(\underline{U}_2, \widetilde{U}\right)$. Therefore, if $\Pr\{W_{t+k} = 0 | W_t = 1\} = 0$ $\forall t$ and $\forall k > 0$, then $\Pr\left\{v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = 0$. **Q.E.D.**

7.3.5 Proof of Proposition 3

Consider a solution which satisfies (20). Since (17) binds for $v \geq \widetilde{U}$, then $\Pr\{v_{t+1} = v/\beta | v_t = v\} = 1 - \pi > 0$. If $\Pr\left\{v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = 0$, then $\Pr\left\{v_{t-1} \in \left(\beta\underline{U}_2, \beta\widetilde{U}\right)\right\} = 0$ and $\Pr\left\{v_{t-2} \in \left(\beta^2\underline{U}_2, \beta^2\widetilde{U}\right)\right\} = 0$ and so on. Therefore, $\Pr\{v_t \notin \{\underline{U}_2, \beta\underline{U}_2, ...\}\} = 0$. Choose $v_0 \geq v^{\max}$ s.t. $v_0 \notin \{\underline{U}_2, \beta\underline{U}_2, ...\}$. This yields $\Pr\{W_{t+k} = 0 | W_t = 1\} > 0$. **Q.E.D.**

7.3.6 Proof of Theorem 1

We can formally write the theorem as follows:

If $w_2 \geq \widetilde{w}_2$, $\nexists$ a solution to (12) s.t. $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$,

Step 1. If $w_2 \geq -\overline{x}/\beta$, imagine if $\exists$ a solution to (12) s.t. $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$, and consider a potential long run distribution of v. Since $\Pr\{v_{t+1} = \underline{U}_2 | v_t = \underline{U}_2\} = 1$, by step 2 of the proof of Lemma 4, then $\Pr\{v_t = \underline{U}_2\} = 0$ under this long run distribution.

Step 2. If $v \in (\underline{U}_2, \beta\underline{U}_2)$, then from Proposition 5, $v_z^{W*}(v) \leq \beta\underline{U}_2$ $\forall z$ and $v_z^{L*}(v) = \underline{U}_2$ $\forall z$. Therefore,

$$\begin{aligned} \Pr\{v_{t+1} = \underline{U}_2 | W_t = 0, v_t \in (\underline{U}_2, \beta\underline{U}_2)\} &= 1 - \pi, \text{ and} && (30) \\ \Pr\{v_{t+1} \leq \beta\underline{U}_2 | W_t = 1, v_t \in (\underline{U}_2, \beta\underline{U}_2)\} &= 1 && (31) \end{aligned}$$

under the long run distribution of v. **Step 3.** From (30),

$$\Pr\{v_{t+1} = \underline{U}_2\} \geq \Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} \times \Pr\{W_t = 0 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} \times (1 - \pi)$$

under the long run distribution. In order that $\Pr\{v_{t+1} = \underline{U}_2\} = 0$, it is necessary that $\Pr\{W_t = 0 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 0$. This is because $\Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} > 0$ since $\Pr\{v_t = \underline{U}_2\} = 0$ and since Proposition 1 and Lemma 1 imply that $\Pr\{v_t \in [\underline{U}_2, \beta\underline{U}_2)\} = \Pr\{v_t = \underline{U}_2\} + \Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} > 0$. **Step 4.** The fact that $\Pr\{W_t = 1 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1$ combined with (31) implies $\Pr\{v_{t+1} \in (\underline{U}_2, \beta\underline{U}_2) | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1$, and by forward induction

$$\Pr\{W_k = 1 \ \forall k \geq t + 1 | v_t \in (\underline{U}_2, \beta\underline{U}_2)\} = 1.$$

Since $\Pr\{v_t \in (\underline{U}_2, \beta\underline{U}_2)\} > 0$, then $\Pr\{W_k = 1 \ \forall k \geq t + 1\} = \Pr\{v_{t+1} = \underline{U}_2\} > 0$ which is a contradiction. **Q.E.D.**

7.3.7 Proof of Theorem 2

We can formally write the theorem as follows:

(i) If $w_2 < \widetilde{w}_2$, $\exists$ a solution to (12) s.t. $\lim_{t\to\infty} \Pr\{W_t = 0\} = 0$, and
(ii) If $w_2 < \widetilde{w}_2$, $\exists$ a solution to (12) s.t. $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$.

Step 1. If $w_2 < -\overline{x}/\beta$, by Corollary 1, $\forall v \in \left(\underline{U}_2, \widetilde{U}\right)$, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) = \frac{\widetilde{U} - v}{\widetilde{U} - \underline{U}_2}$, $v_z^{W*}(v) = \underline{U}_2$, $x_z^*(v) = \overline{x}$, $v_z^{H*}(v) = \frac{\widetilde{U} + \overline{x}}{\beta}$, and $v_z^{L*}(v) = \frac{\widetilde{U}}{\beta}$ $\forall z$. **Step 2.** Construct an equilibrium with the property of step 1, and imagine if $\lim_{t\to\infty} \Pr\{W_t = 0\} > 0$. Then it must be that $\Pr\{v_t = \underline{U}_2\} = 0$ under the long run distribution. However, in such an equilibrium, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = \frac{\widetilde{U} - v_t}{\widetilde{U} - \underline{U}_2} > 0$ under the long run distribution. By Proposition 1 and Lemma 1, $\Pr\left\{v_t \in \left[\underline{U}_2, \widetilde{U}\right)\right\} > 0$. Consequently, $\Pr\{v_t = \underline{U}_2\} > 0$ under the long run distribution. This establishes the first part of the theorem. **Step 3.** If $w_2 < -\overline{x}/\beta$, by Corollary 1, $\forall v \in \left(\underline{U}_2, \widetilde{U}\right)$, $\exists \alpha^*(v) \in \Psi(v)$ s.t. $W^*(v) = \frac{\widetilde{U} - v}{\widetilde{U} - (\underline{U}_2 + \epsilon)}$, $v_z^{W*}(v) = v^{W*}(\underline{U}_2 + \epsilon) > \underline{U}_2 + \epsilon$, $x_z^*(v) = \overline{x}$, $v_z^{H*}(v) = \frac{\widetilde{U} + \overline{x}}{\beta}$, and $v_z^{L*}(v) = \frac{\widetilde{U}}{\beta}$ $\forall z$ for $\epsilon > 0$ which is arbitrarily small. **Step 4.** Construct an equilibrium with the property of step 3, and imagine if $\lim_{t\to\infty} \Pr\{W_t = 0\} = 0$. Then it must be that $\Pr\{v_t = \underline{U}_2\} > 0$ under the long run distribution. However, in such an equilibrium, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left(\underline{U}_2, \widetilde{U}\right)\right\} = 0$. Moreover, by Corollary 1, $\Pr\left\{v_{t+1} = \underline{U}_2 | v_t \in \left[\widetilde{U}, \overline{U}_2\right]\right\} = 0$. Since $U_2(\alpha) \geq v^{\max} > \underline{U}_2$, then $\Pr\{v_t = \underline{U}_2\} = 0$ under the long run distribution. **Q.E.D.**

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