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Winners and Losers in Housing Markets*

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Abstract

This paper is a quantitatively-oriented theoretical study into the interaction between housing prices, aggregate production, and household behavior over a lifetime. We develop an overlapping generations model of a production economy in which land and capital are combined into residential and commercial structures. We find that, in the economy where land is more important for structures, households face a higher house price-rental ratio and tend to buy houses later in life in the steady state. Moreover, the housing price reacts more to an exogenous change in fundamentals, causing a larger redistribution effect between net buyers and net sellers of houses.

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1 Introduction

Over the last few decades, we observe considerable fluctuations of real estate value and aggregate economic activities in some economies. In Japan, both the real capital gains on land during the prosperous decade of the 1980s and the losses during the depressed decade of the 1990s are in the order of multiple years worth of GDP for the corresponding periods. Recent fluctuations in housing prices in many developed countries raise concerns. To what extent are these housing price fluctuations consistent with fundamental conditions? Is the Japanese 1990s-style recession a possible outcome? In this paper, we develop a theoretical framework to investigate how the housing price and aggregate production react to changes in technology and financial conditions. At the same time, we use the same framework to explore the life-cycle of home ownership and consumption. By doing so, we can examine which groups of households are most affected by changes in macroeconomic fundamentals through their effect on the housing market. The life-cycle framework also helps us in explaining differences in home-ownership rates across countries and over time.

One unique aspect of housing (and real estates more generally) arises from the fact that land (or location) is an important input for making residential and commercial structures. Because the supply of land is limited, the supply of structures does not grow as fast as final output with steady growth of technology and population, causing an upward trend in the real rental price and the purchase price of real estate. Because per capita land supply decreases with population growth, land scarcity becomes severer with population growth.

Another unique aspect of real estate is incomplete contract enforcement. Often, landlords are afraid that the tenant may modify (or depreciate) the property against their interests. Even if the modifications are beneficial, disputes may arise over splitting the costs. In order to mitigate these problems, the landlord restricts the tenant's discretion over the use and modification of the house, and the tenant enjoys lower utility from renting the house compared to owning and controlling the same house. If there were no other frictions, then the household would buy the house straight away. The household, however, may face a financing constraint, because the creditor fears that the borrowing household may default. The creditor demands the borrower to put his house as collateral for a loan and asks him to provide a downpayment from his own net worth.

In this paper, we take the importance of land for structures, the loss of utility from rented housing and the tightness of collateral constraints as exogenous parameters, and examine how these parameters affect household life-cycle choices, prices and aggregate quantities. For this purpose, we develop an overlapping generations model of a production economy in which land and capital are combined into residential and commercial structures. We are also interested in the way households cope with idiosyncratic and uninsurable shocks to their labour productivity and wage income.

The interaction between the collateral constraint and the loss of utility from renting a house turns out to generate a typical pattern of consumption and housing over the life cycle. When the household is born without any inheritance, it cannot afford a sufficiently high downpayment for buying a house. Thus the household rents and consumes modestly to save for a downpayment. When some net worth has been accumulated, the household buys a house subject to the collateral constraint, which is smaller than a house that would be bought without the collateral constraint. As net worth further rises, the household upgrades along the housing ladder with the collateral constraint continuing to be binding. At some stage, the household finds it better to start repaying the debt rather than maximizing the size of the house, — the collateral constraint is no longer binding. When the time comes for retirement, possibly with idiosyncratic risk attached, the household moves to a smaller house, anticipating a lower income in the future.

Because people tend to save substantially in order to cope with the downpayment requirement and uninsurable shocks to their wage income, there is relatively abundant capital stock in our economy. Thus the rate of return tends to be low relative to the time preference and economic growth rate in equilibrium. Then, after retirement, people tend not to save enough to keep up with economic growth, slowly shrinking their assets relative to the average wage of the working population, if they live long enough.

In equilibrium, the more important land is for structures, the higher is the expected growth rate of the rental price and the higher is the housing price-rental ratio. (The price-rental ratio is an increasing function of the importance of land, also because the effective depreciation rate of structures decreases as land becomes more important for structures since land does not depreciate). This is true for a country like Japan or a metropolitan area. In such an economy, the household needs a larger downpayment relative to wage income in order to buy a house, and tends to buy a house later in life, resulting in lower home ownership rates.

In an economy where land is more important for structures, we also find the housing price to be more sensitive to exogenous changes in the fundamental such as the expected growth rate of labour productivity or the world interest rate, along the perfect foresight path from one steady state to another. Interestingly, a recent paper by Del Negro and Otrok (2007) finds empirical evidence that is consistent with this theoretical prediction of the model: using a factor decomposition of recent house price changes in the U.S., they attribute a higher percentage change to local factors in states where land is relatively more scarce. Relatedly, Morris and Palumbo (2007) find that empirically the value of land in total housing costs has risen in U.S. metropolitan cities and they view this development as contributing to faster home-price appreciation and, possibly, larger swings in house prices. We view both studies as consistent with the theoretical relationship between housing prices and land implied by our model[1].

The exogenous changes in fundamentals affect welfare of various groups differently, causing winners and losers in housing markets. Generally, net house buyers (such as young worker-tenants) lose and net house sellers (such as retiree-home owners) gain from the house price hike[2]. This redistribution effect through housing price changes is larger in an economy in which the share of land value in structures is larger relative to the capital stock value. The total welfare effect on each group also depends upon the underlying shock causing the

[1] Other factors that might be empirically relevant for house price determination (like the effects of inflation through money illusion) are ignored in our framework; see Brunnermeier and Julliard (2007) for relevant empirical evidence along this dimension.

[2] In an overlapping generations framework, the current living population in the aggregate is a net seller of the existing stock of houses to the future population (not born yet). But, the effect of this aggregate net selling position of the present population is quantitatively very small, because the discounted value of selling the existing housing stock to the next unborn populations in 70 to 80 years from now is small. In comparison, the redistribution within the living population between young and old, or between tenants and home owners, is much larger.

housing price change. If a permanent increase in the expected productivity growth rate is the reason for the housing price hike, then people on average gain directly from the higher expected productivity growth, aside from the redistribution effect. In contrast, if the house price hike in a small open economy is due to a permanent decrease in the world real interest rate, then household welfare falls on average, because the gap between the time preference rate and the real interest rate expands in our economy with abundant precautionary saving.

Our work broadly follows two strands of the literature. One is the literature on consumption and saving of a household facing an idiosyncratic and uninsurable earnings shock and a borrowing constraint, which includes Bewley (1977), Deaton (1991), Carroll (1997), Attanasio et. al. (1999) and Gourinchas and Parker (2002). Huggett (1993), Aiyagari (1994), den Haan (1994), and Krusell and Smith (1997, 1998)) have examined the general equilibrium implications of such models. The second strand is the literature on the investment behavior of firms under liquidity constraints. In particular, Kiyotaki and Moore (1997) is closely related since they study the dynamic interaction between asset prices, collateral value, credit limits and aggregate economic activity for an economy in which entrepreneurs face collateral constraints. When many households borrow substantially against their collateral assets (houses), consume, and move up and down along the housing ladder, these households are more like small businesses rather than simple consumers.

Our attention to housing collateral is in line with substantial micro-level evidence in the UK (Campbell and Cocco (2004)) and the US (Hurst and Stafford (2004)) which suggests that dwellings are an important source of collateral for households. Given the empirical connection between housing prices, home equity and aggregate consumption, there has been substantial research on building models that capture these relationships, either with a representative agent (Aoki et. al. (2004), Davis and Heathcote (2005) and Iacoviello (2005)), or with heterogeneous agents (Ortalo-Magne and Rady (2006), Fernandez-Villaverde and Krueger (2001, 2006), Chambers, Garriga and Schlagenhauf (2004) and Rios-Rull and Sanchez (2005)). Distinguishing features of our analysis include an investigation of the interaction between household life-cycle choices and the aggregate economy, an explicit account of the role of land as a limiting factor in a general equilibrium production economy and evaluating welfare changes across heterogeneous agents stemming from shocks to fundamentals.

Our paper is organized as follows. Section 2 lays out the model and section 3 presents long-run observations about US and other economy aggregate time series facts relevant for housing markets. Section 4 investigates the individual and aggregate predictions of the model using calibration, Section 5 performs the welfare evaluations and Section 6 concludes.

2 The Model

2.1 Framework

We consider an economy with homogeneous product and labour, and homogeneous reproducible capital stock and non-reproducible land. Capital and land are combined for structures. The structures are fully furnished or equipped, and can be used as houses or productive structures (such as offices and factories) interchangeably. There is a continuum of heterogeneous households of population size $\overline{N}_t$ in period t, a representative foreigner, and a representative firm.

The representative firm has a constant returns to scale production technology to produce output (Y_t) from labour (N_t) and productive structures (Z_{Yt}) as:

$$Y_t = F(A_t N_t, Z_{Yt}) = (A_t N_t)^{1-\eta} Z_{Yt}^{\eta}, \quad 0 < \eta < 1, \tag{1}$$

where A_t is aggregate labour productivity which grows at a constant rate, $A_{t+1}/A_t = G_A$. Structures (Z_t) are produced according to a constant returns to scale production function using aggregate capital (K_t) and land (L), and become either a productive structure or a house[3]:

$$\begin{aligned} Z_t &= L^{1-\gamma} K_t^{\gamma}, \quad 0 < \gamma < 1, \\ &= Z_{Yt} + \int_0^{\overline{N}_t} h_t(i) di, \end{aligned} \tag{2}$$

[3] We abstract from productivity growth in this sector for simplicity. This assumption is consistent with the observed low productivity growth rates in the U.S. construction sector. Davis and Heathcote (2005) calculate productivity rates that are lower than the ones in services and manufacturing and are close to zero (-0.27 percent per annum).

where $h_t(i)$ is housing used by household i in period t[4]. With this production function of structures, the firm can continually adjust entire stock of land and capital without friction.[5] Without loss of generality, we normalize the aggregate supply of land L to be unity. The capital stock depreciates at a constant rate $1-\lambda \in (0,1)$ every period, but can be accumulated through investment of goods (I_t) as:

$$K_t = \lambda K_{t-1} + I_t \tag{3}$$

Structures built this period can be used immediately.

Households are heterogeneous in labour productivity, and can have either low productivity, medium productivity, high productivity, or be retired. Every period, there is a flow of new households born with low productivity without any inheritance of the asset. Each low productivity household may switch to medium productivity in the next period with a constant probability δ^l. Each medium productivity household has a constant probability δ^m to become a high productivity one in the next period. Once a household has switched to high productivity it remains at this high productivity until retirement. All the households with low, medium and high productivity are called *workers*, and the flow of new born workers is $G_N - \omega$ fraction of the workforce in the previous period, where $G_N > \omega > \delta^i$ for $i = l, m$. All the workers have constant probability $1 - \omega \in (0,1)$ of retiring next period. Once retired, each household has constant probability $1 - \sigma \in (0,1)$ of dying before the next period. (In other words, a worker continues to work with probability ω, and a retiree survives with probability σ in the next period). The productivity level of the individual household is private information (so that the low productivity household can pretend to be retired, for example). All the transitions are i.i.d. across a continuum of households and over time, and thus there is no aggregate uncertainty on the distribution of individual labour productivity. Let N_t^l, N_t^m and N_t^h be populations of low, medium and high productivity workers, respectively, and let N_t^r be the population size of retired households in period t.

[4] Setting $\gamma = 1$ means that land is not a factor of production and we have the more standard specification where $Z_t = K_t$.

[5] Davis and Heathcote (2005) use a production function in which only a fixed flow of new vacant land can be used for building new houses. Perhaps, in reality, the allocation of land and capital is not as flexible as in our model but not as inflexible as Davis and Heathcote (2005).

Then, we have:

$$
\begin{aligned}
N_t^l &= (G_N - \omega)\left(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h\right) + (\omega - \delta^l) N_{t-1}^l \\
N_t^m &= \delta^l N_{t-1}^l + (\omega - \delta^m) N_{t-1}^m \\
N_t^h &= \delta^m N_{t-1}^m + \omega N_{t-1}^h \\
N_t^r &= (1-\omega)(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + \sigma N_{t-1}^r
\end{aligned}
$$

Solving these equations for the different population group shares, we can see that the fraction of low productivity workers is larger, the larger is the entry of new low productivity workers, and the more difficult it is for them to transit to the medium productivity state.[6]

Each household derives utility from the consumption of output (c_t) and housing services (h_t) of rented or owned housing, and suffers disutility from supplying labour (n_t). (We suppress the index of household i when we describe a typical household). We assume that, when the household rents a house rather than owns and controls the same house, she enjoys smaller utility by a factor $\psi \in (0,1)$. This disadvantage of rented housing reflects the tenant's limited discretion over the way the house is used and modified according to her tastes. The preference of the household is given by the expected discounted utility as:

$$
E_0 \left(\sum_{t=0}^{\infty} \beta^t u(c_t - v(n_t, \varepsilon_t), \left[1 - \psi I(rent_t)\right] h_t) \right), \ 0 < \beta < 1, \tag{4}
$$

where $I(rent_t)$ is an indicator function which takes the value of unity when the household rents the house in period t and zero when she owns it.[7] Disutility of labour $v(n_t, \varepsilon_t)$ is a convex and increasing function of labour supply in terms of efficiency n_t, and is subject to idiosyncratic shocks to its labour productivity ε_t. The value of ε_t is either high (ε^h), medium (ε^m), low (ε^l), or 0, depending on whether the household has high, medium or

[6] We choose to formulate the household's life-cycle in this stylized way, following Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) and Gertler (1999), because we are mainly interested in the interaction between the life-cycles of households and the aggregate economy. The three levels of wage income give us enough flexibility to mimic a typical life-cycle of wage income for our aggregate analysis.

[7] We assume that, in order to enjoy full utility of the house, the household must own and control the entire house used. If the household rents a fraction of the house used, then she will not enjoy full utility even for the fraction of the house owned.

low productivity, or is retired, and follows the stationary Markov process described above. $E_0(X_t)$ is the expected value of X_t conditional on survival at date t and conditional on information at date 0. For most of our computation, we choose a particular utility function with inelastic labour supply as:

$$u\left(c_t, h_t\right) = \frac{\left(\left(\frac{c_t - v_t}{\alpha}\right)^{\alpha}\left(\frac{[1-\psi I(rent_t)h_t]}{1-\alpha}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho}, \tag{5}$$

and $v_t = 0$ if $n_t \leq \varepsilon_t$, and v_t becomes arbitrarily large if $n_t > \varepsilon_t$. We normalize the labour productivity of the average worker to unity as:

$$N_t^l \varepsilon^l + N_t^m \varepsilon^m + N_t^h \varepsilon^h = N_t^l + N_t^m + N_t^h. \tag{6}$$

The parameter $\rho > 0$ is the coefficient of relative risk aversion (as well as the inverse of the elasticity of intertemporal substitution) and $\alpha \in (0, 1)$ reflects the share of consumption of goods (rather than housing services) in total expenditure.

We focus on the environment in which there are problems in enforcing contracts and thus there are restrictions on trades in markets. There is no insurance market against the idiosyncratic shock to labour productivity of each household. The only assets that households hold and trade are the equities of structures, and the annuity contract upon this equity. Because we analyze the economy under the assumption of perfect foresight about the aggregate states, this restriction on tradeable assets is not important (because all the tradeable assets would earn the same rate of return), except for the case of an unanticipated aggregate shock. Because the production function of structures is constant returns to scale, the equity holder of a structure effectively holds capital and land together and receives the returns from both land and the capital stock. There is no separate market for equities of capital and land. Each household can own and control the house, which is partly financed by outside equity held by outside creditors. But the outside creditors only provide credit up to a fraction $1 - \theta \in [0, 1)$ of the house. Thus, in order to control the house and enjoy full utility of a house of size h_t as a home owner, the household must hold equity s_t at least:

$$s_t \geqslant \theta h_t. \tag{7}$$

We can think of this constraint as a collateral constraint for a residential mortgage — even though in our economy the mortgage is financed by equity rather than debt[8] —, and we take θ as an exogenous parameter of the collateral constraint. Also, because the tenant household does not have a collateral asset, we assume the tenant cannot borrow (or issue equities):

$$s_t \geq 0. \tag{8}$$

Although we do not attempt to derive these restrictions on market transactions explicitly as the outcome of an optimal contract, the restrictions are broadly consistent with our environment in which agents can default on contracts, misrepresent their labour productivity, and can trade assets anonymously (if they wish).[9] The rented house yields lower utility than the house which the resident controls as a home owner, because landlords try to mitigate the disputes over the house modification by limiting tenant discretion. There is no separate market for equities of land and capital upon it, because people prefer to control land and capital together in order to avoid the complications. The outside creditor asks the home owners to maintain some fraction of the housing equity to prevent default.

Let w_t be the real wage rate, r_t be the rental price of structures, and q_t be the price of equity of structures of unit size at the beginning of this period (before used in this period). The equity holder of the unit size structure of this period receives rental income r_t this period and gross dividend d_{t+1} in the next period and no payoffs afterwards. (In order to receive income from the structure after that, the agent has to buy another equity in the next

[8] Caplin, Chan, Freeman and Tracy (1997) provide arguments that equity is superior to debt for housing finance. Recently, we have observed the rapid development of a market for shares of real estate trust funds in many developed countries. Later, we are going to examine the effects of an unanticipated shock on the economy in which only debt is used for financing in order to compare the effects on the economy with only equity.

[9] Cole and Kocherlakota (2001) show that, if agents can misrepresent their idiosyncratic income and can save privately, the optimal contract is a simple debt contract with a credit limit. For more explicit analysis of optimal contracts with tangible assets as collateral, see Lustig (2004) and Lustig and Van Nieuwerburgh (2005).

period).[10] The flow-of-funds constraint of the worker is given by:

$$c_t + r_t h_t + q_t s_t = (1-\tau) w_t \varepsilon_t + r_t s_t + d_t s_{t-1}, \tag{9}$$

where τ is a constant tax rate on wage income[11]. The left hand side (LHS) of this equation is consumption, the rental cost of housing (or the opportunity cost of using a house rather than renting it out), and purchases of equities of structures. The right hand side (RHS) is gross revenue, which is the sum of after tax wage income, the rental income from equities purchased this period, and the gross dividend for the equity purchased in the previous period.

For the retiree who only survives until the next period with probability σ, there is a competitive annuity market in which the owner of a unit annuity will receive the gross dividend d_{t+1}/σ if and only if the owner survives, and receive nothing if dead.[12] The retiree also receives the benefit b_t per person from the government, which is financed by the tax revenue on wage income of the workers as[13]

$$b_t N_t^r = \tau w_t (N_t^l + N_t^m + N_t^h). \tag{10}$$

Because the productivity of each household is private information and a low productivity worker can pretend to be retired, the viable retirement benefit does not exceed after-tax

[10] With this dividend policy, we can keep counting an equity of unit size as the claim to the returns from a structure of unit size. However, alternative dividend policies do not change the allocation because the Modigliani and Miller Theorem holds in our economy under perfect foresignt.

[11] The firm pays uniform payroll taxes before paying wages to the workers. The firm observes each worker's labour contribution to its production, but it does not observe whether the worker works elsewhere as well.

[12] When the retiree who owned the house dies, then the house becomes the creditor's – similar to a reverse mortgage.

[13] More generally, if the governement consumes C_t^G and purchases the equity S_t^G, then the flow-of-funds constraint of the government is given by

$$C_t^G + b_t N_t^r + q_t S_t^G = \tau_t w_t N_t + r_t S_t^G + d_t S_{t-1}^G$$

wage income of the low productivity worker[14], or:

$$b_t/w_t = \tau \frac{G_N - \sigma}{1-\omega} \leq (1-\tau)\varepsilon^l.$$

The flow-of-funds constraint for the retiree is

$$c_t + r_t h_t + q_t s_t = b_t + r_t s_t + (d_t/\sigma)s_{t-1}. \tag{11}$$

Each household takes the equity from the previous period (s_{t-1}) and the joint process of prices, dividends and idiosyncratic labour productivity shocks $\{w_t, r_t, q_t, d_t, \varepsilon_t\}$ as given, and chooses the plan of consumption of goods and housing, and the equity holding $\{c_t, h_t, s_t\}$ to maximize the expected discounted utility subject to the constraints of flow-of-funds and collateral.

The representative firm takes the wage rate, the rental price of structures and the rate of return on equity

$$R_t = \frac{d_{t+1}}{q_t - r_t} \tag{12}$$

as given. The firm owns and controls land and capital from last period, and chooses a production plan $\{N_t, Z_{Yt}, Y_t, I_t, K_t\}$ to maximize the value of the firm, that is, the present value of net cash flows from production[15]:

$$V_t = Y_t - w_t N_t + r_t(Z_t - Z_{Yt}) - I_t + \sum_{s=t+1}^{\infty} \frac{1}{R_t R_{t+1} \cdot \cdot R_{s-1}} \left[Y_s - w_s N_s + r_s(Z_s - Z_{Ys}) - I_s\right]$$

subject to the constraints of technology (1), (2) and (3). The net cash flow consists of output net of wage costs as well as net rental income minus investment cost. The gross dividend is defined as the value of the firm per unit of the equities of the structure from last period:

$$d_t = V_t / Z_{t-1}$$

[14] Although the government does not observe the productivity of each household, it observes whether the household works or not, at least with some probability by random monitoring. We assume that the penalty of getting caught for cheating is sufficiently high (say, a prohibition to receive any benefit in the future), so that no worker receives the benefit while working.

[15] The firm owns and controls total land and capital. Firm cash flows in period t from final goods production equal $Y_t - w_t N_t - r_t Z_{Yt}$ and cash flows from structures production equal $r_t Z_t - I_t$.

The representative foreigner purchases goods C_t^* and equities on home structures S_t^* in net (thus both C_t^* and S_t^* can be negative), subject to the international flow-of-funds constraint against home agents as:

$$C_t^* + q_t S_t^* = r_t S_t^* + d_t S_{t-1}^*. \tag{13}$$

The LHS is gross expenditure of foreigners on home goods and equities, which means gross inflow of funds to home agents. The RHS is the gross receipts of foreigners. Although the foreigner maximizes their objective subject to their technological constraint and the flow-of-funds constraint, here we posit the reduced form demand function for home equities of the representative foreigner as an increasing function of the gap between the rate of return on home equities and the rate of return on foreign asset, R_t^*, as:

$$S_t^* = S^*(R_t,\ R_t^*) = \overline{S}^* + \xi(R_t - R_t^*),$$

where $\xi > 0$ is the sensitivity of demand with respect to the gap in the rates of returns, and $\overline{S}^*$ is the parameter which summarizes the other determinants of their demand[16]. One special case is a small open economy in which $\xi \to \infty$, and another special case is a closed economy in which $\overline{S}^* = \xi = 0$.

Given the above choice of many households and a representative firm and a foreigner, the competitive equilibrium of our economy is characterized by the prices $\{w_t, r_t, q_t\}$ which clear the markets for labour, output, equity and the rental of structures as:

$$N_t = \int_0^{\bar{N}_t} n_{it} di = \varepsilon^l N_t^l + \varepsilon^m N_t^m + \varepsilon^h N_t^h = N_t^l + N_t^m + N_t^h, \tag{14}$$

$$Y_t = \int_0^{\bar{N}_t} c_{it} di + I_t + C_t^*, \tag{15}$$

$$Z_t = \int_0^{\bar{N}_t} s_{it} di + S_t^*. \tag{16}$$

[16] The rates of returns on home and foreign assets can differ under perfect foresight because of the transaction costs.

and (2)[17]. Because of Walras' Law, one of these four market clearing conditions is not independent.

2.2 Behavior of Representative Firm

The representative firm chooses a plan of production to maximize the value of the firm. The first order conditions for the maximization are:

$$w_t = (1-\eta)Y_t/N_t \tag{17}$$

$$r_t = \eta Y_t/Z_{Yt} = \eta\left(\frac{N_t'}{f_t Z_t}\right)^{1-\eta}, \text{ where } N_t' \equiv A_t N_t \text{ and } f_t \equiv Z_{Yt}/Z_t \tag{18}$$

$$1-\frac{\lambda}{R_t} = r_t \gamma K_t^{\gamma-1} = \gamma\eta\left(\frac{N_t'}{f_t}\right)^{1-\eta} K_t^{\gamma\eta-1} \tag{19}$$

The first two equations are the familiar equality of price and marginal products of factors of production. The value of N_t' is the labour in efficiency unit, and f_t is a fraction of structures used for production. The last equation says that the opportunity cost of holding capital for one period – the cost of capital – should be equal to the marginal value product of capital. (Note the total supply of land is unity). Thus we have

$$K_t = \left[\frac{\gamma\eta}{1-\frac{\lambda}{R_t}}\left(\frac{N_t'}{f_t}\right)^{1-\eta}\right]^{1/(1-\gamma\eta)} \tag{20}$$

$$Y_t = f_t\left[\left(\frac{\gamma\eta}{1-\frac{\lambda}{R_t}}\right)^{\gamma\eta}\left(\frac{N_t'}{f_t}\right)^{1-\eta}\right]^{1/(1-\gamma\eta)} \tag{21}$$

Because the production function of output is constant returns to scale, there is no profit associated with regular production. The resulting value of the firm is:

$$\begin{aligned} d_t Z_{t-1} &= V_t = r_t Z_t - (K_t - \lambda K_{t-1}) + \frac{1}{R_t}\left[r_{t+1}Z_{t+1} - (K_{t+1} - \lambda K_t)\right] + ... \qquad (22) \\ &= \lambda K_{t-1} + \eta(1-\gamma)\left(\frac{Y_t}{f_t} + \frac{1}{R_t}\frac{Y_{t+1}}{f_{t+1}} + \frac{1}{R_t R_{t+1}}\frac{Y_{t+2}}{f_{t+2}} + ...\right) \end{aligned}$$

[17] The name of individual household i is such that a fraction of new-born housholds named after the names of the deceiced households and the remaining fraction of new-borns are given new names for $i \in \left(\overline{N}_{t-1}, \overline{N}_t\right]$. In this way, the name of households are always destributed unifromely in $\left[0, \overline{N}_t\right]$ at date t.

The second term of the RHS is the value of land, which is proportional to the present value of augmented output Y_t/f_t. Thus, the value of the representative firm is equal to the sum of the capital stock inherited from last period and the value of land, and the equity holders receive returns from capital and land indirectly through their holdings of shares in the entire structure.

2.3 Household Behavior

The household chooses one among three modes of housing - becoming a tenant, a credit constrained home-owner, and an unconstrained home-owner. The flow-of-funds constraint of the worker and retiree can be rewritten as

$$\begin{aligned} c_t + r_t h_t + (q_t - r_t)s_t &= (1-\tau)w_t\varepsilon_t + d_t s_{t-1} \equiv x_t, \\ c_t + r_t h_t + (q_t - r_t)s_t &= b_t + (d_t/\sigma)s_{t-1} \equiv x_t, \end{aligned}$$

where x_t is the liquid wealth of the household. Liquid wealth is the wealth of the household, excluding illiquid human capital (the expected discounted value of future wage and pension income). We call liquid wealth "net worth" hereafter.

2.3.1 The tenant

The tenant chooses consumption of goods and housing services to maximize the utility, which leads to:

$$c_t : r_t h_t = \alpha : 1 - \alpha$$

Using the flow-of-funds constraint we can express housing and consumption as functions of current expenditure:

$$c_t = \alpha[x_t - (q_t - r_t)s_t]$$

and

$$h_t = \frac{(1-\alpha)\left[x_t - (q_t - r_t)s_t\right]}{r_t}$$

Substituting these into the utility function we get the following indirect utility function:

$$u^T(s_t, x_t; r_t, q_t) = \left(\frac{r_t}{1-\psi}\right)^{(\alpha-1)(1-\rho)} \frac{[x_t - (q_t - r_t)s_t]^{1-\rho}}{1-\rho}$$

Due to the lower utility from living in a rented house, the tenant effectively faces a higher rental price than the home owner for the same utility, i.e., $[r_t/(1-\psi)]$ rather than r_t.

2.3.2 The constrained home-owner

The constrained home owner faces a binding collateral constraint as:

$$s_t = \theta h_t$$

Thus he consumes $h_t = s_t/\theta$ amount of housing services, and spends the remaining on goods as:

$$c_t = x_t - \left(q_t - r_t + \frac{r_t}{\theta}\right) s_t$$

The indirect period utility of the constrained home owner is now:

$$u^C\left(s_t, x_t; r_t, q_t\right) = \left\{ \left[\frac{x_t - \left(q_t - r_t + \frac{r_t}{\theta}\right) s_t}{\alpha} \right]^{\alpha} \left[\frac{s_t/\theta}{1-\alpha} \right]^{1-\alpha} \right\}^{1-\rho} /(1-\rho)$$

2.3.3 The unconstrained home-owner

The unconstrained home-owner does not face a binding collateral constraint. Her intra-temporal choice is identical to the tenant's but she does not suffer from the limited discretion associated with renting a house.

$$u^U\left(s_t, x_t; r_t, q_t\right) = r_t^{(\alpha-1)(1-\rho)} \frac{[x_t - (q_t - r_t)s_t]^{1-\rho}}{1-\rho}$$

2.3.4 Value functions

Let $\overline{A}_t$ be the vector of variables characterizing the aggregate state of the economy at the beginning of period t

$$\overline{A}_t = (A_t,\ N_t^l,\ N_t^m, N_t^h,\ N_t^r,\ K_{t-1},\ S_{t-1}^*,\ \Phi_t(\varepsilon_t(i), s_{t-1}(i)))',$$

where $\Phi_t(\varepsilon_t(i), s_{t-1}(i))$ is the date-t joint distribution function of current productivity and equity holdings from the previous period across households. Each household has perfect foresight about the future evolution of this aggregate state, even if each faces idiosyncratic risks on her labour productivity. The prices and dividend (w_t, r_t, q_t, d_t) would be a function of this aggregate state in equilibrium. We can express the value functions of the retiree, high, medium and the low productivity worker by $V^r(x_t, \overline{A}_t)$, $V^h(x_t, \overline{A}_t)$, $V^m(x_t, \overline{A}_t)$, and $V^l(x_t, \overline{A}_t)$ as functions of the individual net worth and the aggregate state.

First consider the choice of the retiree. The retiree chooses the mode of housing and an annuity contract on equities, s_t, subject to the flow-of-funds constraint. Then, the retiree's value function satisfies the Bellman equation:

$$\begin{aligned} V^r(x_t, \overline{A}_t) \quad = \quad & Max(\max_{s_t} \left\{ u^T\left(s_t, x_t; r_t, q_t\right) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}, \\ & \max_{s_t} \left\{ u^C\left(s_t, x_t; r_t, q_t\right) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}, \\ & \max_{s_t} \left\{ u^U\left(s_t, x_t; r_t, q_t\right) + \beta\sigma V^r(b_{t+1} + (d_{t+1}/\sigma)s_t, \overline{A}_{t+1}) \right\}) \end{aligned}$$

Now consider the choice of the worker. The worker chooses whether to own or rent a house, and whether to consume or save to buy the equities. Let us denote $\epsilon^i = \varepsilon^i(1-\tau)$ after tax labour productivity of the worker of type i (high, medium or low). Then the value function of the worker of high productivity satisfies the Bellman equation:

$$V^h(x_t, \overline{A}_t) = Max($$

$$\begin{aligned} & \max_{s_t} \left\{ \begin{array}{c} u^T\left(s_t, x_t; r_t, q_t\right) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}, \\ & \max_{s_t} \left\{ \begin{array}{c} u^C\left(s_t, x_t; r_t, q_t\right) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}, \\ & \max_{s_t} \left\{ \begin{array}{c} u^U\left(s_t, x_t; r_t, q_t\right) + \beta[\omega V^h(\epsilon^h w_{t+1} + d_{t+1}s_t, \overline{A}_{t+1}) \\ +(1-\omega)V^r(b_{t+1} + d_{t+1}s_t, \overline{A}_{t+1})] \end{array} \right\}). \end{aligned}$$

The high productivity worker may retire with probability $1-\omega$ next period, and continues to work with probability ω.

The value function of a medium productivity worker satisfies:

$$V^m(x_t, A_t) = Max($$

$$\max_{s_t}\left\{\begin{array}{c} u^T(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\},$$
$$\max_{s_t}\left\{\begin{array}{c} u^C(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\},$$
$$\max_{s_t}\left\{\begin{array}{c} u^U(s_t,x_t)+\beta[(\omega-\delta^m)V^m(\epsilon^m w_{t+1}+d_{t+1}s_t,A_{t+1}) \\ +\delta^m V^h(\epsilon^h w_{t+1}+d_{t+1}s_t,A_{t+1})+(1-\omega)V^r(b_{t+1}+d_{t+1}s_t,A_{t+1})] \end{array}\right\}).$$

Next period, the medium productivity worker switches to high productivity with probability δ^m, retires with probability $1-\omega$, and remains with medium productivity with probability $\omega-\delta^m$.

The value function of a low productivity worker is similar to the value function of a medium productivity worker, except for the fact that m is replaced by l and h is replaced by m. (Remember a low productivity worker may become a medium productivity worker with probability δ^l).

Growth in the economy with land presents a unique problem for the solution of the individual agent problem because wages grow at different rates from the rental price and the purchase price of structures even in the steady state. This means that we need to transform the non-stationary per capita variables in the model into stationary per capita units. In Appendix B, we describe how to convert the value functions of the household into a stationary representation in the growing economy with scarce land.

2.4 Steady State Growth

Before calibrating, it is useful to examine the steady state growth properties of our economy. Let $G_X = X_{t+1}/X_t$ be the steady state growth factor of variable X_t. In the following we simply call the growth factor as the "growth rate". In steady state, the growth rate of aggregate output variables should be equal:

$$\frac{Y_{t+1}}{Y_t}=\frac{I_{t+1}}{I_t}=\frac{K_{t+1}}{K_t}=G_Y.$$

The growth rate of structures need not be equal the growth rate of output, but it should be equal to the growth rate of productive structures:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_{Yt+1}}{Z_{Yt}} = G_Z.$$

Then, from the production functions, these growth rates depend upon the growth rates of aggregate labour productivity and population as:

$$G_Y = (G_A G_N)^{1-\eta}\ G_Z^{\eta}, \text{ and } G_Z = G_Y^{\gamma}.$$

Thus

$$\begin{aligned} G_Y &= (G_A G_N)^{(1-\eta)/(1-\gamma\eta)} \qquad (23) \\ G_Z &= (G_A G_N)^{\gamma(1-\eta)/(1-\gamma\eta)} \end{aligned}$$

Because the supply of land is fixed, to the extent that land is an important input for structures, the growth rates of output and structures are both smaller than the growth rate of labour in efficiency units. Moreover, because structures are more directly affected by the limitation of land than output, the growth rate of structures lags behind the growth rate of output.

In the steady state of the competitive economy, the real rental price and the purchase price of structures grow are a rate approximately equal to the difference between the growth rate of output and the growth rate of structures, keeping the share of rent in total expenditure constant under our Cobb Douglas utility function:

$$G_r \equiv \frac{r_{t+1}}{r_t} = \frac{q_{t+1}}{q_t} = \frac{G_Y}{G_Z} = G_Y^{1-\gamma}. \qquad (24)$$

Because land is scarce ($\gamma < 1$), the rate of increase of the rental price and the purchase price of structures is an increasing function of the growth rate of workers in efficiency units in steady state. The wage rate grows in the steady state with the same rate as the per capita output as

$$G_w = \frac{G_Y}{G_N} = \left[G_A^{1-\eta} G_N^{-\eta(1-\gamma)}\right]^{1/(1-\gamma\eta)}$$

Because the per capita supply of land decreases with population growth, the growth rate of the wage rate is a decreasing function of the population growth rate.

3 Observations

Here, we gather some observations, which give us some guidance for our calibrations.

3.1 Features of U.S. Economy

Table 1 summarizes the features of the US. economy, relevant for our aggregate economy

Table 1: Long run aggregate features of the U.S. economy

	1900	1939	1958	Average
Reproducible tangible assets/GDP	3.07	3.34	2.92	3.3
Fraction of productive structures	0.47	0.43	0.42	0.53
Land/GDP	1.61	0.96	0.66	-
Residential Land/GDP	-	0.28	0.18	0.39
Market Value of Homes/GDP	-	1.30	1.10	1.28

Notes to Table 1: Reproducible tangible assets, the fraction of productive structures and total land from 1900 to 1958 are from Raymond Goldsmith, (1962). GDP is from GDP - Millennial Edition Series of Table Ca9-19 of Volume 3 of Carter et. al. (2006). The fraction of productive structures is defined as the ratio of nonfarm nonresidential structures plus producer durables to the sum of nonfarm residential and nonresidential structures and producer and consumer durables. Average for the first two rows refers to the average quarterly estimates between 1952:Q1 and 2005:Q5 for the US economy based on Flow of Funds data (see data appendix for details on the construction of these variables). The numbers for the last two rows are from Heathcote and Morris (2007) and are annual averages between 1930 and 2000.

We observe that the ratio of land value to annual GDP falls from 1.61 in 1900 to 0.66 in 1958. This is largely due to a decline of the share of agricultural land. If we look at the ratio of private nonfarm land to GDP, it only falls from 0.57 in 1900 to 0.36 in 1958 (according to Goldsmith (1962)). This remaining decline in the United States suggests that the elasticity of substitution between land and reproducible capital in production of fully equipped structures may exceed unity, because the share of land decreases as the ratio of prices of land and capital increases. (Roughly speaking, the scarcity of land is relatively easily overcome by using technology with higher capital-land ratio). Thus, our assumption of a Cobb-Douglas production function (equation (2)) is only a rough approximation of the production of structures.[18] On the other hand, the fraction of productive structures (our Z_{Yt}/Z_t) shows only small change over the long period of time.

3.2 Evolution of home-ownership rates

There exists considerable variation in home ownership rates across countries and over time. *Table* 2 shows the home ownership rates (fraction of households that own and control houses) of selected developed countries between 1970 and 2003 taken from IMF World Economic Outlook. The table shows a general upward trend in home-ownership rates across countries since 1970.

Table 2: Home ownership rates in %	1970	1980	1990	2003
United States	64.2	65.6	64.0	68.3
Germany	-	41.0	39.0	43.6
Italy	-	59.0	68.0	80.0
United Kingdom	50.0	55.0	66.0	70.0
Japan	-	60.0	61.0	62.0

Notes to Table 2: See Table 2.1 in page 73 of World Economic Outlook (September 2004).

Focussing on the U.S., *Table* 3 shows the evolution of home ownership rates for white and black households for the 1900-1990 period derived from Collins and Margo (2001).

Table 3: U.S. Home-Ownership Rates (in %)						
	1900	1920	1940	1960	1980	1990
whites	48.5	47.1	42.1	64.0	68.6	66.5
blacks	24.1	24.6	20.5	35.8	43.8	40.9

We observe that there is a substantial gap between white and black households, reflecting the difference in their income and access to the credit market. The home ownership rates

[18] For Japan, Kiyotaki and West (2006) provide evidence that the elasticity of substitution between land and capital is not significantly larger than unity for the period 1961-1995.

for both whites and blacks declined during the Great Depression, before increasing after WWII. During the 1980s, average home ownership rate declined, perhaps because of the high nominal and real interest rates[19].

4 Calibrations

4.1 Parameters for Calibration

The parameters for the baseline calibration are as in *Table* 4.

Table 4: Parameters for Baseline Calibration
$\eta = 0.258$: share of productive structures in production of output
$\gamma = 0.9$: share of capital in the production of structures
$\lambda = 0.9 : 1-$ depreciation rate
$\overline{S}^* = 0$: exogenous foreign demand for domestic equities
$\xi = 0$: elasticity of foreign demand with respect to return gap
$\beta = 0.96$: utility discount factor
$\alpha = 0.75$: share of nondurables in total expenditure
$\rho = 2$: coefficient of relative risk aversion
$\psi = 0.09$: fraction of utility loss from renting a house
$\theta = 0.3$: fraction of house that needs downpayment
$\delta^l = 0.08,\ \delta^m = 0.014$: probability of switching to a higher wage
$\varepsilon^l = 0.331,\ \varepsilon^m = 0.663$ and $\varepsilon^h = 2.650$: labour productivities
$\frac{b}{w} = 0.2$: ratio of retirement benefit to pre-tax wages of average worker
$\omega = 0.978$: probability of continuing working
$\sigma = 0.945$: surviving probability
$G_A = 1.02$: labour productivity growth
$G_N = 1.01$: population growth

[19] The high nominal interest rate often tightens the credit constraint, because lenders tend to restrict loans to households with a high ratio of mortgage payments to disposable income, and because the payment of traditional fixed interest mortgage in earlier stage increases with a higher nominal interest rate.

We consider one period of our model to be roughly one year and think of the baseline economy as the U.S.. The share of productive structures in the production of final output ($\eta = 0.258$) is a bit lower than the one used in other studies because the theoretical model includes explicitly housing tangible assets. Consistent with the Cooley and Prescott (1995) methodology of aligning the data to their theoretical counterparts, Appendix D outlines how the U.S. Flow of Funds and NIPA data for the period 1952:Q1 to 2005:Q5 are used to derive an estimate for η, which ends up being a bit lower than the usual $0.3 - 0.4$ range.

A key parameter in our model is the importance of land in the production of structures $(1-\gamma)$. When γ approaches 1, land plays a very limited role in the model (land is plentiful). A higher γ therefore captures a state like Nebraska instead of a city like New York or a country like the U.S. instead of a country like Japan. This parameter will be the most important parameter we will explore when performing comparative statics results across countries. Thinking of the U.S. economy as our baseline, we set $\gamma = 0.9$ since Haughwout and Inman (2001) calculate the share of land in capital income between 1987 and 2005 to be about 10.9%, while Heathcote and Morris (2005) also use $\gamma = 0.9$. It should be noted that in a more recent empirical paper on the price and quantity of land in the U.S., Heathcote and Morris (2007) note that the share of land in residential housing values has risen and it is close to 50% in major metropolitan areas like Boston and San Francisco. In our comparative statics experiments, we will use $\gamma = 0.5$ to understand the role of land scarcity on the economy's responses to fundamental shocks[20].

The depreciation rate of capital stock $(1 - \lambda)$ is set at 10 percent per annum, while the annual discount factor is set at 0.96 and the coefficient of relative risk aversion at 2, all standard parameter choices. For the baseline, we consider a closed economy so that both $\overline{S}^*$ and ξ are set to be zero. Recent papers have calibrated α (the share of non-durables in total expenditure) at around 0.8 (Diaz and Luengo-Prado (2007) use 0.83 and Li and Yao (2006) use 0.8 based on the average share of housing expenditure found in the 2001 Consumer Expenditure Survey). We use a slightly lower number since we think of housing as inclusive of other durables.

[20] Caselli and Freyer (2007) note that, in recent World Bank data, land shares in total capital range between 12 and 27 percent for a range of countries but rise to 51 percent for Japan.

The utility loss from renting a house is set to generate reasonable implications for aggregate home-ownership/tenant rates: a small value for ψ at around 0.09 worked well to generate the observed fraction of renters in the data. The fraction of a house that needs a downpayment (θ) is set at 30% but we perform extensive comparative statics relative to this parameter since one of our goals is to better understand the role of collateral constraints on home-ownership rates, house prices and allocations.

The probability (δ^l, δ^m) of switching earnings states is set so that population ratio of low, medium and high productive workers is approximately equal to 30%, 50%, and 20%. The ratio of the earnings shocks are calibrated to have mean normalized to one and the relative shares are chosen to reflect substantial earnings heterogeneity. We use the levels used by Castaneda, Diaz-Gimenez and Rios-Rull (2003), while ignoring their fourth state that captures the wealth distribution of the super wealthy for simplicity. The probability of continuing to work (ω) is set so that the expected duration of working life is 45.5 years, while the conditional probability of surviving (σ) implies an expected retirement duration of 18.2 years. The replacement ratio (b) implies that the ratio of the government retirement benefit to the after-tax wage is equal to $b/\left[(1-\tau)\,\varepsilon^l\right] = 0.647$ for a low productivity worker, and is equal to $b/\left[(1-\tau)\,\varepsilon^h\right] = 0.081$ for a high productivity worker. Thus, the retirement benefit is roughly equal to two-thirds of after-tax earnings for the low-wage worker, while it is about one-twelfth of the after-tax wage of the highly productive worker, generating the intended redistribution of the pension system. Labour productivity (G_A) and population growth (G_N) are set to two and one percent, respectively.

4.2 General Features of Household Behavior

The household chooses present consumption, saving, and mode of housing, taking into account its net worth – the result of past saving – and its expectations of future income. $Figure$ $1A$ illustrates the consumption of goods, housing services and the mode of housing of the worker with low productivity as a function of net worth. In order to explore the stable relationship between the household choice and the state variable, we detrend all variables using their own theoretical trend as in *Appendix B*. When the worker does not have much net worth, $x < x_{1l}$, he does not have enough to pay for a downpayment of even a tiny house. He chooses to rent a modest house and consume a modest amount. Hoping to become more productive in the future, the low productivity worker hardly saves. $Figure\ 1B$ shows the transition of the equity-holdings for the low productivity worker. The locus $s' = s(s, q, yl)$ shows the equity-holding at the end of the present period as a function of the equity-holding at the end of the last period for the low productivity worker. Everyone enters the labour market with low productivity and no inheritance $s_0 = 0$. As long as the worker continues to be with low productivity, he does not save, and continues to live in a rented house.[21]

$Figure\ 2A$ shows the choice of a worker in the medium productivity state. When she does not have much net worth to pay for a downpayment to buy a house, $x < x_{1m}$, she chooses to rent a place, a similar behaviour with the low productivity worker. The main difference is that the medium productivity worker saves vigorously to accumulate the downpayment to buy a house in the future. In $Figure\ 2B$, the $s' = s(s, q, ym)$ locus (the transition of equity-holdings of the medium productive worker from this to the next period) lies above the 45-degree line for $s < sm^*$, so that the equity holding at the end of this period is larger than at the end of the last period. When the medium productivity worker accumulates modest net worth, $x \in [x_{1m}, x_{2m}]$ in $Figure\ 2A$, she buys her own house subject to the binding collateral constraint. The size of the house at net worth $x = x_{1m}$ is smaller than the house rented at net worth slightly below x_{1m}, because she can only afford to pay downpayment on a smaller house. (Nonetheless, she is happier than before, because she derives more utility from the owned home than a rented place). Consumption is lower too, because she tries to mitigate the collateral constraint by saving vigorously. For $x \in [x_{1m}, x_{2m}]$, the size of an owned house is a sharply increasing function of net worth, because the worker maximizes the size of the house subject to the downpayment constraint. When the medium

[21] No saving by a low productivity worker is not always true for an economy with different parameters. If the income gap between low productivity and high productivity workers is not so large and/or the transition probability from less to more productive states is not so high, then the low productivity worker saves to accumulate net worth to buy a house. The low income worker also would save if the pension were very limited. If the low productivity households had substantial inheritance, then they would decumulate the share-holding until $s = 0.12$ at the intersection between $s(s, q, yl)$ locus and the 45-degree line.

productivity worker has substantial net worth $x > x_{2m}$, she becomes an unconstrained home owner, using her saving partly to repay the debt (or increase the housing equity ownership). In $Figure$ $2B$, the medium productivity worker continues to accumulate her equity holding along $s' = s(s, q, ym)$ until she reaches the level of equity holding at sm^*, the intersection of $s(s, q, ym)$ and the 45-degree line.

The behavior of the high productivity worker is similar to the medium productivity one, except that she accumulates more equities: $s' = s(s, q, yh)$ lies above $s' = s(s, q, ym)$ and her converging equity-holding sh^* is larger than that of medium productive worker sm^*. Therefore, the equity holding of all the workers is distributed in $s \in [0, sh^*]$, with mass of workers at both $s = 0$, $s = sm^*$ and $s = sh^*$.

$Figure$ $3A$ illustrates the consumption and housing choices of the retiree. Because pension income and the probability of death are the same for every retiree (by assumption), he consumes goods and housing services as a function of only net worth. $Figure$ $3B$ illustrates the transition of equity-holding of the retiree. Because in our economy, the productive workers have strong incentives to save for retirement and mitigate the collateral constraint, the equilibrium level of capital stock and structures tend to be fairly large. Then, for a large set of parameters, the rate of return on equity-holding (in terms of utility) is not high relative to the time preference rate, taking into account the effect of growth. (Note that the real rate of return should be sufficiently higher than the time preference rate in a growing economy for the retiree to maintain their relative equity holding). Thus, the transition of equity-holding of the retirees, the locus $s' = s(s, q, b)$, lies below the 45-degree line for $s > sr^*$. Thus the retiree slowly decreases his equity-holding along the locus $s(s, q, b)$ until $s = sr^*$. The relative decumulation of equity holding of the retiree stops at $s = sr^*$, the threshold for him to become a constrained home owner, and his holding stays at sr^* afterwards.[22]

[22] In the Baseline economy, there is a small population of the retirees who never had medium productivity during the working period and thus retire without any net worth. Because the low productivity workers give up hope of becoming productive at the time of retirement and their pension is not much lower than their after-tax wage income in the Baseline economy, they actually save to become a constrained home owner by accumulating equity holding along the locus of $s' = s(s, q, b)$ (which lies above 45-degree line for $s < sr^*$).

This behavior will disappear in an economy in which there are sufficient incentives for low wage workers

Putting together these arguments, we can draw a picture of a typical life-cycle. *Figure* 4 illustrates a typical life for a household. The horizontal axis is age, and the vertical axis measures housing consumption (h) and equity-holding (s). Starting from no inheritance, he chooses to live in a rented house without saving during the young and low wage periods until the $19th$ year. When he becomes a medium productivity wage worker at the $20th$ year, he starts saving vigorously. Quickly, he buys a house subject to the collateral constraint. Then he moves up fast the housing ladder to become a unconstrained home owner at the $22nd$ year.[23] Afterwards, he starts increasing the fraction of his own equity of the house (similar to repaying the debt), instead of moving to the maximum size house within the collateral constraint. By the time of retirement, he has repaid all the mortgage and has accumulated equities more than the value of the his own house under his control. (Remember that the aggregate equity-holding of structures of all the households is the sum of all the houses and productive structures in equilibrium). When the worker hits the wall of retirement (with the arrival of a retirement shock) at the $51st$ year, his permanent income drops, and he moves to a smaller house. He also sells all the equities to buy an annuity contract on the equities, because the annuity earns the gross rate of return which is $(1/\sigma) > 1$ times as much as the straightforward equity-holding, (as long as he survives with probability σ). But his effective utility discount factor shrinks by a factor σ too. Thus as the rate of return on the annuity is not sufficiently high (relative to the effective time preference rate) to induce the retiree to save enough, he decumulates slowly the relative equity-holding, downsizing his consumption

to save (because of a small pension, for example). In such an economy, the equilibrium real interest rate is low and the retiree's equity-holding rule $s' = s(s, q, b)$ lies below the 45-degree line for all $s > 0$. Then, the retiree becomes a constrained home owner and then becomes a tenant as he gets older. Eventually, the shareholding of the retiree will stop when he eats up all the equities at point $s = sr^* = 0$. After that, the retiree will rely entirely on the benefit to pay for rent and consumption. Then the retiree goes back to a real shirtsleeves again if he lives long enough.

[23] The worker moves to a bigger house every period in our model because there are no transaction costs. With a transaction cost, the worker moves infrequently, and changes housing consumption by discrete amounts, rather than continuously. The housing ladder would become a true ladder, instead of having a continual upward slope. He may even buy first a larger house than the house rented before, anticipating the transaction cost. But the basic features remain the same.

of goods and housing services as he gets older relative to the working population. When he dies, his assets drop to zero, according to the annuity contract (which pays zero if the contract holder dies).

4.3 Comparison of Steady States

4.3.1 Closed Economy

We present our results of the baseline calibration in the first column of $Table$ 5. In the baseline calibration the fraction of tenants in the population is about 25%, which is substantial but a bit lower than the number from Collins and Margo (2001). The fraction of constrained home owners is 8.3%. The fraction of houses lived in by tenants and constrained home owners is smaller than the fraction of their population, because they live in smaller houses than the unconstrained home owners, on average. The average size of a tenant's house is about 34.6% (= 8.69/25.16) of the average house size of the economy, and the average house size of constrained home owners is about 29% of the economy average. The tenants and the constrained home owners live in smaller houses than the unconstrained home owners, mainly because the former have lower permanent income. The constrained home owners, in addition, tend to choose smaller housing in order to meet the collateral constraint. The distribution of equity-holding is even more unequal among the groups of households in different modes of housing. The fraction of total equities held by tenants is negligible (0.05%), the fraction of total equities held by constrained home owners is 0.33%, and the remainder is held by unconstrained home owners. This is consistent with the conventional wisdom that the distribution of wealth is much more skewed than the distribution of income. Perhaps a new insight would be that, when the distribution of wealth and income are difficult to observe, we can infer inequality by looking at the home ownership rates across different groups of people, as Collins and Margo (2001) do.

Turning to prices and aggregate variables, the gross rate of return on equity-holding is 1.068 in terms of goods, and is equal to $1.068 \div G_r^{1-\alpha} = 1.067$ in terms of the consumption basket. The latter is smaller than the inverse of the time preference rate, which, adjusted for growth effects, equals $(1/\beta)\left(G_w/G_r^{1-\alpha}\right)^{\rho} = 1.080$. This is not because people are impatient, but because people tend to save substantially during the working period in order to cope with idiosyncratic shocks to the labour productivity and to mitigate the collateral constraint. Many general equilibrium models with uninsurable idiosyncratic risk have such a feature, including Bewley (1977) and Aiyagari (1994). Even though some aggregate variables are not the same as the numbers in *Table* 3, they are broadly consistent with the main features of the US economy. The ratio of average housing value to the average wage is 2.6 years, while the housing price to rental ratio is 8.5 years in the baseline economy. The ratio of value of total structures to GDP is 2.9 years, while the share of housing in total structures is 46%.

Columns 2 and 3 of *Table* 5 report the results for a different level of financial development. Column 2 is the case of a more advanced financial system, where the fraction of house that needs downpayment is 0.1 instead of 0.3 (the baseline number). The main difference relative to the baseline economy is that now there are more constrained home owners instead of tenants. Intuitively, because borrowing becomes easier, relatively poor households buy a house with high leverage (outside equity ownership) instead of renting. Column 3, by comparison, is the case of no housing mortgage ($\theta = 1$) so that the household must buy the house from its own net worth. In this economy, the fraction of tenants is significantly larger. Financial development affects substantially the home-ownership rate. On the other hand, financial development by itself has limited effects on prices and aggregate quantities in steady state. This result arises because the equity holding of tenants and constrained households is a small fraction of aggregate wealth, and because the required adjustment is mostly achieved through conversion of houses between being rented to being owned.

In column 4 we present an economy in which the growth rate of the population is two percent, instead of one per cent. A higher population growth rate leads to a higher real rate of return on equity in the closed economy. This encourages saving, despite having a greater fraction of low productivity workers from the larger inflow of new low productivity workers. (The housing price-rental ratio is slightly lower, because the higher real return offsets the higher expected growth rate of the future rental rate). Home-ownership rises, particularly for the constrained home owners due to higher savings.

In column 5, we consider an economy in which the growth rate of labor productivity is three percent instead of two percent. A higher growth rate of productivity leads to a substantially higher rate of return on equity, given the low elasticity of intertemporal substitution. The housing price-rental ratio is lower because of the higher real interest rate which dominates the effect of a larger expected growth rate of the rental price. The effects on workers' saving rate and home-ownership rate are limited, because the higher rate of return encourages saving while higher expected wages in the future discourage saving.

In Column 6, we decrease the ratio of the retirement benefit to average pre-tax wage to 0.1 from 0.2 (in baseline). This has significant overall effects on both the distribution of the mode of housing and aggregate allocations because households save more in preparing for the retirement shock. As a result of the more vigorous saving among workers, the rate of return on equities is much lower than the rate of time preference, and the home ownership rate increases in the new steady state.

In Columns 7, we consider an economy in which land is more important for structures than in the Baseline: $\gamma = 0.5$, instead of 0.9. (Remember $1 - \gamma$ is the share of land in structures). Because land is more important, the housing price-rental ratio is substantially higher (11.7 years instead of 8.5 years in the baseline), reflecting the higher expected growth rate of future rental rates. The rate of return in terms of output is substantially higher, even though the rate of return on the consumption basket is muted by the growth in rents – $R/G_r^{1-\alpha} = 1.088$ instead of 1.092. The home-ownership rate is higher because a higher real rate of return on equity encourages saving, which outweighs the negative effect of a higher housing price-rental ratio.

4.3.2 Small Open Economy

We can conduct the above comparative steady state for the case of a small open economy, i.e., $\xi = \infty$ instead of $\xi = 0$, by keeping the real interest rate at the exogenous level of the world interest rate $R^* = 1.0683$ in *Table* 6. This exercise is useful to examine a regional economy within a country (like London in the U.K. or New York in the U.S.).

The baseline results (column 1) in table 6 are identical to their closed economy counterparts (column 1 of table 5) since the world real return is chosen to be the same as the baseline of the closed economy. Changing the downpayment requirement (θ) again predominantly affects home-ownership rates rather than prices, in a similar way as in the previous section. A substantial number of differences arise, however, in the response of endogenous variables to a one percent productivity growth. Faced with higher productivity growth, and with the real return not adjusting, there is a pronounced increase in the housing price-rental ratio and a substantial decrease in the home-ownership rate. This contrasts sharply with the results from the economy where the real return was allowed to adjust, and suggests that the level of international capital market integration may be key in assessing the way in which fundamentals affect housing prices, home-ownership rates and aggregate allocations. Similar results arise from a one percent reduction in the world real rate, with an even higher increase in the house price to rental rate. The other experiments generate similar effects as in the closed economy model.

5 Winners and Losers in Housing Markets

We now examine how the small open economy reacts to a once-for-all change in different fundamental conditions in technology and the financial environment. We change a parameter once-and-for-all unexpectedly and solve for the path of prices and quantities that lead the economy to the new steady state. Here, we assume perfect foresight except for the initial surprise. Details of the numerical procedure can be found in Appendix A but the basic idea involves guessing a set of house prices over a period of 50 years, solving backwards the household problem based on these prices and then updating this price vector until convergence. To highlight the importance of land, we compare the reaction of the economy in which land is more important for residential and commercial structures ($\gamma = 0.5$) with the reactions of the baseline economy ($\gamma = 0.9$). This gives us a sense of how the housing market in an economy like Japan or the UK might respond differently to shocks, relative to the U.S. baseline (where the share of land value in structures is small).

5.1 Welfare Evaluations

We are particularly interested in how an unanticipated change in fundamentals affects the welfare of different groups of households differently. Here, using the joint distribution of current productivity and equity holdings from the previous period $\Phi\left(\varepsilon_t(i), s_{-1}(i)\right)$ in the steady state before the shock hits, we define the group as the set I_g of individual households of a particular labor productivity (low, medium, high, and retired), and a particular range of equity holdings of the previous period which corresponds to a particular home-ownership mode (tenant, constrained owner or unconstrained owner). For example, the tenant low-wage worker group is a group of agents with low labor productivity who choose to be tenants in the previous period under the old steady state.

One simple measure of the welfare gain would be the average rate of increase in consumption after the shock relative to the old steady state for each group of households:

$$\mu_g = \text{average of } \left(\frac{c^n(i)}{c^o(i)} - 1\right) \text{ for all } i \in I_g,$$

where $c^n(i)$ is the consumption of individual household i immediately after the shock and $c^o(i)$ is the consumption in the old steady state for the same level of net worth and the same level of idiosyncratic productivity. Because the real rate of return on equity is fixed in the small open economy, the consumption change roughly corresponds to the change in permanent income, except for the case of reducing the world interest rate.

Alternatively, we can use the value functions. Given that we have solved for the prices and value functions for all the periods in the transition, we know that the value functions at the period when the change in fundamentals takes place is a sufficient statistic for welfare in the transition. Let $V_o^{j(i)}(x(i))$ be the value function at the old steady state and $V_n^{j(i)}(x(i))$ be the value function in the first period after the shock hits as a function of revalued net worth $x(i)$ and labour productivity($j(i) = h, m, l$ and r) of individual i.[24] The net worth is equal to:

$$x(i) = w\epsilon^{j(i)} + ds'_{-1}(i),$$

[24]Note that V_n is the value function that has been derived after the full perfect foresight transition has been solved for and therefore includes all this information about the transition to the new steady state.

where $\epsilon^j = (1-\tau)\varepsilon^j$ for worker of productivity j and $\epsilon^j = (b/w)$ for $j = r$, $s'_{-1}(i) = s_{-1}(i)$ if i was a worker and $s'_{-1}(i) = s_{-1}(i)/\sigma$ if i was a retiree in the previous period. Let (w_o, d_o) be the wage rate and gross dividend in the old steady state, and let (w_n, d_n) be their respective endogenously solved equivalents immediately after the shock. We compute a measure of welfare change for the group I_g as:

$$\overline{\mu}_g = \text{average of } \left[\left(\frac{V_n^{j(i)}([w_n \epsilon^{j(i)} + d_n s'_{-1}(i)])}{V_o^{j(i)}([w_o \epsilon^{j(i)} + d_o s'_{-1}(i)])} \right)^{\frac{1}{1-\rho}} - 1 \right] \text{ for all } i \in I_g. \tag{25}$$

We call this measure as the certainty expenditure equivalent, because we convert the change of the value into the dimension of expenditure before taking the average.[25]

We also find informative and report the change in non-human wealth due to a revaluation of structures when the shock hits as

$$\text{average of } \left(\frac{[w_n \epsilon^{j(i)} + d_n s'_{-1}(i)]}{[w_o \epsilon^{j(i)} + d_o s'_{-1}(i)]} - 1 \right) \text{ for all } i \in I_g \tag{26}$$

5.2 Transition of Small Open Economy following a Change in Fundamentals

Figure 5 shows the responses to a once-for-all increase in the growth rate of labor productivity from 2% to 3%. Because the economy is growing, all the following figures show the percentage difference from the steady state growth path of the baseline economy. In both

[25] We also computed the net worth equivalent that would make a household indifferent between the period before and after the shock as the value of $\lambda(i)$ such that

$$V_o^{j(i)}([w_o \epsilon^{j(i)} + d_o s'_{-1}(i)]) = V_n^{j(i)}(\lambda(i)[w_n \epsilon^{j(i)} + d_n s'_{-1}(i)])$$

The value of $\lambda(i)$ measures how much the initial net worth must be multiplied immediately after the shock in order to maintain the same level of the expected discounted utility as the old steady state. We know that the value functions are monotonically increasing and therefore we can find the net worth equivalent uniquely. We can then compute the average of individual $\lambda(i) - 1$ for a particular group g of agents as $\widetilde{\mu}_g$.

This welfare measure suffers from the drawback that net worth does not include the value of human capital. Thus, if two groups have different ratios of net worth (liquid wealth) to human capital, a difference in $\widetilde{\mu}_g$ may reflect the difference of human to non-human wealth rather than the difference in the welfare effect.

economies the housing price increases substantially initially and continues to increase afterwards. In the economy where land is more important, the increase in house prices is larger, and real house price inflation afterwards is higher. The housing price-rental ratio is going to be higher (especially for the economy with land being more important), anticipating the increase in the rental price in the future (as we discussed in the previous section on the small open economy in the steady state). The home-ownership rate gradually declines because young workers take a longer time to accumulate a sufficient downpayment to buy a house.

Table 7 reports the average rate of change of welfare (25) in Panel A and the average rate of change of non-human wealth (26) in Panel B for each group against changes in the fundamentals for the baseline small open economy ($\gamma = 0.9$) and for the economy with land being more important ($\gamma = 0.5$). The first and second columns report the average rate of changes from an increase in the growth rate of labour productivity from 2% to 3% for the cases of ($\gamma = 0.9$) and ($\gamma = 0.5$). Given the higher productivity growth, households are on average better off from a higher permanent income. (Remember the retiree's benefit is proportional to the wage rate of present workers). The higher housing price, however, affects the welfare of different groups of households differently. Those who buy (or expand) houses in the future lose from the housing price hike, while those who sell houses in the future gain. This redistribution effect is larger in the economy in which land is more important ($\gamma = 0.5$) since the house price rise is bigger in this economy. We can observe the change in non-human wealth in Panel B. In the aggregate, the non-human wealth in the economy rises, and it rises by more in the land-important economy ($\gamma = 0.5$), but the distribution of the wealth increase is unequal. The non-human wealth of renter workers falls because higher human wealth causes a short-run consumption increase, generating a switch from productive to residential structures that reduces current wages. On the other hand, workers with higher holdings of shares (constrained and unconstrained homeowners) and retirees experience an increase in non-human wealth with the house price rise. This is why the retirees, who tend to be net sellers of houses, gain more on average than workers (who tend to be net buyers of houses) for the land-important economy ($\gamma = 0.5$). When land is not so important ($\gamma = 0.9$), the workers gain more on average than the retirees, because their permanent income increases relatively more from the higher productivity growth rate.

Figure 6 shows how these two economies react to a once-for-all fall in the world real interest rate by 1%. In both economies, housing prices and output increase, and the adjustment of housing prices is fast. In the economy with land being more important, the swing of net exports and consumption is larger, output takes a longer time to increase despite the large increase in the capital stock, because a large amount of structures gets allocated to housing in the early stages of the transition. The home-ownership rate declines gradually because the lower real interest rate discourages saving, delaying the age of switching from renting to owning a house over the life cycle. The third and fourth columns of *Table* 7 report the reaction of welfare to this decrease in the world real interest rate for ($\gamma = 0.9$) and ($\gamma = 0.5$). Because the real interest rate is lower than the time preference rate in our old steady state economy, a further decrease in the real interest rate hurts a majority of people who save to finance living expenses after retirement, while the higher house prices have important redistribution effects between net buyers and net sellers of houses. Low-wage workers as a group lose more in the land-important economy ($\gamma = 0.5$) than in the ($\gamma = 0.9$) economy. Retirees (particularly unconstrained homeowner retirees) who tend to be net sellers of houses, gain on average (especially in the land-important economy ($\gamma = 0.5$)), because they benefit from the house price hike. Looking at the value of non-human wealth in Panel B, we find that despite the different signs on welfare changes in Panel A for different groups, all groups are wealthier from a higher house price coming from the lower world real interest rate. Moreover, the wealth increase is higher for each group (except for low income workers) in the ($\gamma = 0.5$) rather than the ($\gamma = 0.9$) economy.

The fifth and sixth columns of *Table* 7 report the welfare effects from down-sizing the pension system. Given the constant world interest rate, there is a small increase in house prices reflecting the higher private saving in the economy. Because now people have to save more privately to finance consumption during retirement, welfare tends to fall significantly, and the fall is more dramatic for the currently retired households (whose present value of pension falls dramatically). Workers are also affected but less than retirees, while most of the fall is absorbed by the renter workers who not only have to save more for retirement, they have to suddenly save even more to finance a more expensive house purchase. The revaluation effects show up as very little effects in panel B, however, illustrating that most of the welfare change is coming from a change in savings behavior in this instance.

We have also done the experiment of lowering the downpayment requirement from 30% to 10%. This provides extra liquidity for households, especially for constrained home owners, and encourages consumption initially. At the same time, with a less stringent collateral constraint, low wage workers and tenants of the previous period cut back their consumption initially to buy houses. Overall, however, the welfare effects from this are not substantial. This might seem surprising given that homeownership rises quite substantially in the transition from the repressed to the more open financial system. The explanation for this finding comes from the fact that it is the poor people who are renters in the model. Even though they become better off by locking the utility gain by moving from rental to homeownership status, the fact that the houses they own are very small means that the welfare gain in the aggregate (and even within this group) are not substantial.

Putting together the simulation results from these experiments, we can conclude that, if we were to explain the large increase in housing prices in many developed countries in the last decades, we have to look for increases in the expected growth rate of labor productivity and for decreases in the real interest rate. Suppose that the expected growth rate of labor productivity rises from 2% to 3%. Then in the baseline economy, housing prices would increase initially by 9% and the real housing inflation rate would afterwards increase from 0.29% to 0.38% in terms of output[26]. In an economy in which land is more important ($\gamma = 0.5$), the housing price would initially increase by 20%, followed by the real housing price inflation of 1.9% in terms of output.

Suppose next that the world real rate of return on assets falls from 6.83% to 5.83%, in addition to the above 1% increase in the growth rate of labor productivity. Then, in the baseline economy, the housing price would increase initially by approximately 18%, followed by an annual real housing price inflation of 0.38% in terms of output. In the economy where land is more important, the initial increase in the real housing price would be 44%, followed by the real housing price inflation of 1.9% annually in terms of output. In 10 years, the cumulative increase in real housing price in terms of output would be about 22% in the

[26] Here we use equation(24) for computing the growth rate of housing price in the steady state.

baseline economy and would be 74% in the economy where land is more important ($\gamma = 0.5$). Thus, if half the population lives and works in the area in which land is important ($\gamma = 0.5$) and another half lives and works in the area in which land is not important for structures, *and* the mobility of labor is restricted while capital freely moves between the areas, then the cumulative housing price increase would be roughly $(22 + 74)/2 = 48\%$. Arguably, this is a very crude calculation, ignoring how regional agglomeration takes place. Nonetheless, it gives us some guidance that a significant fraction of the increase in real housing prices may be explained by a combination of an increase in the growth rate of labour productivity, a decrease in the real interest rate, and the fact that the largest fraction of economic activity is taking place in the area in which land has a larger share in the value of residential and commercial structures.

6 Conclusions

This paper develops an aggregate life-cycle model to investigate the interaction between housing prices, aggregate production, and household behavior over a lifetime. A key innovation involves the explicit introduction of land as a fixed factor of production and analyzing the implications for time series (aggregate) and cross sectional (life-cycle) outcomes. Comparing two small open economies with differences in the importance of land, the economy with a larger share of land value for structures has a higher housing price-rental ratio and a lower home-ownerhsip rate in the steady state. On the other hand, the development of the financial system (more relaxed collateral constraints) has a limited impact on housing prices and aggregate production, even though it encourages households to buy houses earlier in life. The transitions of the small open economy along the perfect foresight path illustrate that, where land is more important for structures, once-for-all shocks to the growth rate of labor productivity and/or the world interest rate generate more volatile movement in housing prices and swings in aggregate expenditures.

We argue that credible welfare comparisons in housing markets need the ingredients introduced in the model: land scarcity generates sharper (and more reasonable) housing price responses to fundamental shocks, while the redistribution from house price changes can only be captured by a reasonable lifecycle model that creates different groups endogenously. We find that the welfare effect of house price changes can be substantial, they depend critically on the fundamental shocks causing the change and substantial redistribution effects arise depending on the broad categorization between "net house buyers" and "net house sellers".

Appendix A: Solving the model

Solving the household's decision problem

We discretize the net worth (x_t^i) using 200 grid points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The grid range for the continuous state variable is verified ex-post by comparing it with the values obtained in the simulations. For points which do not lie on state space grid, we evaluate the value function using cubic spline interpolation along the net worth. We simulate the idiosyncratic exogenous productivity shock from its three-point distribution. The realizations of these exogenous random variables are held constant when searching for the market clearing prices (q and r). We use the policy functions to simulate the behavior of 10000 agents over 600 (the exact number depends on the probability of exiting working life and the survival probability) generations and aggregate the individual housing and equity demands to determine the market clearing rental and housing price and the equilibrium household allocations.

Solving perfect foresight model

We guess a sequence of structure rental rates $\{r_t\}_{t=1}^T$ such that the rental rate has converged to the new steady state. Use (19) to calculate a sequence of capital stocks $\{K_t\}_{t=1}^T$ and then use (12) to compute the sequence of structure prices $\{q_t\}_{t=1}^T$. Given these guessed prices, we solve the household's problem backwards from period T when the economy is assumed to have converged to the new steady state. Households are assumed to know the realization of the entire path of structure prices and rental rates. The value function in period T is the value function for the new steady state. Then the value function in period T-1 is computed as follows:

$$V_{T-1}\left(x_{T-1}|r_{T-1},q_{T-1}\right)=\max_{c_T,h_T}\left[u\left(c_{T-1},h_{T-1}\right)+\beta V_T\left(x_T|r_T,q_T\right)\right]$$

We simulate the model forward, starting from the capital stock and the joint distribution of labour productivity and equity of the original steady state. In each period, we simulate a cross-section of 10000 agents over 600 generations and aggregate their individual housing choices, computing the excess demand for structure in each period. We increase the rental rate in periods with a positive excess demand and decrease the rental rate in periods with a negative excess demand, generating a new path $\{r_t\}_{t=1}^T$ for the rental rate. We repeat this until successive paths of the rental rate are less than 0.0001% from each other.

Appendix B: Stationary Representation of Value Functions

The stationary representation of the household's problem

Using the property of the steady state equilibrium of Section 2.4, we normalize the quantities and prices using the power function of labour in efficiency units $N_t' \equiv A_t N_t$ and population N_t. Both variables are exogenous state variables, and there can be a jump or a kink in the trend if labour productivity experiences a once-for-all change in its level or growth rate. Let us denote the normalized variable X_t as $\widetilde{X}_t$. Then we have:

$$\begin{aligned}
\widetilde{K}_t &= K_t / N_t'^{\,\frac{1-\eta}{1-\gamma\eta}}, \quad \widetilde{S}_t^* = S_t^* / N_t'^{\,\gamma\frac{1-\eta}{1-\gamma\eta}} \\
(\widetilde{w}_t, \widetilde{x}_t) &= (w_t, x_t) / (N_t'^{\,\frac{1-\eta}{1-\gamma\eta}} / N_t) \\
(\widetilde{h}_t, \widetilde{s}_t) &= (h_t, s_t) / (N_t'^{\,\gamma\frac{1-\eta}{1-\gamma\eta}} / N_t) \\
\left(\widetilde{r}_t, \widetilde{q}_t, \widetilde{d}_t\right) &= (r_t, q_t, d_t) / N_t'^{\,(1-\gamma)\frac{1-\eta}{1-\gamma\eta}} \\
\widetilde{V}_t^i &= V_t^i / \left[\frac{N_t'^{\,\frac{1-\eta}{1-\gamma\eta}} / N_t}{N_t'^{\,(1-\alpha)(1-\gamma)\frac{1-\eta}{1-\gamma\eta}}} \right]^{1-\rho}, \text{ for } i = l, m, h, \text{ or } r
\end{aligned}$$

We also define the normalized discount factor as:

$$\widetilde{\beta} = \beta \left(\frac{G_w}{G_r^{1-\alpha}} \right)^{1-\rho}.$$

Let us assume population grows along the steady state path. Let $\widetilde{A}_t$ be deviation of labour productivity from the trend. Then the vector of normalized state variables adjusted by the productivity change are:

$$\overline{\widetilde{A}}_t = \left(\widetilde{A}_t, \widetilde{K}_{t-1}, \widetilde{S}_{t-1}^*, \widetilde{\Phi}_t \left(\varepsilon_t, \widetilde{s}_{t-1}(i) \right) \right)'.$$

Using these normalized variables, we can define the normalized value function. For an example, the stationary representation of the retiree's problem is

$$\widetilde{V}^r(\widetilde{x}, \overline{\widetilde{A}}_t) = Max($$

$$
\max_{\widetilde{s}} \left\{ \begin{array}{c} (1-\psi)^{(1-\alpha)(1-\rho)} \frac{(\widetilde{x}-(\widetilde{q}_t-\widetilde{r}_t)\widetilde{s})^{1-\rho}}{1-\rho} \left(\frac{\widetilde{w}_t}{\widetilde{r}_t^{1-\alpha}}\right)^{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1}) \end{array} \right\},
$$

$$
\max_{\widetilde{s}} \left\{ \begin{array}{c} \left\{\left[\frac{\widetilde{x}-\left(\widetilde{q}_t-\widetilde{r}_t+\frac{\widetilde{r}_t}{\theta}\right)\widetilde{s}}{\alpha}\right]^{\alpha} \left[\frac{\widetilde{s}\widetilde{r}/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho} /(1-\rho) \\ +\widetilde{\beta}\sigma\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1}) \end{array} \right\}, \cdot
$$

$$
\max_{\widetilde{s}} \left\{ \begin{array}{c} \frac{(\widetilde{x}-(\widetilde{q}_t-\widetilde{r}_t)\widetilde{s})^{1-\rho}}{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r(\widetilde{b}+\left(\widetilde{d}/\sigma G_z\right)\widetilde{s},\overline{\widetilde{A}}_{t+1}) \end{array} \right\})
$$

Appendix C: Representative Agent Model

We consider a special case of the economy in which there is no heterogeneity of labour productivity, retirement , nor death, i.e., $\delta^l = \delta^m = 0$ and $\omega = 1$. Everyone lives forever with the same labour productivity (no idiosyncratic shock to labour productivity), and population is equal to the number of workers. Because there are no retirees, there is no pension and no tax to finance to pension. The technology can be written in per capita terms as:

$$
y_t = A_t^{1-\eta} z_{Yt}^{\eta}
$$

$$
z_t = \left(\frac{L}{N_t}\right)^{1-\gamma} k_t^{\gamma}
$$

$$
i_t = k_t - \frac{\lambda}{G_N} k_{t-1}
$$

Consider a closed economy with the representative agent who maximizes utility under certainty. Because nobody lends or borrows in equilibrium, the collateral constraint does not bind.

The competitive equilibrium corresponds to the solution of the planner's problem, in which a planner maximizes the social welfare function:

$$
N_t V(k_{t-1}, \overline{A}_t) = \sum_{j=t}^{\infty} \beta^{j-t} N_j u(c_j, h_j)
$$

subject to the constraint of technology and resource allocation. Let $\overline{A}_t = (A_t, N_t)'$ be the exogenous state variables. The value function of the planner would be:

$$\begin{aligned} V(k_{t-1}, \overline{A}_t) &= Max\left\{u(c_t, h_t) + \beta G_N V(k_t, \overline{A}_{t+1})\right\} \\ &= \underset{k_{Yt}, k_t}{Max}\left\{ \begin{array}{c} u\left(A_t z_{Yt}{}^{\eta} - k_t + \frac{\lambda}{G_N} k_{t-1}, \left(\frac{L}{N_t}\right)^{1-\gamma} k_t^{\gamma} - z_{Yt}\right) \\ +\beta G_N V\left(k_t, \overline{A}_{t+1}\right) \end{array} \right\} \end{aligned}$$

The first order conditions are:

$$\eta \frac{y_t}{z_{Yt}} = \frac{u_{h_t}}{u_{c_t}} = \frac{1-\alpha}{\alpha}\frac{c_t}{h_t}, \tag{A1}$$

$$\begin{aligned} 1 &= \gamma \frac{z_t}{k_t}\frac{1-\alpha}{\alpha}\frac{c_t}{h_t} + \beta G_N \frac{\partial V(k_t, A_{t+1})}{\partial k_t} \\ &= \gamma \frac{z_t}{k_t}\frac{1-\alpha}{\alpha}\frac{c_t}{h_t} + \beta \frac{u_{c_{t+1}}}{u_{c_t}}\lambda. \end{aligned} \tag{A2}$$

In the steady state, per capita quantities satisfy:

$$G_y = G_Y/G_N, \quad \text{and } G_z = G_Z/G_N.$$

Then,

$$\frac{u_{c_{t+1}}}{u_{c_t}} = \left(G_y^{\alpha}\ G_z^{1-\alpha}\right)^{1-\rho} G_y^{-1} = G_u\ G_y^{-1}$$

where $G_u = \left(G_y^{\alpha}\ G_z^{1-\alpha}\right)^{1-\rho}$ is the growth rate of utility. Let f_K be the capital-output ratio and f be share of productive structures in the steady state:

$$\begin{aligned} f_K &= \frac{k_t}{y_t}, \\ f &= \frac{z_{Yt}}{z_t}. \end{aligned}$$

Then we learn:

$$\frac{c_t}{y_t} = 1 - \frac{i_t}{k_t}\frac{k_t}{y_t} = 1 - \left(1 - \frac{\lambda}{G_K}\right) f_K.$$

From the two first order conditions $(A1), (A2)$, we learn:

$$\begin{aligned} \frac{\eta\alpha}{1-\alpha} &= \frac{c_t}{y_t}\frac{z_{Yt}}{h_t} = \left[1 - \left(1 - \frac{\lambda}{G_K}\right) f_K\right]\frac{f}{1-f}, \\ 1 &= \frac{\gamma\eta}{f_K f} + \beta\lambda\frac{G_u}{G_y}. \end{aligned}$$

Solving these with respect to f_K and f, we get

$$\begin{aligned} f &= \eta \frac{\frac{\alpha}{1-\alpha} + \gamma \frac{1-\frac{\lambda}{G_K}}{1-\beta\lambda G_u/G_y}}{\frac{\eta\alpha}{1-\alpha}+1}, \\ f_K &= \gamma \frac{\frac{\eta\alpha}{1-\alpha}+1}{\frac{\alpha}{1-\alpha}\left[1-\beta\lambda G_u/G_y\right] + \gamma\left(1-\frac{\lambda}{G_K}\right)}. \end{aligned}$$

Then, we learn

$$\frac{u_{c_t}}{\beta u_{c_{t+1}}} = \frac{G_y}{\beta G_u} = R = \frac{d_{t+1}}{q_t - r_t} = \frac{G_K{}^{\gamma} q_t - (G_K - \lambda)\frac{k_t}{z_t}}{q_t - \frac{\eta}{f_K f}\frac{k_t}{z_t}}.$$

Thus

$$Q \equiv \frac{q_t z_t}{k_t} = \frac{1}{\gamma}\frac{R-(1-\gamma)\lambda - \gamma G_K}{R - G_K{}^{\gamma}}.$$

We can also compute price-rental ratio as

$$\frac{q_t}{r_t} = \frac{q_t}{\eta y_t / z_{Yt}} = \frac{q_t z_t / k_t}{\eta}\frac{z_{Yt}}{z_t}\frac{k_t}{y_t} = \frac{1}{\eta} f_K f Q.$$

The ratio of housing value to wage is:

$$\frac{q_t h_t}{w_t} = \frac{q_t (1-f)\, z_t}{(1-\eta)\, y_t} = \frac{1}{1-\eta} f_K (1-f) Q.$$

Thus, in our Baseline calibration, we learn

$$\begin{aligned} f &= \frac{z_{Yt}}{z_t} = 0.543 \\ f_K &= \frac{k_t}{y_t} = 2.78 \\ R &= 1.0636 \\ Q &= \frac{q_t z_t}{k_t} = 1.56 \\ \frac{q_t}{r_t} &= 9.12 \\ \frac{q_t h_t}{w_t} &= 2.64. \end{aligned}$$

Appendix D: Data sources and definitions

We use quarterly data from the US Flow of Funds accounts and from the NIPA for the 1952 Q1 - 2005Q4 period. We follow Cooley and Prescott (1995) in aligning the model economy with the data. A crucial parameter we need to calibrate is the productive structures' share in production (η). Given that the model economy includes residential structures explicitly we need to adjust GDP and flow of funds data to derive a reasonable estimate for this parameter.

We define unambiguous capital income as the sum of rental income (r), corporate profits (π) and net interest (i) from the NIPA (table 1.12). We allocate the share of proprietors' income (Y_P, NIPA, Table 1.12) arising from productive structures using η, while a measure for the depreciation of capital (DEP) is given by the consumption of fixed capital (NIPA, table 1.14). Defining Y_{KP} as income from productive structures, Y_{KP} can be computed as the sum of unambiguous capital income plus $\eta *$Proprietors' Income plus DEP. $Y_{KP} = \eta Y$, where Y is GDP excluding explicit and implicit rents from housing. Solving for η, we have

$$\eta = \frac{r + \pi + i + DEP}{Y - Y_P}$$

which is a similar expression for the share of capital in output found in Cooley and Prescott (1995, p.19).

Averaging the quarterly data for the U.S. from 1952 to 2005, we obtain a value of η equal to 0.26. This is lower than the share of capital in output in the real business cycle literature (estimates there range between 0.3 and 0.4) because we have explicitly included housing services in the theoretical model and, consistent with the logic in Cooley and Prescott (1995), we are not including housing in this computation.

The presence of housing in both the firm and household side also allows us to decompose economy-wide tangible assets between the household and the firm. The exact definitions in the data and their counterparts in the theoretical model are given in the following table:

Economic concept	Flow of Funds concept
qK_y	Non-farm, non-financial tangible assets (Non-residential structures+Equipment+software+Inventories) Flow of funds, Tables B.102 and B.103 FL102010005.Q+FL112010005
qH	Household tangible assets (Residential structures+Equipment+software+Consumer durables) Flow of funds, Table B.100 FL152010005.Q

Using this definitions, we compute the average numbers between 1952:Q1 and 2005:Q4. The ratio of household tangible assets to firm tangible assets (H/K_y) is 0.913 (equivalently the ratio of household tangible assets to total capital is 0.47) and the ratio of total capital to GDP ($q\left(H+K_y\right)/Y$) is 3.3. If farm corporate and non-corporate tangible assets (FL132010005.Q in the Flow of Funds)[27] are added to the non-farm tangible assets, then the ratio of household tangible assets to total capital falls from 0.47 to 0.44 while the ratio of total capital to GDP rises from 3.3 to 3.6.

[27] Thanks to Michael Palumbo (Board of Governors) of kindly sending us this series in private correspondence.

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FIGURE 1A: Policy functions for a low productivity worker

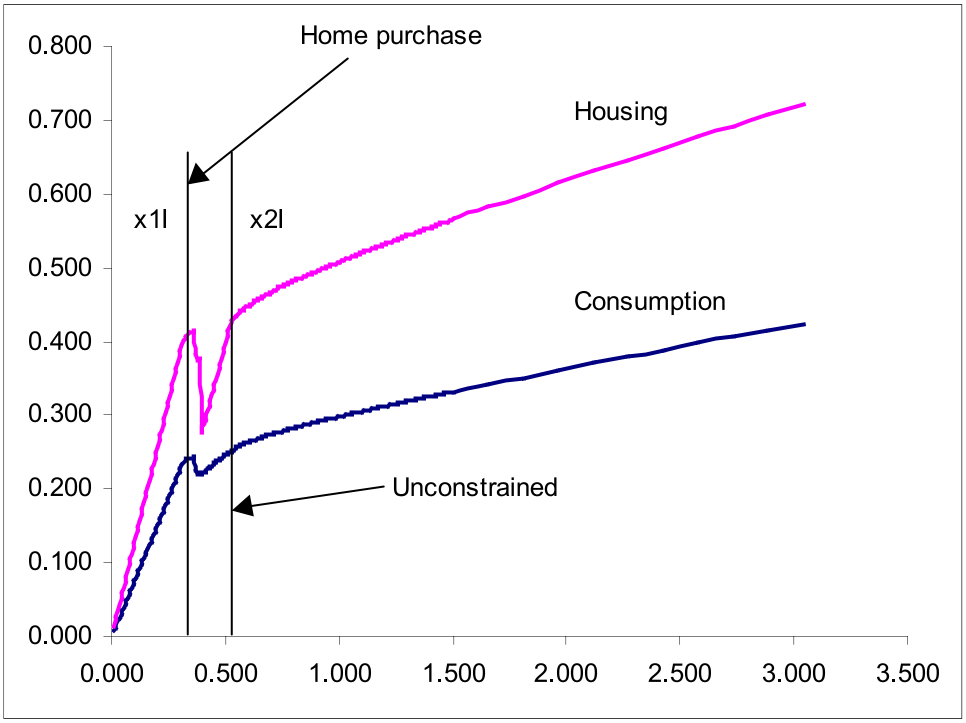

FIGURE 1B: Evolution of savings for a low productivity household

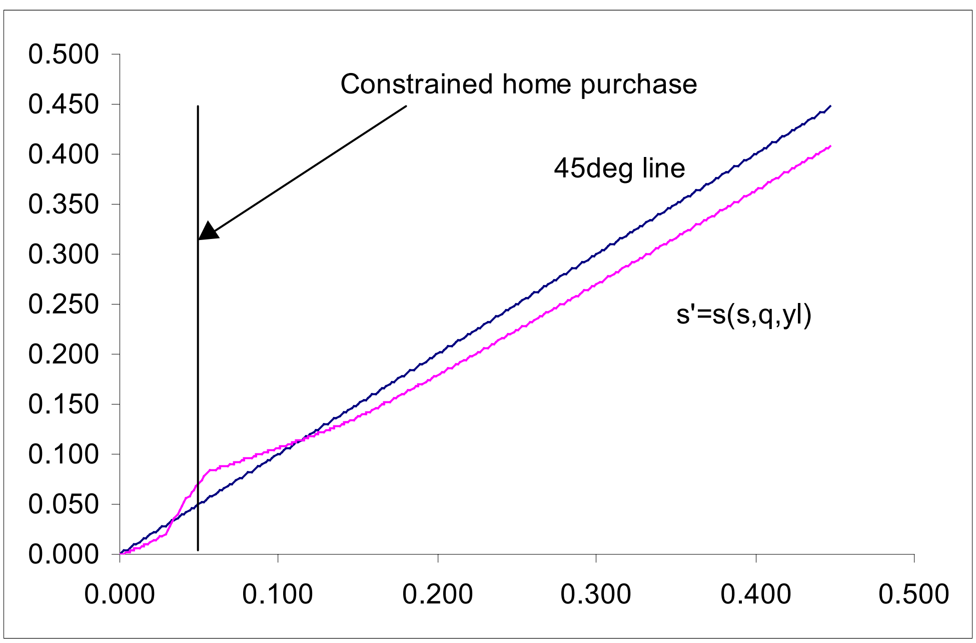

FIGURE 2A: Policy functions for a high productivity worker

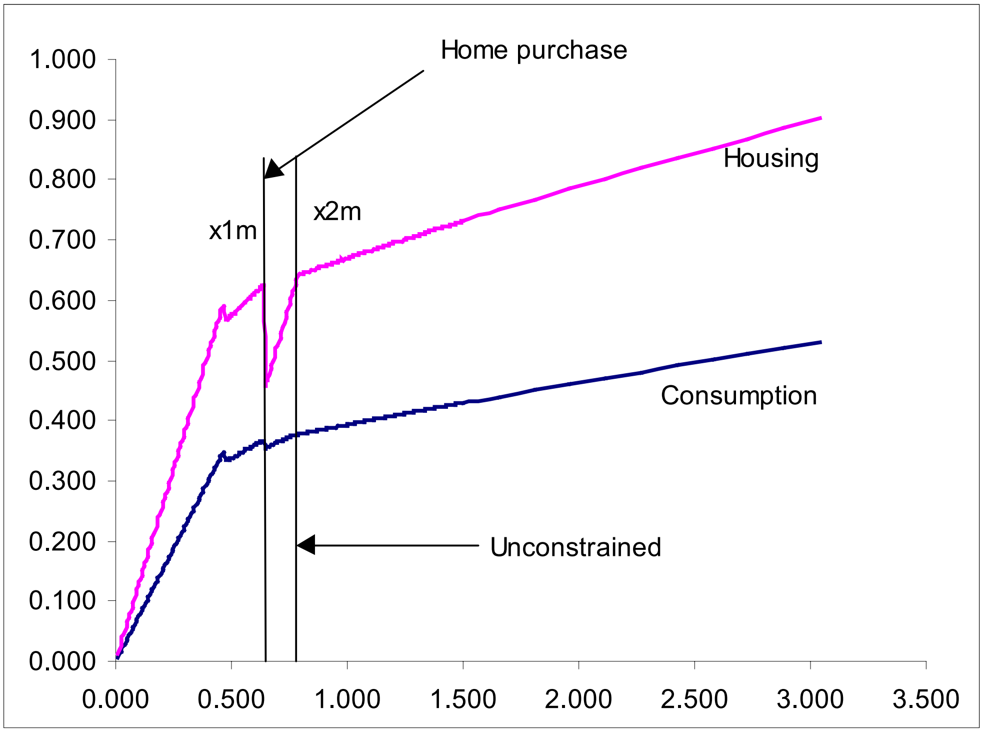

FIGURE 2B: Evolution of savings for a high productivity household

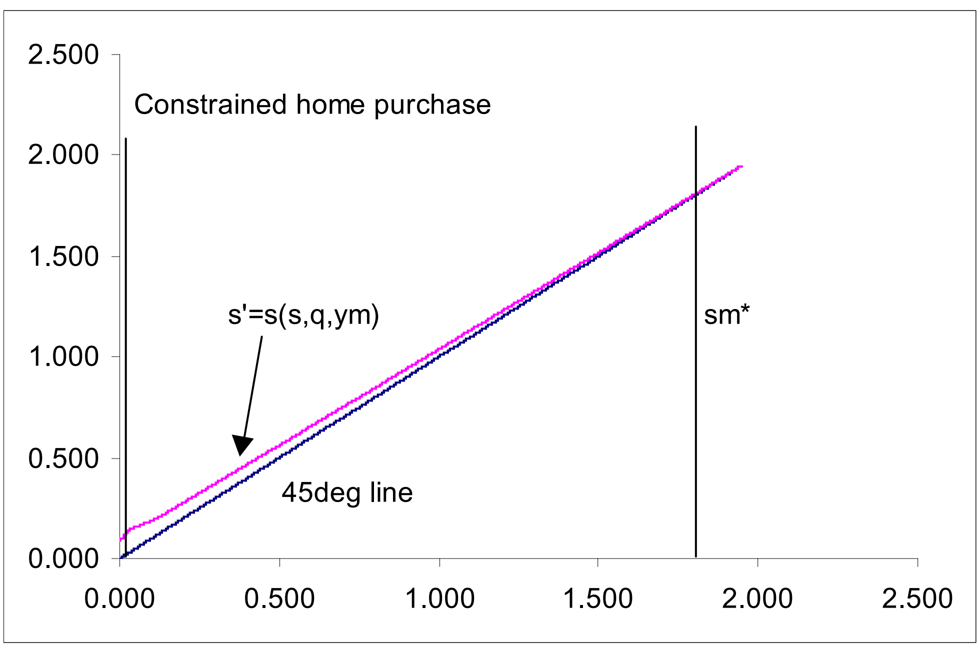

FIGURE 3A: Policy functions for the Retiree

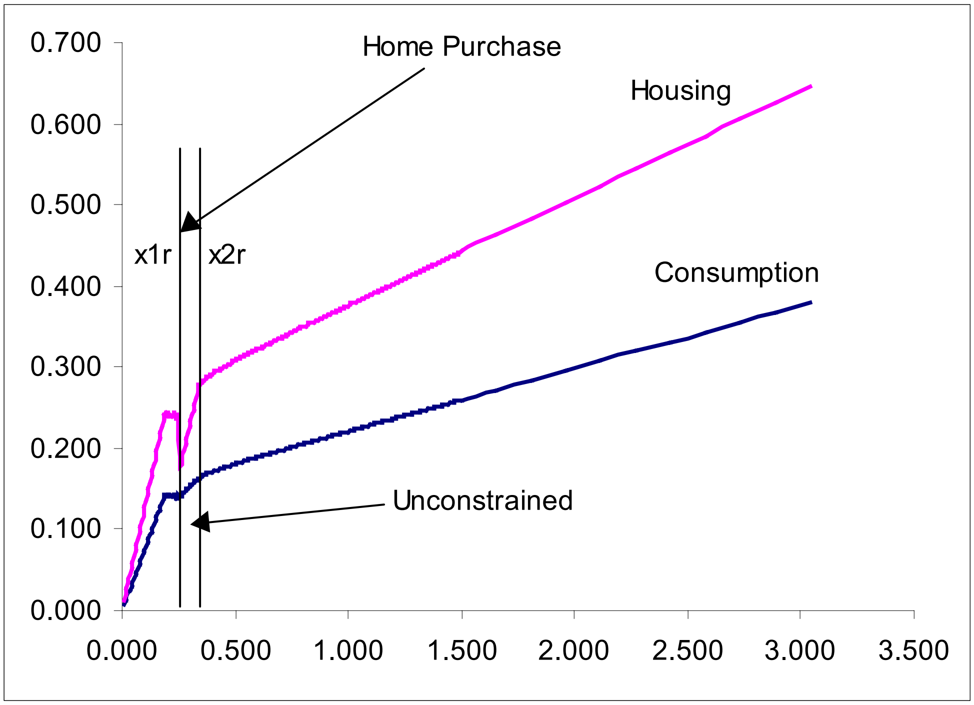

FIGURE 3B: Evolution of savings for the retiree

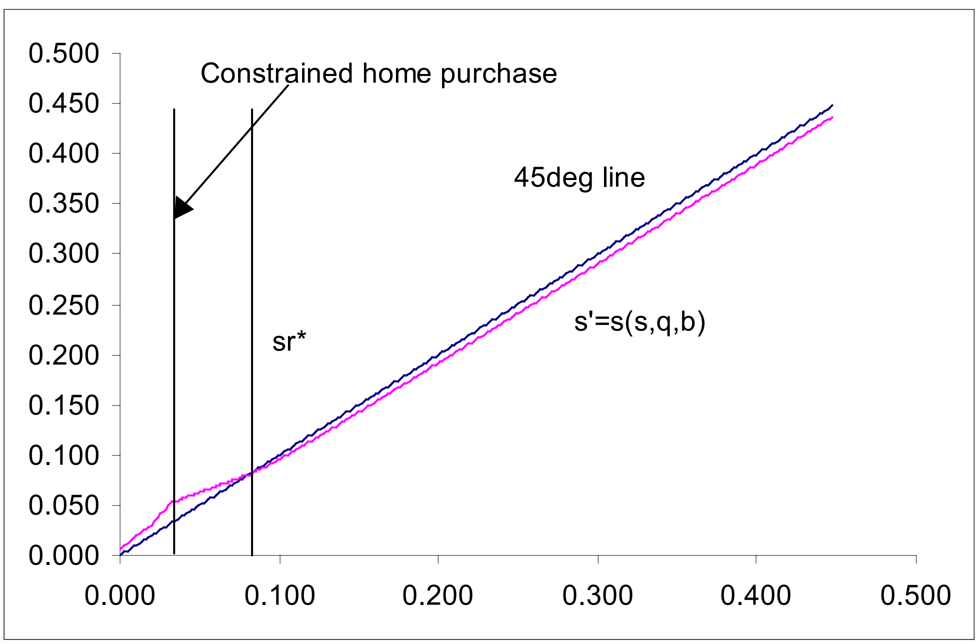

FIGURE 4: An example life time

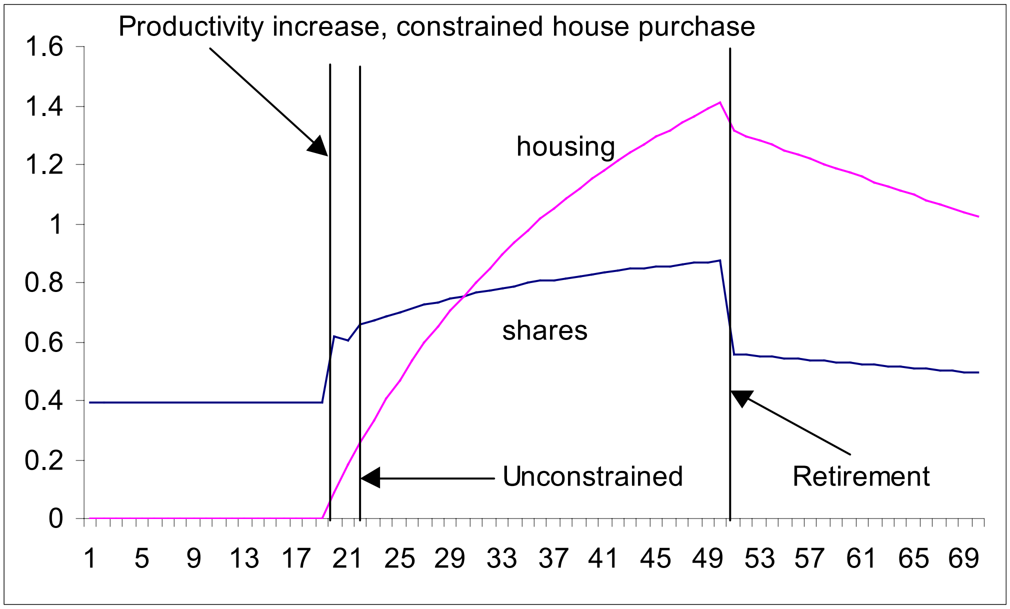

Figure 5
Transition dynamics from a 1% increase in labour productivity growth
(solid line: γ=0.9, dotted line: γ=0.5)

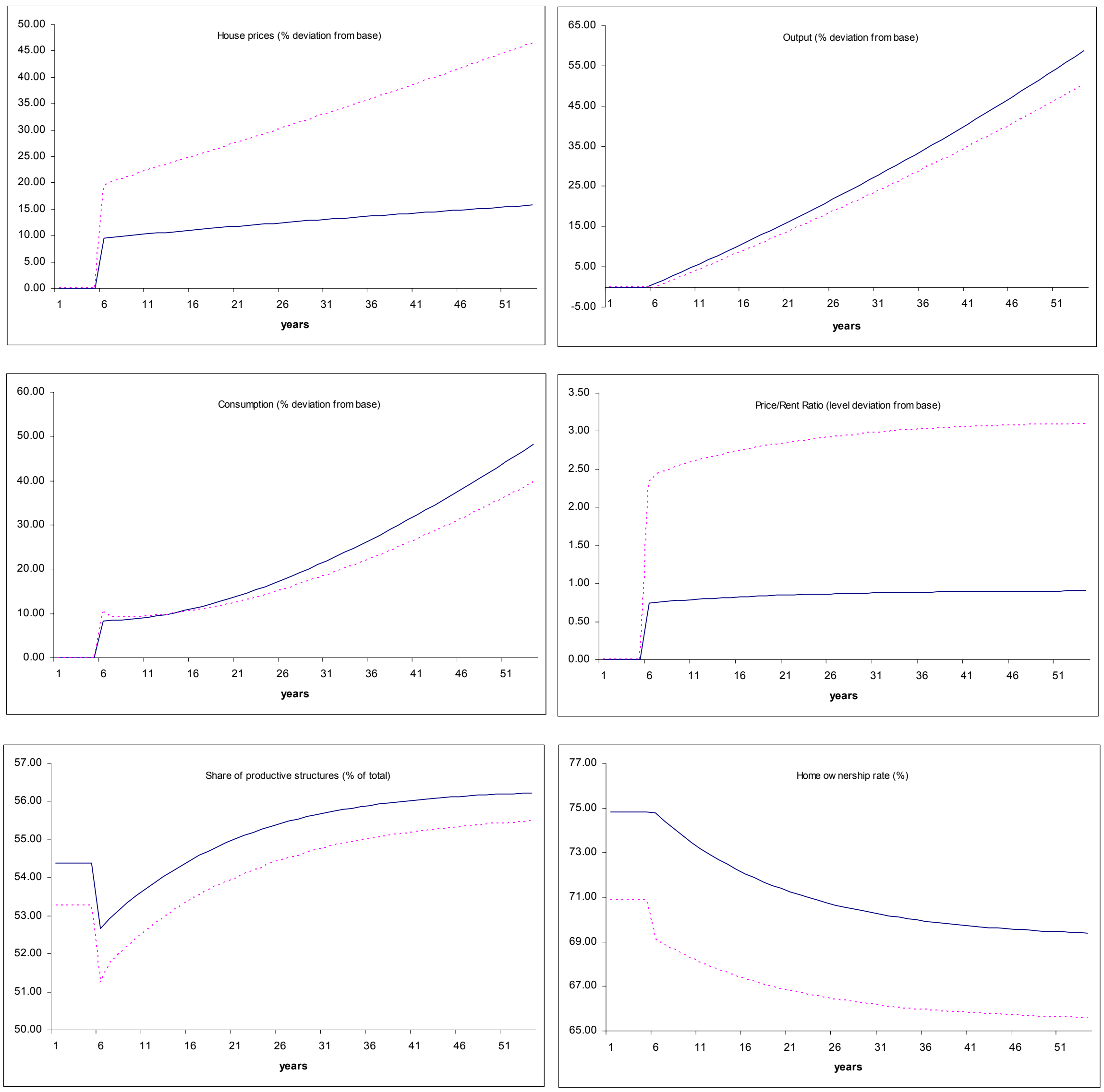

Figure 6
Transition dynamics from a 1% reduction in the world real interest rate
(solid line: γ=0.9, dotted line: γ=0.5)

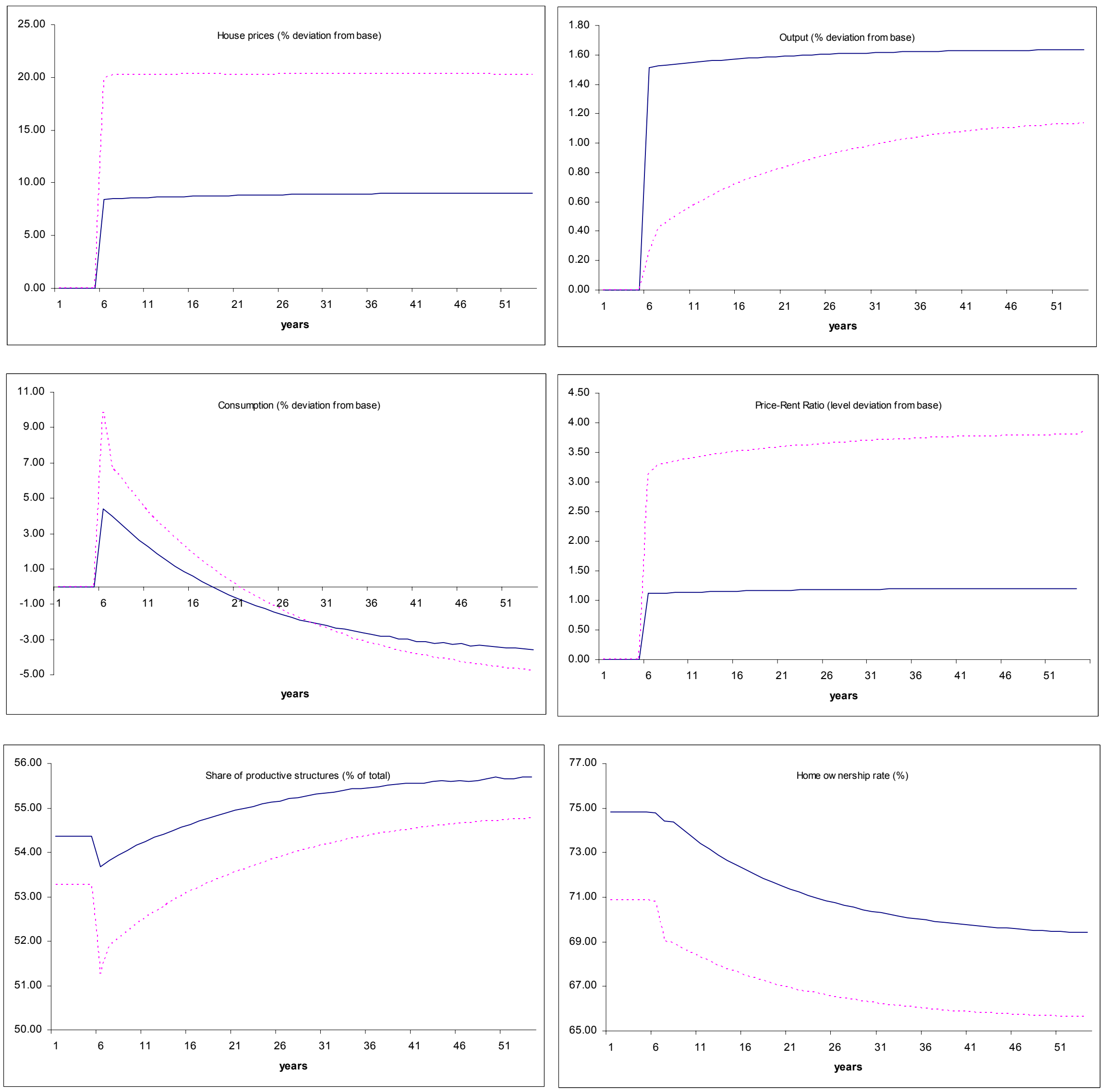

Table 5

	baseline	*θ=0.1*	*θ=1.0*	*gn=1.02*	*ga=1.03*	*b=0.1*	*γ=0.5*
Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*
% of tenants	25.16	2.95	37.48	11.05	25.16	5.07	10.30
% of constrained households	8.34	25.71	10.94	20.99	8.56	4.65	8.50
% of unconstrained homeowners	66.50	71.35	51.58	67.95	66.28	90.28	81.20
% of shares owned by tenants	0.05	0.02	0.82	0.19	0.08	0.10	0.12
% of shares owned by constrained	0.33	0.37	2.95	1.01	0.34	0.12	0.32
% of housing used by tenants	8.69	0.57	13.17	3.32	8.67	1.34	2.45
% of housing used by constrained	2.42	8.11	6.48	7.44	2.49	0.90	2.22
Current account as % of GDP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Net foreign Assets as % of GDP	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Value of total structures to GDP	2.90	2.91	2.91	2.86	2.62	3.31	4.23
Housing structures to total structures	0.46	0.46	0.46	0.45	0.46	0.45	0.52
Value of housing to wages	2.57	2.46	2.46	2.41	2.27	2.72	4.16
Housing price to rental rate	8.45	8.46	8.48	8.36	7.70	9.54	11.08
Real return	6.83	6.81	6.79	7.65	8.40	5.90	9.20
House price (N=An=1)	1.63	1.64	1.64	1.66	1.60	1.77	4.20
Output (N=An=1)	1.11	1.11	1.11	1.10	1.08	1.12	0.87

Notes to Table 5: Results from the closed economy with zero demand for domestic shares by the representative foreigner. In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. The results from reducing γ from its baseline value of 0.9 to 0.5 are reported in column (7) labeled {γ=0.5, (7)}.

Table 6

	baseline	*θ=0.1*	*θ=1.0*	*gn=1.02*	*ga=1.03*	*b=0.1*	*R*=5.62*	*γ=0.5*
Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
% of tenants	25.16	2.95	37.48	15.09	30.81	2.95	30.81	29.12
% of constrained households	8.34	25.70	9.87	19.39	6.61	3.66	3.71	5.79
% of unconstrained homeowners	66.50	71.36	52.65	65.53	62.58	93.39	65.48	65.09
% of shares owned by tenants	0.05	0.10	0.89	0.47	0.20	0.02	-0.01	1.08
% of shares owned by constrained	0.33	0.37	2.54	1.46	1.03	0.07	0.34	1.22
% of housing used by tenants	8.69	0.75	13.76	5.80	10.83	0.65	10.60	9.06
% of housing used by constrained	2.42	8.10	5.69	6.68	2.68	0.69	1.33	1.70
Current account as % of GDP	0.00	-0.07	-0.16	3.56	4.79	-4.18	3.56	13.05
Net foreign Assets as % of GDP	0.00	1.53	3.40	-73.00	-124.34	86.37	-92.34	-251.26
Value of total structures to GDP	2.90	2.90	2.90	3.21	3.19	2.94	3.32	5.81
Housing structures to total structures	0.46	0.46	0.46	0.44	0.44	0.47	0.44	0.47
Value of housing to wages	2.57	2.57	2.57	2.69	2.68	2.68	2.78	4.82
Housing price to rental rate	8.45	8.45	8.45	9.36	9.35	8.45	9.65	15.71
Real return	6.83	6.83	6.83	6.83	6.83	6.83	5.83	6.83
House price (N=An=1)	1.63	1.63	1.63	1.79	1.81	1.64	1.78	5.46
Output (N=An=1)	1.11	1.11	1.11	1.11	1.11	1.10	1.12	0.90

Notes to Table 6: Results from the small open economy with a given demand for domestic shares by a representative foreigner (world interest rate is 6.83% and γ=0.9). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. R* is the world real return. γ=0.5 (column (8)) reports the results from setting γ equal to 0.5 at this given world interest rate.

Table 7

Scarcity of Land Parameter	γ=0.9	γ=0.5	γ=0.9	γ=0.5	γ=0.9	γ=0.5
Column	1	2	3	4	5	6
Panel A: Certainty expenditure equivalent	*ga+1%*	*ga+1%*	*R*-1%*	*R*-1%*	*b=0.15*	*b=0.1*
Workers	9.69	8.14	-0.89	-0.21	-5.06	-4.16
Renter Workers	9.67	7.33	-0.45	-1.22	-11.38	-10.51
Constrained Homeowner Workers	9.64	7.14	-1.45	-1.87	-8.23	-6.55
Unconstrained Homeowner Workers	10.21	9.12	-1.53	-0.33	-2.78	-1.68
Low Income Workers	9.60	7.30	-0.53	-1.14	-11.40	-10.75
Middle Income Workers	10.36	8.73	-1.19	-0.36	-4.65	-3.49
High Income Workers	9.73	9.36	-1.79	0.00	0.90	1.62
Retirees	7.56	8.45	0.00	3.47	-29.86	-26.90
Renter Retirees	6.93	5.60	-0.30	-0.33	-43.79	-44.06
Constrained Homeowner Retirees	7.42	5.80	-0.05	0.15	-42.60	-40.73
Unconstrained Homeowner Retirees	8.61	9.38	0.79	4.53	-22.02	-18.77
Panel B: Wealth change						
Workers	4.09	6.94	5.94	10.44	0.00	0.00
Renter Workers	-0.16	-0.97	1.51	0.58	0.03	0.25
Constrained Homeowner Workers	1.51	4.13	3.18	6.93	0.21	0.45
Unconstrained Homeowner Workers	6.53	10.96	8.12	15.39	0.47	0.47
Low Income Workers	-0.15	-1.20	1.50	0.24	0.10	0.23
Middle Income Workers	5.81	9.54	7.23	13.77	0.42	0.58
High Income Workers	7.67	12.32	9.16	16.99	0.00	0.26
Retirees	5.66	9.92	7.22	14.25	0.00	0.00
Renter Retirees	0.89	1.25	2.30	3.36	0.45	0.45
Constrained Homeowner Retirees	3.43	6.81	4.41	9.95	0.50	0.64
Unconstrained Homeowner Retirees	7.65	11.97	8.15	16.60	0.58	0.68

Notes to Table 7: Panel A reports the certainty expenditure equivalent changes (in percent) from shifts in the specified fundamentals relative to the baseline steady state in the small open economy (details of this computation are given in the text). In the baseline economy, the collateral constraint is denoted by θ and is equal to 0.3, gn denotes population growth and is equal to 1.01 (one percent per annum), ga=1.02 denotes a two percent annual productivity growth, and b=0.2 denotes a twenty percent gross replacement rate during retirement. R* is the world real return. Panel B reports the Wealth Change (in percent) right after the unexpected change in the specified fundamentals for the same cases (details for these calculations are given in the text).